Computer algebra independent integration tests

Summer 2022 edition

7-Inverse-hyperbolic-functions/7.6-Inverse-hyperbolic-cosecant/202-7.6.1-u-a+b-arccsch-c-x-^n

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [178]. This is test number [202].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (178)	0.00 (0)
Mathematica	96.63 (172)	3.37 (6)
Fricas	64.04 (114)	35.96 (64)
Maple	57.87 (103)	42.13 (75)
Maxima	47.19 (84)	52.81 (94)
Mupad	27.53 (49)	72.47 (129)
Giac	25.84 (46)	74.16 (132)
Sympy	20.79 (37)	79.21 (141)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

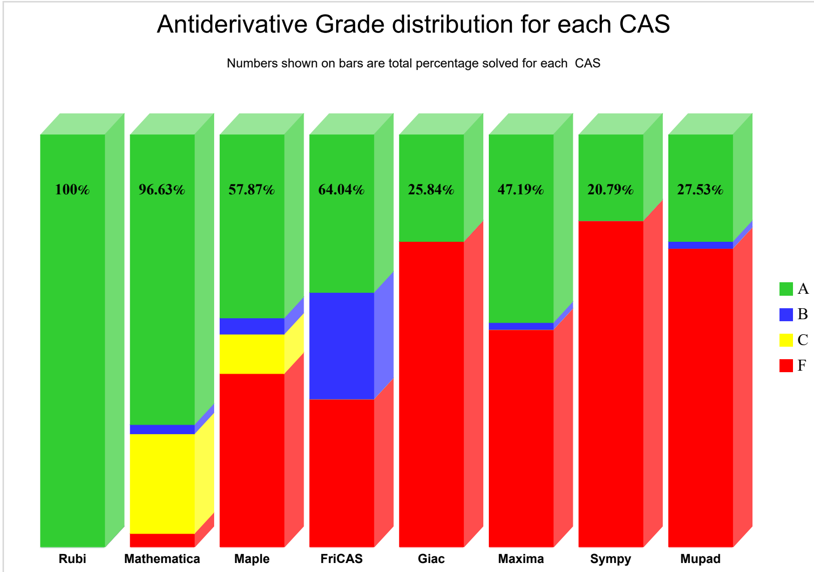
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

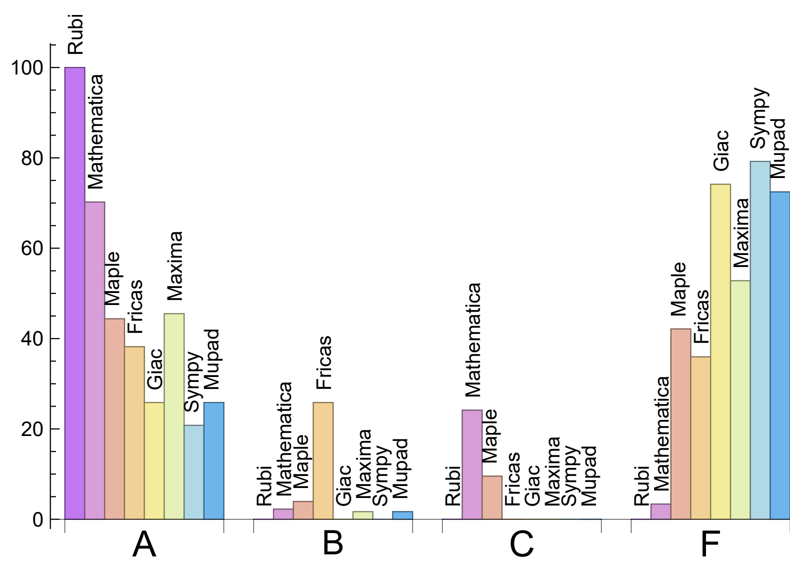
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	70.22	2.25	24.16	3.37
Maxima	45.51	1.69	0.00	52.81
Maple	44.38	3.93	9.55	42.13
Fricas	38.20	25.84	0.00	35.96
Mupad	N/A	1.69	0.00	72.47
Giac	25.84	0.00	0.00	74.16
Sympy	20.79	0.00	0.00	79.21

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	6	83.33 %	16.67 %	0.00 %
Maple	75	100.00 %	0.00 %	0.00 %
Fricas	64	71.88 %	14.06 %	14.06 %
Giac	132	98.48 %	0.00 %	1.52 %
Maxima	94	98.94 %	0.00 %	1.06 %
Sympy	141	75.89 %	21.28 %	2.84 %
Mupad	129	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

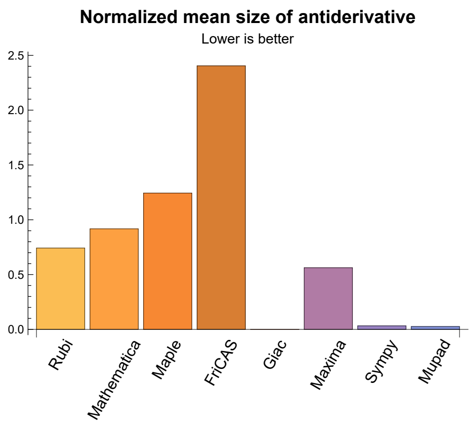
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

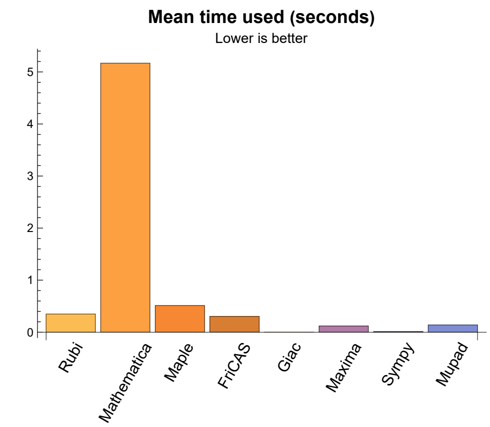
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.35	218.12	0.74	132.50	1.00
Mathematica	5.17	289.94	0.92	129.00	0.87
Maple	0.51	393.42	1.24	75.00	0.99
Maxima	0.12	65.65	0.56	0.00	0.00
Fricas	0.30	407.01	2.40	141.00	1.23
Sympy	0.01	0.97	0.03	0.00	0.00
Giac	0.00	0.00	0.00	0.00	0.00
Mupad	0.14	1.61	0.03	-1.00	-0.04

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
2.5
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
5
4
3
2
1
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{51, 57, 72, 75, 103, 104, 107, 108, 109, 110, 111, 115, 116, 117\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

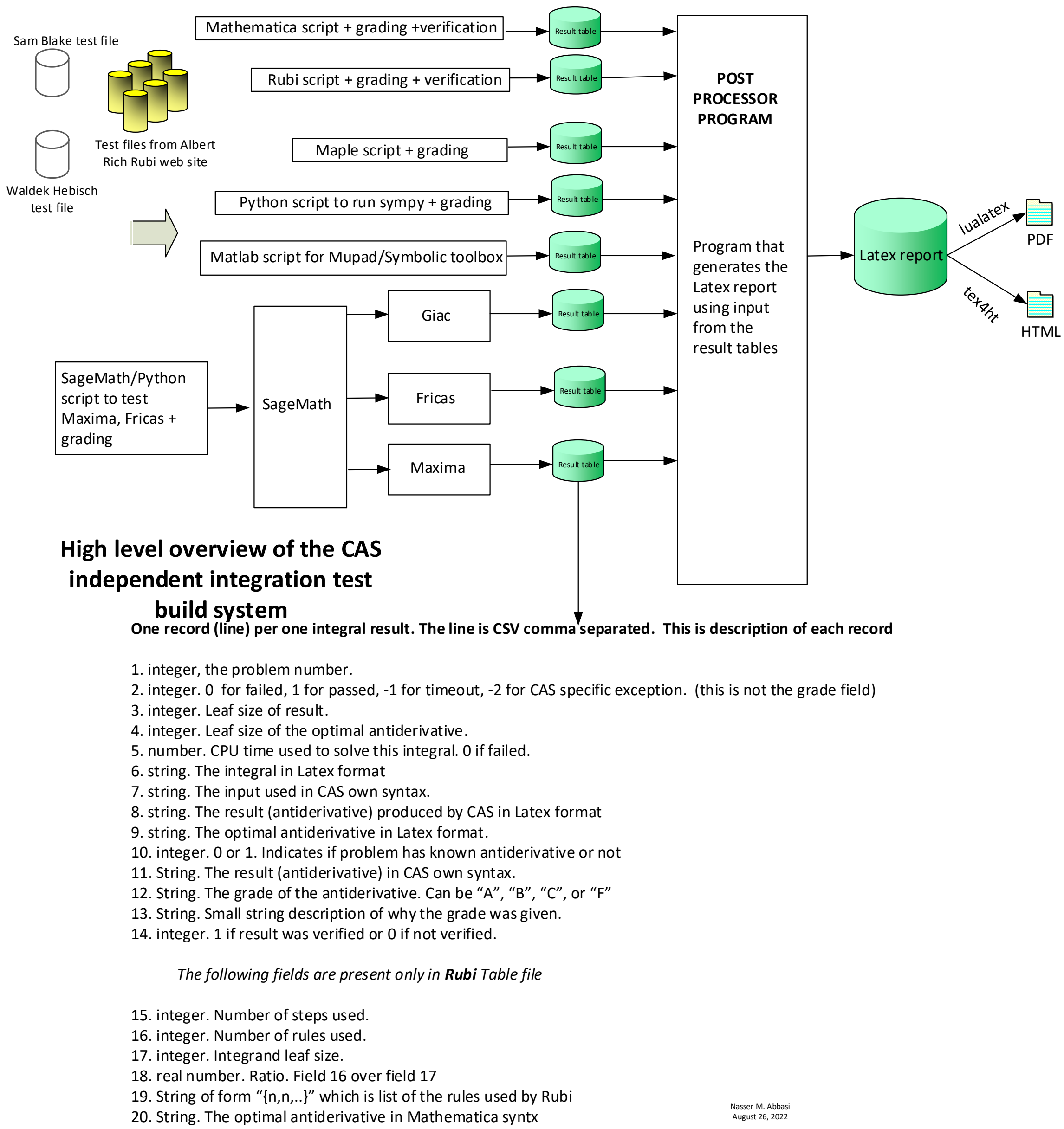
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 55, 61, 62, 67, 68, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 118, 119, 120, 121, 122, 123, 124, 125, 128, 129, 130, 131, 132, 133, 134, 135, 138, 139, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 152, 153, 154, 156, 157, 158, 159, 160, 161, 162, 163, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178 }

B grade: { 7, 25, 27, 47 }

C grade: { 48, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 126, 127, 136, 137, 146, 155, 164 }

F grade: { 54, 106, 114, 165, 166, 167 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 11, 12, 13, 14, 33, 34, 35, 39, 40, 42, 43, 44, 45, 46, 47, 54, 55, 61, 62, 67, 68, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 9, 10, 49, 50, 105, 112, 113 }

C grade: { 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75 }

F grade: { 8, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 48, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 11, 13, 17, 20, 29, 33, 34, 35, 39, 40, 42, 43, 44, 45, 46, 47, 54, 55, 61, 62, 67, 68, 73, 74, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 10, 12, 14 }

C grade: { }

F grade: { 8, 15, 16, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 36, 37, 38, 41, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

2.1.5 FriCAS

A grade: { 2, 4, 10, 11, 12, 13, 14, 23, 32, 33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 80, 81, 82, 83, 84, 85, 92, 93, 94, 95, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 154, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 174, 175, 177, 178 }

B grade: { 1, 3, 5, 6, 7, 9, 15, 17, 20, 21, 22, 29, 30, 31, 44, 45, 46, 47, 49, 50, 76, 77, 78, 79, 88, 89, 90, 91, 105, 112, 113, 118, 119, 120, 128, 129, 138, 139, 140, 147, 148, 149, 156, 157, 158, 176 }

C grade: { }

F grade: { 8, 16, 18, 19, 24, 25, 26, 27, 28, 36, 37, 38, 41, 48, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 86, 87, 96, 97, 98, 99, 100, 101, 102, 103, 104, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 126, 127, 136, 137, 145, 146, 155, 163, 164, 165, 166, 167 }

2.1.6 Sympy

A grade: { 9, 33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 121, 122, 123, 124, 125, 130, 131, 133, 134, 135, 141, 142, 143, 144, 150, 152, 153, 168, 171, 172, 173, 177, 178 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 132, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 151, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 174, 175, 176 }

2.1.7 Giac

A grade: { 33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

2.1.8 Mupad

A grade: { 33, 34, 35, 39, 40, 42, 43, 54, 55, 61, 62, 67, 68, 73, 74, 121, 122, 123, 124, 125, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 168, 169, 170, 171, 172, 173, 177, 178 }

B grade: { 6, 9, 10 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 7, 8, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 37, 38, 41, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 63, 64, 65, 66, 69, 70, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 126, 127, 128, 129, 136, 137, 138, 139, 140, 145, 146, 147, 148, 149, 154, 155, 156, 157, 158, 163, 164, 165, 166, 167, 174, 175, 176 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	107	127	158	208	0	0	-1
N.S.	1	1.00	0.97	1.15	1.44	1.89	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.105	0.220	0.260	0.467	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	72	83	77	97	0	0	-1
N.S.	1	1.00	0.84	0.97	0.90	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.090	0.191	0.255	0.474	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	97	108	128	199	0	0	-1
N.S.	1	1.00	1.13	1.26	1.49	2.31	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.033	0.200	0.263	0.545	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	62	74	57	87	0	0	-1
N.S.	1	1.00	1.00	1.19	0.92	1.40	0.00	0.00	-0.02
time (sec)	N/A	0.019	0.068	0.191	0.251	0.427	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	85	88	96	186	0	0	-1
N.S.	1	1.00	1.37	1.42	1.55	3.00	0.00	0.00	-0.02
time (sec)	N/A	0.023	0.039	0.211	0.256	0.435	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	50	65	35	70	0	0	39
N.S.	1	1.00	1.32	1.71	0.92	1.84	0.00	0.00	1.03
time (sec)	N/A	0.010	0.019	0.207	0.259	0.359	0.000	0.000	2.217

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	78	36	49	143	0	0	-1
N.S.	1	1.00	2.60	1.20	1.63	4.77	0.00	0.00	-0.03
time (sec)	N/A	0.015	0.095	0.105	0.264	0.474	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	47	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.097	0.032	0.026	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	40	62	32	64	36	0	35
N.S.	1	1.00	1.33	2.07	1.07	2.13	1.20	0.00	1.17
time (sec)	N/A	0.016	0.022	0.240	0.338	0.520	0.441	0.000	2.323

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	66	99	105	76	0	0	51
N.S.	1	1.00	1.32	1.98	2.10	1.52	0.00	0.00	1.02
time (sec)	N/A	0.025	0.025	0.225	0.265	0.408	0.000	0.000	2.265

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	59	75	56	77	0	0	-1
N.S.	1	1.00	1.02	1.29	0.97	1.33	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.036	0.213	0.255	0.465	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	78	120	147	89	0	0	-1
N.S.	1	1.00	1.05	1.62	1.99	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.032	0.210	0.260	0.450	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	69	83	73	87	0	0	-1
N.S.	1	1.00	0.87	1.05	0.92	1.10	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.042	0.200	0.253	0.524	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	88	139	185	99	0	0	-1
N.S.	1	1.00	0.90	1.42	1.89	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.043	0.053	0.197	0.269	0.419	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	122	0	0	272	0	0	-1
N.S.	1	1.00	1.16	0.00	0.00	2.59	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.140	0.020	0.000	0.447	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	224	0	0	0	0	0	-1
N.S.	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.991	0.022	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	87	0	82	234	0	0	-1
N.S.	1	1.00	1.61	0.00	1.52	4.33	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.084	0.023	0.271	0.576	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	121	0	0	0	0	0	-1
N.S.	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.160	0.020	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	115	0	0	0	0	0	-1
N.S.	1	1.00	1.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.076	0.020	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	70	0	78	139	0	0	-1
N.S.	1	1.00	1.43	0.00	1.59	2.84	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.103	0.020	0.257	0.404	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	100	0	0	163	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	1.87	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.086	0.020	0.000	0.399	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	106	0	0	178	0	0	-1
N.S.	1	1.00	1.06	0.00	0.00	1.78	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.120	0.022	0.000	0.462	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	147	0	0	202	0	0	-1
N.S.	1	1.00	1.11	0.00	0.00	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.114	0.020	0.000	0.449	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	271	0	0	0	0	0	-1
N.S.	1	1.00	1.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.676	0.020	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	461	0	0	0	0	0	-1
N.S.	1	1.00	2.38	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	5.658	0.003	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	171	0	0	0	0	0	-1
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.299	0.022	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	246	0	0	0	0	0	-1
N.S.	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.215	0.022	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	198	0	0	0	0	0	-1
N.S.	1	1.00	1.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	0.121	0.020	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	132	0	144	222	0	0	-1
N.S.	1	1.00	1.69	0.00	1.85	2.85	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.149	0.021	0.272	0.573	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	182	0	0	267	0	0	-1
N.S.	1	1.00	1.48	0.00	0.00	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.200	0.021	0.000	0.389	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	200	0	0	301	0	0	-1
N.S.	1	1.00	1.20	0.00	0.00	1.81	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.193	0.020	0.000	0.441	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	277	0	0	346	0	0	-1
N.S.	1	1.00	1.36	0.00	0.00	1.70	0.00	0.00	-0.00
time (sec)	N/A	0.131	0.235	0.018	0.000	0.491	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.011	2.637	0.027	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	13	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08
time (sec)	N/A	0.004	0.020	0.021	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.018	0.213	0.023	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.081	0.056	0.023	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	56	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.109	0.053	0.023	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	91	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.120	0.022	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	4.583	0.020	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.017	3.025	0.020	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	81	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.165	0.018	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.019	0.631	0.021	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.018	1.307	0.022	0.000	0.000	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	165	269	259	879	0	0	-1
N.S.	1	1.00	0.99	1.61	1.55	5.26	0.00	0.00	-0.01
time (sec)	N/A	0.268	0.164	0.298	0.272	0.530	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	204	192	543	0	0	-1
N.S.	1	1.00	1.00	1.67	1.57	4.45	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.106	0.262	0.263	0.513	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	133	115	89	252	0	0	-1
N.S.	1	1.00	1.64	1.42	1.10	3.11	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.275	0.204	0.263	0.428	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	78	36	49	143	0	0	-1
N.S.	1	1.00	2.60	1.20	1.63	4.77	0.00	0.00	-0.03
time (sec)	N/A	0.016	0.052	0.100	0.253	0.381	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	506	0	0	0	0	0	-1
N.S.	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.256	0.450	0.112	0.000	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	134	210	0	687	0	0	-1
N.S.	1	1.00	1.37	2.14	0.00	7.01	0.00	0.00	-0.01
time (sec)	N/A	0.106	0.152	1.812	0.000	0.414	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	204	929	0	2579	0	0	-1
N.S.	1	1.00	1.25	5.70	0.00	15.82	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.319	2.321	0.000	1.023	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	918	918	1094	2517	0	0	0	0	-1
N.S.	1	1.00	1.19	2.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.172	39.583	1.094	0.000	0.000	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	679	679	418	1966	0	0	0	0	-1
N.S.	1	1.00	0.62	2.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.669	10.973	1.046	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	926	840	0	0	0	0	-1
N.S.	1	1.00	2.16	1.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.454	29.452	0.912	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.050	180.001	0.115	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	6.000	0.117	0.000	0.000	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	486	380	1939	0	0	0	0	-1
N.S.	1	1.00	0.78	3.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.684	10.929	0.864	0.000	0.000	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	939	939	1098	2545	0	0	0	0	-1
N.S.	1	1.00	1.17	2.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.965	29.455	0.928	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	707	707	1012	1991	0	0	0	0	-1
N.S.	1	1.00	1.43	2.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.476	29.691	0.997	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	474	474	343	868	0	0	0	0	-1
N.S.	1	1.00	0.72	1.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.202	10.830	0.822	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	307	395	0	0	0	0	-1
N.S.	1	1.00	1.08	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.287	5.453	0.784	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.051	4.517	0.118	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.054	7.026	0.118	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	731	731	1042	2021	0	0	0	0	-1
N.S.	1	1.00	1.43	2.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.790	29.868	0.874	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	499	499	820	896	0	0	0	0	-1
N.S.	1	1.00	1.64	1.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.364	27.555	0.845	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	264	425	0	0	0	0	-1
N.S.	1	1.00	0.83	1.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.139	11.028	0.783	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	166	328	0	0	0	0	-1
N.S.	1	1.00	1.11	2.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.188	0.471	0.748	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.058	14.467	0.114	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.063	16.000	0.127	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	777	777	1108	2728	0	0	0	0	-1
N.S.	1	1.00	1.43	3.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.145	29.611	0.872	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	569	569	1076	2497	0	0	0	0	-1
N.S.	1	1.00	1.89	4.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.684	29.618	0.819	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	390	2106	0	0	0	0	-1
N.S.	1	1.00	0.99	5.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.484	11.619	0.795	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	784	2079	0	0	0	0	-1
N.S.	1	1.00	2.12	5.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.377	18.870	0.802	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.064	45.927	0.154	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.067	43.960	0.144	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	648	785	1217	3782	0	0	0	0	-1
N.S.	1	1.21	1.88	5.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.728	30.053	0.984	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	138	211	291	386	0	0	-1
N.S.	1	1.00	0.64	0.99	1.36	1.80	0.00	0.00	-0.00
time (sec)	N/A	0.091	0.170	0.352	0.260	0.404	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	119	171	229	357	0	0	-1
N.S.	1	1.00	0.71	1.02	1.37	2.14	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.125	0.386	0.256	0.434	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	169	126	150	312	0	0	-1
N.S.	1	1.00	1.47	1.10	1.30	2.71	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.374	0.333	0.266	0.376	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	123	107	86	273	0	0	-1
N.S.	1	1.00	1.35	1.18	0.95	3.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.163	0.240	0.256	0.386	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	68	122	93	133	0	0	-1
N.S.	1	1.00	0.62	1.12	0.85	1.22	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.064	0.234	0.269	0.356	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	93	140	134	169	0	0	-1
N.S.	1	1.00	0.59	0.89	0.85	1.07	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.088	0.245	0.253	0.362	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	109	158	167	196	0	0	-1
N.S.	1	1.00	0.53	0.77	0.81	0.96	0.00	0.00	-0.00
time (sec)	N/A	0.082	0.108	0.249	0.264	0.371	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	114	152	178	224	0	0	-1
N.S.	1	1.00	0.56	0.75	0.87	1.10	0.00	0.00	-0.00
time (sec)	N/A	0.111	0.175	0.361	0.268	0.421	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	97	267	139	196	0	0	-1
N.S.	1	1.00	0.61	1.68	0.87	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.146	0.448	0.265	0.368	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	77	195	97	162	0	0	-1
N.S.	1	1.00	0.53	1.34	0.66	1.11	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.076	0.440	0.255	0.369	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	93	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.140	0.024	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	138	0	0	0	0	0	-1
N.S.	1	1.00	1.08	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.713	0.026	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	182	286	396	679	0	0	-1
N.S.	1	1.00	0.70	1.10	1.52	2.61	0.00	0.00	-0.00
time (sec)	N/A	0.173	0.449	0.463	0.273	0.484	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	149	217	287	614	0	0	-1
N.S.	1	1.00	0.76	1.10	1.46	3.12	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.134	0.335	0.259	0.460	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	134	189	191	580	0	0	-1
N.S.	1	1.00	0.79	1.11	1.12	3.41	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.132	0.341	0.258	0.408	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	123	190	152	559	0	0	-1
N.S.	1	1.00	0.75	1.16	0.93	3.41	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.164	0.323	0.274	0.486	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	126	191	175	278	0	0	-1
N.S.	1	1.00	0.67	1.01	0.93	1.47	0.00	0.00	-0.01
time (sec)	N/A	0.112	0.126	0.332	0.261	0.420	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	152	223	232	347	0	0	-1
N.S.	1	1.00	0.61	0.90	0.93	1.39	0.00	0.00	-0.00
time (sec)	N/A	0.136	0.156	0.336	0.263	0.352	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	159	377	244	412	0	0	-1
N.S.	1	1.00	0.64	1.51	0.98	1.65	0.00	0.00	-0.00
time (sec)	N/A	0.165	0.229	0.567	0.270	0.379	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	123	280	183	346	0	0	-1
N.S.	1	1.00	0.61	1.38	0.90	1.70	0.00	0.00	-0.00
time (sec)	N/A	0.106	0.176	0.552	0.267	0.373	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	148	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.292	0.250	0.119	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	187	0	0	0	0	0	-1
N.S.	1	1.00	1.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.288	0.594	0.109	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	1221	0	0	0	0	0	-1
N.S.	1	1.00	2.38	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.841	1.130	0.128	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	467	467	1103	0	0	0	0	0	-1
N.S.	1	1.00	2.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.793	0.331	0.117	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	477	477	1055	0	0	0	0	0	-1
N.S.	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.571	0.360	0.127	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	387	0	0	0	0	0	-1
N.S.	1	1.00	0.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.587	0.634	0.128	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	1211	0	0	0	0	0	-1
N.S.	1	1.00	2.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.729	1.147	0.133	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	591	591	1447	0	0	0	0	0	-1
N.S.	1	1.00	2.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.912	3.912	0.134	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	1410	0	0	0	0	0	-1
N.S.	1	1.00	2.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.842	1.561	0.128	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	271	356	0	1060	0	0	-1
N.S.	1	1.00	1.95	2.56	0.00	7.63	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.536	5.201	0.000	0.390	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.768	40.567	0.122	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	756	756	1583	0	0	0	0	0	-1
N.S.	1	1.00	2.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.668	6.042	13.288	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	719	719	1442	0	0	0	0	0	-1
N.S.	1	1.00	2.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.838	1.649	0.154	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	713	713	1437	0	0	0	0	0	-1
N.S.	1	1.00	2.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.512	2.262	0.151	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	758	758	1487	0	0	0	0	0	-1
N.S.	1	1.00	1.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.559	2.024	8.598	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	694	694	2023	0	0	0	0	0	-1
N.S.	1	1.00	2.91	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.016	7.230	0.139	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	375	1881	0	3944	0	0	-1
N.S.	1	1.00	2.25	11.26	0.00	23.62	0.00	0.00	-0.01
time (sec)	N/A	0.147	0.951	6.336	0.000	0.658	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	368	1838	0	3307	0	0	-1
N.S.	1	1.00	1.80	8.97	0.00	16.13	0.00	0.00	-0.00
time (sec)	N/A	0.139	0.621	6.437	0.000	0.578	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	657	657	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.859	61.314	0.133	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1106	1106	2045	0	0	0	0	0	-1
N.S.	1	1.00	1.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.064	6.047	0.158	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1106	1106	2053	0	0	0	0	0	-1
N.S.	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.044	6.048	0.157	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1096	1096	2038	0	0	0	0	0	-1
N.S.	1	1.00	1.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.505	6.033	0.156	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	319	0	0	2155	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	5.22	0.00	0.00	-0.00
time (sec)	N/A	0.915	3.856	0.153	0.000	1.526	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	255	0	0	1524	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	5.05	0.00	0.00	-0.00
time (sec)	N/A	0.279	2.358	0.164	0.000	0.853	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	194	0	0	1113	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	5.48	0.00	0.00	-0.00
time (sec)	N/A	0.139	1.495	0.154	0.000	0.624	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.067	3.383	0.154	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.067	3.397	0.143	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.064	5.961	0.173	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.024	1.577	0.168	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	1.100	0.166	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	237	0	0	0	0	0	-1
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.278	7.459	0.144	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	527	527	314	0	0	0	0	0	-1
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.432	10.074	0.138	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	293	0	0	2121	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	5.52	0.00	0.00	-0.00
time (sec)	N/A	0.357	3.283	0.141	0.000	1.644	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	233	0	0	1487	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	5.51	0.00	0.00	-0.00
time (sec)	N/A	0.197	2.582	0.132	0.000	0.860	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.080	4.037	0.135	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.081	3.358	0.149	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.076	6.253	0.135	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.029	2.125	0.131	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.069	3.353	0.148	0.000	0.000	0.000	0.000	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.071	9.256	0.135	0.000	0.000	0.000	0.000	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	492	492	291	0	0	0	0	0	-1
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.389	10.186	0.133	0.000	0.000	0.000	0.000	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	643	643	372	0	0	0	0	0	-1
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.546	10.534	0.129	0.000	0.000	0.000	0.000	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	257	0	0	1559	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	4.74	0.00	0.00	-0.00
time (sec)	N/A	0.774	2.145	0.152	0.000	0.903	0.000	0.000	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	198	0	0	1136	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	4.96	0.00	0.00	-0.00
time (sec)	N/A	0.211	1.170	0.153	0.000	0.591	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	137	0	0	885	0	0	-1
N.S.	1	1.00	1.01	0.00	0.00	6.56	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.542	0.151	0.000	0.517	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.064	1.094	0.138	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.075	15.788	0.135	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.063	3.802	0.135	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.022	0.700	0.141	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	139	0	0	0	0	0	-1
N.S.	1	1.00	0.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.172	0.888	0.138	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	239	0	0	0	0	0	-1
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.319	5.508	0.138	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	229	0	0	1705	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	6.66	0.00	0.00	-0.00
time (sec)	N/A	0.768	1.289	0.132	0.000	0.544	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	147	0	0	1146	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	7.16	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.951	0.128	0.000	0.450	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	94	0	0	561	0	0	-1
N.S.	1	1.00	1.15	0.00	0.00	6.84	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.503	0.126	0.000	0.429	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.077	22.129	0.141	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.087	31.967	0.151	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.073	5.371	0.148	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.067	2.976	0.147	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	113	0	0	165	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	1.49	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.668	0.137	0.000	0.105	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	201	0	0	0	0	0	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.216	3.892	0.131	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	241	0	0	3102	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	12.36	0.00	0.00	-0.00
time (sec)	N/A	0.847	1.164	0.125	0.000	0.644	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	173	0	0	1858	0	0	-1
N.S.	1	1.00	1.02	0.00	0.00	10.99	0.00	0.00	-0.01
time (sec)	N/A	0.173	0.161	0.157	0.000	0.564	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	157	0	0	1488	0	0	-1
N.S.	1	1.00	1.09	0.00	0.00	10.33	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.140	0.132	0.000	0.495	0.000	0.000	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.085	38.234	0.155	0.000	0.000	0.000	0.000	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.102	50.238	0.161	0.000	0.000	0.000	0.000	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.082	8.533	0.148	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.073	7.418	0.138	0.000	0.000	0.000	0.000	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	189	0	0	0	0	0	-1
N.S.	1	1.00	0.53	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.211	0.186	0.131	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	248	0	0	0	0	0	-1
N.S.	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.125	3.231	0.148	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	596	577	0	0	0	0	0	0	-1
N.S.	1	0.97	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.592	0.090	0.134	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	379	360	0	0	0	0	0	0	-1
N.S.	1	0.95	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.331	0.072	0.121	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	208	0	0	0	0	0	0	-1
N.S.	1	0.95	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.149	0.056	0.030	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.050	1.227	0.145	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.050	4.457	0.149	0.000	0.000	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.074	0.662	0.149	0.000	0.000	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.067	0.065	0.153	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.067	0.857	0.135	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.073	1.089	0.138	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	214	0	0	382	0	0	-1
N.S.	1	1.00	0.54	0.00	0.00	0.97	0.00	0.00	-0.00
time (sec)	N/A	1.532	0.189	0.152	0.000	0.401	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	180	0	0	324	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	1.23	0.00	0.00	-0.00
time (sec)	N/A	1.309	0.263	0.132	0.000	0.363	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	133	141	0	0	265	0	0	-1
N.S.	1	1.02	1.08	0.00	0.00	2.04	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.216	0.135	0.000	0.394	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.059	0.275	0.133	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.071	2.286	0.129	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [174] had the largest ratio of [26]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.00	12	0.417
2	A	4	3	1.00	12	0.250
3	A	6	5	1.00	12	0.417
4	A	3	3	1.00	12	0.250
5	A	5	5	1.00	12	0.417
6	A	2	2	1.00	10	0.200
7	A	5	4	1.00	8	0.500
8	A	6	6	1.00	12	0.500
9	A	2	2	1.00	12	0.167
10	A	4	4	1.00	12	0.333
11	A	4	3	1.00	12	0.250
12	A	5	4	1.00	12	0.333
13	A	4	3	1.00	12	0.250
14	A	6	4	1.00	12	0.333
15	A	5	5	1.00	14	0.357
16	A	8	6	1.00	14	0.429
17	A	4	4	1.00	12	0.333
18	A	7	5	1.00	10	0.500
19	A	6	6	1.00	14	0.429
20	A	4	3	1.00	14	0.214
21	A	4	3	1.00	14	0.214
22	A	5	5	1.00	14	0.357
23	A	5	3	1.00	14	0.214
24	A	10	10	1.00	14	0.714
25	A	11	8	1.00	14	0.571

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	7	7	1.00	12	0.583
27	A	9	6	1.00	10	0.600
28	A	7	7	1.00	14	0.500
29	A	5	3	1.00	14	0.214
30	A	6	6	1.00	14	0.429
31	A	8	6	1.00	14	0.429
32	A	10	6	1.00	14	0.429
33	A	0	0	0.00	0	0.000
34	A	0	0	0.00	0	0.000
35	A	0	0	0.00	0	0.000
36	A	4	4	1.00	14	0.286
37	A	6	6	1.00	14	0.429
38	A	9	5	1.00	14	0.357
39	A	0	0	0.00	0	0.000
40	A	0	0	0.00	0	0.000
41	A	3	3	1.00	14	0.214
42	A	0	0	0.00	0	0.000
43	A	0	0	0.00	0	0.000
44	A	11	9	1.00	16	0.562
45	A	10	9	1.00	16	0.562
46	A	9	9	1.00	14	0.643
47	A	5	4	1.00	8	0.500
48	A	4	2	1.00	16	0.125
49	A	7	7	1.00	16	0.438
50	A	8	8	1.00	16	0.500
51	A	31	16	1.00	21	0.762
52	A	24	15	1.00	19	0.790
53	A	15	11	1.00	18	0.611
54	A	0	0	0.00	0	0.000
55	A	0	0	0.00	0	0.000
56	A	22	13	1.00	18	0.722
57	A	27	17	1.00	21	0.810
58	A	20	15	1.00	21	0.714
59	A	14	13	1.00	19	0.684
60	A	9	9	1.00	18	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	0	0	0.00	0	0.000
62	A	0	0	0.00	0	0.000
63	A	23	15	1.00	21	0.714
64	A	16	13	1.00	21	0.619
65	A	11	11	1.00	19	0.579
66	A	6	6	1.00	18	0.333
67	A	0	0	0.00	0	0.000
68	A	0	0	0.00	0	0.000
69	A	31	18	1.00	21	0.857
70	A	25	17	1.00	21	0.810
71	A	19	14	1.00	19	0.737
72	A	12	11	1.00	18	0.611
73	A	0	0	0.00	0	0.000
74	A	0	0	0.00	0	0.000
75	A	19	14	1.21	18	0.778
76	A	7	7	1.00	19	0.368
77	A	6	7	1.00	19	0.368
78	A	5	5	1.00	16	0.312
79	A	4	5	1.00	19	0.263
80	A	4	5	1.00	19	0.263
81	A	5	6	1.00	19	0.316
82	A	6	6	1.00	19	0.316
83	A	5	5	1.00	19	0.263
84	A	5	5	1.00	19	0.263
85	A	6	5	1.00	17	0.294
86	A	11	11	1.00	19	0.579
87	A	13	13	1.00	19	0.684
88	A	7	8	1.00	21	0.381
89	A	6	7	1.00	18	0.389
90	A	6	7	1.00	21	0.333
91	A	6	7	1.00	21	0.333
92	A	5	6	1.00	21	0.286
93	A	6	7	1.00	21	0.333
94	A	5	6	1.00	21	0.286
95	A	6	5	1.00	19	0.263

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	12	13	1.00	21	0.619
97	A	14	15	1.00	21	0.714
98	A	25	12	1.00	21	0.571
99	A	26	9	1.00	19	0.474
100	A	19	7	1.00	18	0.389
101	A	19	7	1.00	21	0.333
102	A	24	10	1.00	21	0.476
103	A	31	14	1.00	21	0.667
104	A	29	12	1.00	21	0.571
105	A	7	6	1.00	19	0.316
106	A	24	10	1.00	21	0.476
107	A	51	15	1.00	21	0.714
108	A	27	10	1.00	21	0.476
109	A	47	11	1.00	18	0.611
110	A	50	13	1.00	21	0.619
111	A	33	13	1.00	21	0.619
112	A	6	7	1.00	21	0.333
113	A	8	7	1.00	19	0.368
114	A	28	11	1.00	21	0.524
115	A	35	11	1.00	21	0.524
116	A	63	12	1.00	21	0.571
117	A	81	12	1.00	18	0.667
118	A	12	12	1.00	23	0.522
119	A	11	12	1.00	23	0.522
120	A	9	9	1.00	21	0.429
121	A	0	0	0.00	0	0.000
122	A	0	0	0.00	0	0.000
123	A	0	0	0.00	0	0.000
124	A	0	0	0.00	0	0.000
125	A	0	0	0.00	0	0.000
126	A	8	9	1.00	23	0.391
127	A	9	10	1.00	23	0.435
128	A	12	12	1.00	23	0.522
129	A	10	10	1.00	21	0.476
130	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	0	0	0.00	0	0.000
132	A	0	0	0.00	0	0.000
133	A	0	0	0.00	0	0.000
134	A	0	0	0.00	0	0.000
135	A	0	0	0.00	0	0.000
136	A	9	10	1.00	23	0.435
137	A	10	10	1.00	23	0.435
138	A	11	12	1.00	23	0.522
139	A	10	12	1.00	23	0.522
140	A	9	9	1.00	21	0.429
141	A	0	0	0.00	0	0.000
142	A	0	0	0.00	0	0.000
143	A	0	0	0.00	0	0.000
144	A	0	0	0.00	0	0.000
145	A	8	9	1.00	23	0.391
146	A	8	10	1.00	23	0.435
147	A	10	11	1.00	23	0.478
148	A	9	11	1.00	23	0.478
149	A	4	4	1.00	21	0.190
150	A	0	0	0.00	0	0.000
151	A	0	0	0.00	0	0.000
152	A	0	0	0.00	0	0.000
153	A	0	0	0.00	0	0.000
154	A	3	4	1.00	20	0.200
155	A	7	9	1.00	23	0.391
156	A	10	11	1.00	23	0.478
157	A	7	8	1.00	23	0.348
158	A	5	5	1.00	21	0.238
159	A	0	0	0.00	0	0.000
160	A	0	0	0.00	0	0.000
161	A	0	0	0.00	0	0.000
162	A	0	0	0.00	0	0.000
163	A	7	8	1.00	23	0.348
164	A	5	7	1.00	20	0.350
165	A	6	7	0.97	23	0.304

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	6	7	0.95	23	0.304
167	A	5	6	0.95	21	0.286
168	A	0	0	0.00	0	0.000
169	A	0	0	0.00	0	0.000
170	A	0	0	0.00	0	0.000
171	A	0	0	0.00	0	0.000
172	A	0	0	0.00	0	0.000
173	A	0	0	0.00	0	0.000
174	A	16	11	1.00	26	0.423
175	A	13	11	1.00	26	0.423
176	A	8	9	1.02	26	0.346
177	A	0	0	0.00	0	0.000
178	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^6 \left(a + b\text{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=110

$$\frac{5b\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{112c^5}-\frac{5b\sqrt{1+\frac{1}{c^2x^2}}\,x^4}{168c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^6}{42c}+\frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx)\right)-\frac{5b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{112c^7}$$

```
[Out] 1/7*x^7*(a+b*arccsch(c*x))-5/112*b*arctanh((1+1/c^2/x^2)^(1/2))/c^7+5/112*b
*x^2*(1+1/c^2/x^2)^(1/2)/c^5-5/168*b*x^4*(1+1/c^2/x^2)^(1/2)/c^3+1/42*b*x^6
*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.05, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6419, 272, 44, 65, 214}

$$\frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx^6\sqrt{\frac{1}{c^2x^2}+1}}{42c}-\frac{5b\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{112c^7}+\frac{5bx^2\sqrt{\frac{1}{c^2x^2}+1}}{112c^5}-\frac{5bx^4\sqrt{\frac{1}{c^2x^2}+1}}{168c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^6*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (5*b*Sqrt[1 + 1/(c^2*x^2)]*x^2)/(112*c^5) - (5*b*Sqrt[1 + 1/(c^2*x^2)]*x^4)
/(168*c^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^6)/(42*c) + (x^7*(a + b*ArcCsch[c*x
]))/7 - (5*b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(112*c^7)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^6\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx)\right)+\frac{b\int\frac{x^5}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{7c}$$

$$=\frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx)\right)-\frac{b\text{Subst}\left(\int\frac{1}{x^4\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{14c}$$

$$=\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^6}{42c}+\frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx)\right)+\frac{(5b)\text{Subst}\left(\int\frac{1}{x^3\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right.}{84c^3}$$

$$=-\frac{5b\sqrt{1+\frac{1}{c^2x^2}}\,x^4}{168c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^6}{42c}+\frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(5b)\text{Subst}\left(\int\right.}{}$$

$$=\frac{5b\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{112c^5}-\frac{5b\sqrt{1+\frac{1}{c^2x^2}}\,x^4}{168c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^6}{42c}+\frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx\right.$$

$$=\frac{5b\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{112c^5}-\frac{5b\sqrt{1+\frac{1}{c^2x^2}}\,x^4}{168c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^6}{42c}+\frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx\right.$$

$$=\frac{5b\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{112c^5}-\frac{5b\sqrt{1+\frac{1}{c^2x^2}}\,x^4}{168c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^6}{42c}+\frac{1}{7}x^7\left(a+b\text{csch}^{-1}(cx\right.$$

<u>Mathematica</u> [A]
time = 0.10, size = 107, normalized size = 0.97

$$\frac{ax^7}{7}+b\sqrt{\frac{1+c^2x^2}{c^2x^2}}\left(\frac{5x^2}{112c^5}-\frac{5x^4}{168c^3}+\frac{x^6}{42c}\right)+\frac{1}{7}bx^7\text{csch}^{-1}(cx)-\frac{5b\log\left(x\left(1+\sqrt{\frac{1+c^2x^2}{c^2x^2}}\right)\right)}{112c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (a*x^7)/7 + b*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]*((5*x^2)/(112*c^5) - (5*x^4)/(1
68*c^3) + x^6/(42*c)) + (b*x^7*ArcCsch[c*x])/7 - (5*b*Log[x*(1 + Sqrt[(1 +
c^2*x^2)/(c^2*x^2)])])/(112*c^7)
```

Maple [A]
time = 0.22, size = 127, normalized size = 1.15

method	result
derivativedivides	$\frac{\frac{c^7x^7a}{7}+b\left(\frac{c^7x^7\operatorname{arccsch}(cx)}{7}+\frac{\sqrt{c^2x^2+1}\,\left(8c^5x^5\sqrt{c^2x^2+1}-10c^3x^3\sqrt{c^2x^2+1}+15cx\sqrt{c^2x^2+1}-15\operatorname{arcs}\right.}{336\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right.}{c^7}$
default	$\frac{\frac{c^7x^7a}{7}+b\left(\frac{c^7x^7\operatorname{arccsch}(cx)}{7}+\frac{\sqrt{c^2x^2+1}\,\left(8c^5x^5\sqrt{c^2x^2+1}-10c^3x^3\sqrt{c^2x^2+1}+15cx\sqrt{c^2x^2+1}-15\operatorname{arcs}\right.}{336\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right.}{c^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^7*(1/7*c^7*x^7*a+b*(1/7*c^7*x^7*arccsch(c*x)+1/336*(c^2*x^2+1)^(1/2)*(8
*c^5*x^5*(c^2*x^2+1)^(1/2)-10*c^3*x^3*(c^2*x^2+1)^(1/2)+15*c*x*(c^2*x^2+1)^
(1/2)-15*arcsinh(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A]
time = 0.26, size = 158, normalized size = 1.44

$$\frac{1}{7}ax^7+\frac{1}{672}\left(96\,x^7\operatorname{arcsch}(cx)+\frac{\frac{2\left(15\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}+1\right)^3-3c^6\left(\frac{1}{c^2x^2}+1\right)^2+3c^6\left(\frac{1}{c^2x^2}+1\right)-c^6}-\frac{15\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^6}+\frac{15\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^6}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*x^7 + 1/672*(96*x^7*arccsch(c*x) + (2*(15*(1/(c^2*x^2) + 1)^(5/2) - 4
0*(1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) + 1
)^3 - 3*c^6*(1/(c^2*x^2) + 1)^2 + 3*c^6*(1/(c^2*x^2) + 1) - c^6) - 15*log(s
qrt(1/(c^2*x^2) + 1) + 1)/c^6 + 15*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^6)/c)*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(92) = 184$.
time = 0.47, size = 208, normalized size = 1.89

$$\frac{48\,ac^7x^7+48\,bc^7\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-48\,bc^7\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)+15\,b\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx\right)+48\left(bc^7x^7-bc^7\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(8\,bc^6x^6-10\,bc^4x^4+15\,bc^2x^2\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{336\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/336*(48*a*c^7*x^7 + 48*b*c^7*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x
+ 1) - 48*b*c^7*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 15*b*log
(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) + 48*(b*c^7*x^7 - b*c^7)*log((c*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (8*b*c^6*x^6 - 10*b*c^4*x^4 +
15*b*c^2*x^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^6(a + b\,\mathrm{acsch}\,(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**6*(a + b*acsch(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^6\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x^6*(a + b*asinh(1/(c*x))), x)
```

3.2 $\int x^5 \left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=86

$$\frac{4b\sqrt{1+\frac{1}{c^2x^2}}\,x}{45c^5}-\frac{2b\sqrt{1+\frac{1}{c^2x^2}}\,x^3}{45c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^5}{30c}+\frac{1}{6}x^6\left(a+b\text{csch}^{-1}(cx)\right)$$

```
[Out] 1/6*x^6*(a+b*arccsch(c*x))+4/45*b*x*(1+1/c^2/x^2)^(1/2)/c^5-2/45*b*x^3*(1+1
/c^2/x^2)^(1/2)/c^3+1/30*b*x^5*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6419, 277, 197}

$$\frac{1}{6}x^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx^5\sqrt{\frac{1}{c^2x^2}+1}}{30c}+\frac{4bx\sqrt{\frac{1}{c^2x^2}+1}}{45c^5}-\frac{2bx^3\sqrt{\frac{1}{c^2x^2}+1}}{45c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (4*b*Sqrt[1 + 1/(c^2*x^2)]*x)/(45*c^5) - (2*b*Sqrt[1 + 1/(c^2*x^2)]*x^3)/(4
5*c^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^5)/(30*c) + (x^6*(a + b*ArcCsch[c*x]))/
6
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^5\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{1}{6}x^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{b\int\frac{x^4}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{6c}$$

$$=\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^5}{30c}+\frac{1}{6}x^6\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(2b)\int\frac{x^2}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{15c^3}$$

$$=-\frac{2b\sqrt{1+\frac{1}{c^2x^2}}\,x^3}{45c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^5}{30c}+\frac{1}{6}x^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{(4b)\int\frac{1}{\sqrt{1+\cdots}}}{45c^5}$$

$$=\frac{4b\sqrt{1+\frac{1}{c^2x^2}}\,x}{45c^5}-\frac{2b\sqrt{1+\frac{1}{c^2x^2}}\,x^3}{45c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^5}{30c}+\frac{1}{6}x^6\left(a+b\text{csch}^{-1}(cx)\right.$$

Mathematica [A]
time = 0.09, size = 72, normalized size = 0.84

$$\frac{ax^6}{6}+b\sqrt{\frac{1+c^2x^2}{c^2x^2}}\left(\frac{4x}{45c^5}-\frac{2x^3}{45c^3}+\frac{x^5}{30c}\right)+\frac{1}{6}bx^6\text{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (a*x^6)/6 + b*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]*((4*x)/(45*c^5) - (2*x^3)/(45*c
^3) + x^5/(30*c)) + (b*x^6*ArcCsch[c*x])/6
```

Maple [A]
time = 0.19, size = 83, normalized size = 0.97

method	result	size
derivativedivides	$\frac{\frac{c^6x^6a}{6}+b\left(\frac{c^6x^6\text{arccsch}(cx)}{6}+\frac{(c^2x^2+1)(3c^4x^4-4c^2x^2+8)}{90\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^6}$	83
default	$\frac{\frac{c^6x^6a}{6}+b\left(\frac{c^6x^6\text{arccsch}(cx)}{6}+\frac{(c^2x^2+1)(3c^4x^4-4c^2x^2+8)}{90\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^6}$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^6*(1/6*c^6*x^6*a+b*(1/6*c^6*x^6*arccsch(c*x)+1/90*(c^2*x^2+1)*(3*c^4*x^
4-4*c^2*x^2+8)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A]
time = 0.25, size = 77, normalized size = 0.90

$$\frac{1}{6}\,ax^{6}+\frac{1}{90}\left(15\,x^{6}\operatorname{arcsch}\left(cx\right)+\frac{3\,c^{4}x^{5}\left(\frac{1}{c^{2}x^{2}}+1\right)^{\frac{5}{2}}-10\,c^{2}x^{3}\left(\frac{1}{c^{2}x^{2}}+1\right)^{\frac{3}{2}}+15\,x\sqrt{\frac{1}{c^{2}x^{2}}+1}}{c^{5}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6 + 1/90*(15*x^6*arccsch(c*x) + (3*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2)
- 10*c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2) + 1))/c^5)*b
```

Fricas [A]
time = 0.47, size = 97, normalized size = 1.13

$$\frac{15\,bc^{5}x^{6}\log\left(\frac{cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+1}{cx}\right)+15\,ac^{5}x^{6}+\left(3\,bc^{4}x^{5}-4\,bc^{2}x^{3}+8\,bx\right)\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}}{90\,c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*b*c^5*x^6*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 15*
a*c^5*x^6 + (3*b*c^4*x^5 - 4*b*c^2*x^3 + 8*b*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2
)))/c^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5}(a+b\operatorname{acsch}\left(cx\right))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**5*(a + b*acsch(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left(a + b \operatorname{asinh}\left(\frac{1}{c x}\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*asinh(1/(c*x))),x)
[Out] int(x^5*(a + b*asinh(1/(c*x))), x)
```

3.3 $\int x^4\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=86

$$-\frac{3b\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{40c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^4}{20c}+\frac{1}{5}x^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{3b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{40c^5}$$

```
[Out] 1/5*x^5*(a+b*arccsch(c*x))+3/40*b*arctanh((1+1/c^2/x^2)^(1/2))/c^5-3/40*b*x
^2*(1+1/c^2/x^2)^(1/2)/c^3+1/20*b*x^4*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.03, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6419, 272, 44, 65, 214}

$$\frac{1}{5}x^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx^4\sqrt{\frac{1}{c^2x^2}+1}}{20c}+\frac{3b\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{40c^5}-\frac{3bx^2\sqrt{\frac{1}{c^2x^2}+1}}{40c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (-3*b*Sqrt[1 + 1/(c^2*x^2)]*x^2)/(40*c^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^4)/(
20*c) + (x^5*(a + b*ArcCsch[c*x]))/5 + (3*b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])
/(40*c^5)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx=\frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{b\int\frac{x^3}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{5c}$$

$$=\frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{b\mathrm{Subst}\left(\int\frac{1}{x^3\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{10c}$$

$$=\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^4}{20c}+\frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{(3b)\mathrm{Subst}\left(\int\frac{1}{x^2\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x}\right.}{40c^3}$$

$$=-\frac{3b\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{40c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^4}{20c}+\frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(3b)\mathrm{Subst}\left(\right.}{}$$

$$=-\frac{3b\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{40c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^4}{20c}+\frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{(3b)\mathrm{Subst}\left(\right.}{}$$

$$=-\frac{3b\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{40c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^4}{20c}+\frac{1}{5}x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{3b\tanh^{-1}\left(\right.}{4}$$

Mathematica [A]
time = 0.03, size = 97, normalized size = 1.13

$$\frac{ax^5}{5}+b\sqrt{\frac{1+c^2x^2}{c^2x^2}}\left(-\frac{3x^2}{40c^3}+\frac{x^4}{20c}\right)+\frac{1}{5}bx^5\operatorname{csch}^{-1}(cx)+\frac{3b\log\left(x\left(1+\sqrt{\frac{1+c^2x^2}{c^2x^2}}\right)\right)}{40c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (a*x^5)/5 + b*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]*((-3*x^2)/(40*c^3) + x^4/(20*c)
) + (b*x^5*ArcCsch[c*x])/5 + (3*b*Log[x*(1 + Sqrt[(1 + c^2*x^2)/(c^2*x^2)])
])/(40*c^5)
```

Maple [A]
time = 0.20, size = 108, normalized size = 1.26

method	result	size
derivativedivides	$\frac{\frac{c^5x^5a}{5}+b\left(\frac{c^5x^5\operatorname{arccsch}(cx)}{5}+\frac{\sqrt{c^2x^2+1}\,\left(2c^3x^3\sqrt{c^2x^2+1}-3cx\sqrt{c^2x^2+1}+3\operatorname{arcsinh}(cx)\right)}{40\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^5}$	108
default	$\frac{\frac{c^5x^5a}{5}+b\left(\frac{c^5x^5\operatorname{arccsch}(cx)}{5}+\frac{\sqrt{c^2x^2+1}\,\left(2c^3x^3\sqrt{c^2x^2+1}-3cx\sqrt{c^2x^2+1}+3\operatorname{arcsinh}(cx)\right)}{40\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^5}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(1/5*c^5*x^5*a+b*(1/5*c^5*x^5*arccsch(c*x)+1/40*(c^2*x^2+1)^(1/2)*(2*
c^3*x^3*(c^2*x^2+1)^(1/2)-3*c*x*(c^2*x^2+1)^(1/2)+3*arcsinh(c*x))/((c^2*x^2
+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A]
time = 0.26, size = 128, normalized size = 1.49

$$\frac{1}{5}\,ax^5+\frac{1}{80}\left(16\,x^5\operatorname{arcsch}(cx)-\frac{\frac{2\left(3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}+1\right)^2-2\,c^4\left(\frac{1}{c^2x^2}+1\right)+c^4}-\frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5 + 1/80*(16*x^5*arccsch(c*x) - (2*(3*(1/(c^2*x^2) + 1)^(3/2) - 5*s
qrt(1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) + 1)^2 - 2*c^4*(1/(c^2*x^2) + 1) +
c^4) - 3*log(sqrt(1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(1/(c^2*x^2) + 1) -
 1)/c^4)/c)*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(72) = 144$.
time = 0.54, size = 199, normalized size = 2.31

$$\frac{8\,ac^5x^5+8\,bc^5\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-8\,bc^5\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)-3\,b\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx\right)+8\,(bc^5x^5-bc^5)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+(2\,bc^4x^4-3\,bc^2x^2)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{40\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*a*c^5*x^5 + 8*b*c^5*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1
) - 8*b*c^5*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) - 3*b*log(c*x*
sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) + 8*(b*c^5*x^5 - b*c^5)*log((c*x*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (2*b*c^4*x^4 - 3*b*c^2*x^2)*sqrt((c^
2*x^2 + 1)/(c^2*x^2)))/c^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4(a + b\,\mathrm{acsch}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**4*(a + b*acsch(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^4\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x^4*(a + b*asinh(1/(c*x))), x)
```

3.4 $\int x^3(a+b\text{csch}^{-1}(cx))\,dx$

Optimal. Leaf size=62

$$-\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)$$

```
[Out] 1/4*x^4*(a+b*arccsch(c*x))-1/6*b*x*(1+1/c^2/x^2)^(1/2)/c^3+1/12*b*x^3*(1+1/
c^2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6419, 277, 197}

$$\frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx^3\sqrt{\frac{1}{c^2x^2}+1}}{12c}-\frac{bx\sqrt{\frac{1}{c^2x^2}+1}}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -1/6*(b*Sqrt[1 + 1/(c^2*x^2)]*x)/c^3 + (b*Sqrt[1 + 1/(c^2*x^2)]*x^3)/(12*c)
 + (x^4*(a + b*ArcCsch[c*x]))/4
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^3 \left(a+b\operatorname{csch}^{-1}(cx)\right) dx = \frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right) + \frac{b\int \frac{x^2}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{4c}$$
$$= \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^3}{12c} + \frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right) - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{6c^3}$$
$$= -\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{6c^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^3}{12c} + \frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)$$

Mathematica [A]
time = 0.07, size = 62, normalized size = 1.00

$$\frac{ax^4}{4} + b\sqrt{\frac{1+c^2x^2}{c^2x^2}}\left(-\frac{x}{6c^3}+\frac{x^3}{12c}\right) + \frac{1}{4}bx^4\operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (a*x^4)/4 + b*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]*(-1/6*x/c^3 + x^3/(12*c)) + (b*
x^4*ArcCsch[c*x])/4
```

Maple [A]
time = 0.19, size = 74, normalized size = 1.19

method	result	size
derivativedivides	$\frac{\frac{c^4x^4a}{4}+b\left(\frac{c^4x^4\operatorname{arccsch}(cx)}{4}+\frac{(c^2x^2+1)(c^2x^2-2)}{12\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^4}$	74
default	$\frac{\frac{c^4x^4a}{4}+b\left(\frac{c^4x^4\operatorname{arccsch}(cx)}{4}+\frac{(c^2x^2+1)(c^2x^2-2)}{12\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^4}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(1/4*c^4*x^4*a+b*(1/4*c^4*x^4*arccsch(c*x)+1/12*(c^2*x^2+1)*(c^2*x^2-
2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A]
time = 0.25, size = 57, normalized size = 0.92

$$\frac{1}{4}\,ax^{4}+\frac{1}{12}\left(3\,x^{4}\operatorname{arcsch}\left(cx\right)+\frac{c^{2}x^{3}\left(\frac{1}{c^{2}x^{2}}+1\right)^{\frac{3}{2}}-3\,x\sqrt{\frac{1}{c^{2}x^{2}}+1}}{c^{3}}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/12*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 3
*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b
```

Fricas [A]
time = 0.43, size = 87, normalized size = 1.40

$$\frac{3\,bc^{3}x^{4}\log\left(\frac{cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+1}{cx}\right)+3\,ac^{3}x^{4}+\left(bc^{2}x^{3}-2\,bx\right)\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}}{12\,c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*c^3*x^4*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 3*a*
c^3*x^4 + (b*c^2*x^3 - 2*b*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}(a+b\operatorname{acsch}\left(cx\right))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^3 \left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right) \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x^3*(a + b*asinh(1/(c*x))), x)
```

3.5 $\int x^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=62

$$\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{6c}+\frac{1}{3}x^3\left(a+b\,\mathrm{csch}^{-1}(cx)\right)-\frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{6c^3}$$

```
[Out] 1/3*x^3*(a+b*arccsch(c*x))-1/6*b*arctanh((1+1/c^2/x^2)^(1/2))/c^3+1/6*b*x^2
*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.02, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {6419, 272, 44, 65, 214}

$$\frac{1}{3}x^3\left(a+b\,\mathrm{csch}^{-1}(cx)\right)+\frac{bx^2\sqrt{\frac{1}{c^2x^2}+1}}{6c}-\frac{b\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*Sqrt[1 + 1/(c^2*x^2)]*x^2)/(6*c) + (x^3*(a + b*ArcCsch[c*x]))/3 - (b*Arc
Tanh[Sqrt[1 + 1/(c^2*x^2)]])/(6*c^3)
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{b\int\frac{x}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{3c}$$

$$=\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)-\frac{b\text{Subst}\left(\int\frac{1}{x^2\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{6c}$$

$$=\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{6c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{b\text{Subst}\left(\int\frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{12c^3}$$

$$=\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{6c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{b\text{Subst}\left(\int\frac{1}{-c^2+c^2x^2}\,dx,x,\sqrt{1+\frac{1}{c^2x^2}}\right)}{6c}$$

$$=\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{6c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)-\frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{6c^3}$$

Mathematica [A]
time = 0.04, size = 85, normalized size = 1.37

$$\frac{ax^3}{3}+\frac{bx^2\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{6c}+\frac{1}{3}bx^3\text{csch}^{-1}(cx)-\frac{b\log\left(x\left(1+\sqrt{\frac{1+c^2x^2}{c^2x^2}}\right)\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (a*x^3)/3 + (b*x^2*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(6*c) + (b*x^3*ArcCsch[c*
x])/3 - (b*Log[x*(1 + Sqrt[(1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

Maple [A]
time = 0.21, size = 88, normalized size = 1.42

method	result	size
derivativedivides	$\frac{\frac{c^3x^3a}{3}+b\left(\frac{c^3x^3\operatorname{arccsch}(cx)}{3}+\frac{\sqrt{c^2x^2+1}\left(cx\sqrt{c^2x^2+1}-\operatorname{arcsinh}(cx)\right)}{6\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^3}$	88
default	$\frac{\frac{c^3x^3a}{3}+b\left(\frac{c^3x^3\operatorname{arccsch}(cx)}{3}+\frac{\sqrt{c^2x^2+1}\left(cx\sqrt{c^2x^2+1}-\operatorname{arcsinh}(cx)\right)}{6\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^3}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/3*c^3*x^3*a+b*(1/3*c^3*x^3*arccsch(c*x)+1/6*(c^2*x^2+1)^(1/2)*(c*x
*(c^2*x^2+1)^(1/2)-arcsinh(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A]
time = 0.26, size = 96, normalized size = 1.55

$$\frac{1}{3}\,ax^3+\frac{1}{12}\left(4\,x^3\operatorname{arcsch}(cx)+\frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}-\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/12*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^
2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x
^2) + 1) - 1)/c^2)/c)*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(52) = 104.

time = 0.43, size = 186, normalized size = 3.00

$$\frac{2\,ac^3x^3 + bc^2x^2\sqrt{\frac{c^2x^2+1}{c^2x^2}} + 2\,bc^3\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - 2\,bc^3\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx - 1\right) + b\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) + 2\left(bc^3x^3 - bc^3\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*x^3 + b*c^2*x^2*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*b*c^3*log(c*
x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - 2*b*c^3*log(c*x*sqrt((c^2*x^2
+ 1)/(c^2*x^2)) - c*x - 1) + b*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x)
 + 2*(b*c^3*x^3 - b*c^3)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))
)/c^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a + b\,\mathrm{acsch}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^2\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x^2*(a + b*asinh(1/(c*x))), x)
```

3.6 $\int x\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=38

$$\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c}+\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)$$

```
[Out] 1/2*x^2*(a+b*arccsch(c*x))+1/2*b*x*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {6419, 197}

$$\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx\sqrt{\frac{1}{c^2x^2}+1}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*c) + (x^2*(a + b*ArcCsch[c*x]))/2
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(a+b\text{csch}^{-1}(cx)\right)\,dx&=\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)+\frac{b\int\frac{1}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{2c}\\&=\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c}+\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 50, normalized size = 1.32

$$\frac{ax^2}{2}+\frac{bx\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{2c}+\frac{1}{2}bx^2\operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCsch[c*x]),x]
[Out] (a*x^2)/2 + (b*x*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(2*c) + (b*x^2*ArcCsch[c*x]
)/2
```

Maple [A]
time = 0.21, size = 65, normalized size = 1.71

method	result	size
derivativedivides	$\frac{\frac{a\,c^2x^2}{2}+b\left(\frac{c^2x^2\operatorname{arccsch}(cx)}{2}+\frac{c^2x^2+1}{2\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^2}$	65
default	$\frac{\frac{a\,c^2x^2}{2}+b\left(\frac{c^2x^2\operatorname{arccsch}(cx)}{2}+\frac{c^2x^2+1}{2\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c^2}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/c^2*(1/2*a*c^2*x^2+b*(1/2*c^2*x^2*arccsch(c*x)+1/2/((c^2*x^2+1)/c^2/x^2)^
(1/2)/c/x*(c^2*x^2+1)))
```

Maxima [A]
time = 0.26, size = 35, normalized size = 0.92

$$\frac{1}{2}\,ax^2+\frac{1}{2}\left(x^2\operatorname{arcsch}\left(cx\right)+\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] 1/2*a*x^2 + 1/2*(x^2*arccsch(c*x) + x*sqrt(1/(c^2*x^2) + 1)/c)*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(32) = 64$.
time = 0.36, size = 70, normalized size = 1.84

$$\frac{bcx^2 \log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + acx^2 + bx\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] 1/2*(b*c*x^2*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + a*c*x^2 +
 b*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b\,\mathrm{acsch}\,(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x)),x)
[Out] Integral(x*(a + b*acsch(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x, x)
```

Mupad [B]
time = 2.22, size = 39, normalized size = 1.03

$$\frac{a\,x^2}{2} + \frac{b\,x^2\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{2} + \frac{b\,x\,\sqrt{\frac{1}{c^2\,x^2}+1}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(1/(c*x))),x)
[Out] (a*x^2)/2 + (b*x^2*asinh(1/(c*x)))/2 + (b*x*(1/(c^2*x^2) + 1)^(1/2))/(2*c)
```

3.7 $\int \left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=30

$$ax+bx\text{csch}^{-1}(cx)+\frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}$$

```
[Out] a*x+b*x*arccsch(c*x)+b*arctanh((1+1/c^2/x^2)^(1/2))/c
```

Rubi [A]
time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6413, 272, 65, 214}

$$ax+\frac{b\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c}+bx\text{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCsch[c*x],x]
```

```
[Out] a*x + b*x*ArcCsch[c*x] + (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/c
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6413

```
Int[ArcCsch[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsch[c*x], x] + Dist[1/c, I
nt[1/(x*Sqrt[1 + 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\int \left(a+b\text{csch}^{-1}(cx)\right)\,dx = ax+b\int \text{csch}^{-1}(cx)\,dx$$

$$= ax+bx\text{csch}^{-1}(cx)+\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x}\,dx}{c}$$

$$= ax+bx\text{csch}^{-1}(cx)-\frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{2c}$$

$$= ax+bx\text{csch}^{-1}(cx)-(bc)\text{Subst}\left(\int \frac{1}{-c^2+c^2x^2}\,dx,x,\sqrt{1+\frac{1}{c^2x^2}}\right)$$

$$= ax+bx\text{csch}^{-1}(cx)+\frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(30) = 60.
time = 0.09, size = 78, normalized size = 2.60

$$ax+bx\text{csch}^{-1}(cx)-\frac{b\sqrt{1+c^2x^2}\,\log\left(-\sqrt{c^2}\,x+\sqrt{1+c^2x^2}\right)}{c\sqrt{c^2}\,\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCsch[c*x],x]
```

```
[Out] a*x + b*x*ArcCsch[c*x] - (b*Sqrt[1 + c^2*x^2]*Log[-(Sqrt[c^2]*x) + Sqrt[1 +
 c^2*x^2]])/(c*Sqrt[c^2]*Sqrt[1 + 1/(c^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 36, normalized size = 1.20

method	result	size

default	$ax+bx\,\mathrm{arccsch}(cx)+\frac{b\ln\left(cx+cx\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}$	36
derivativedivides	$\frac{acx+\mathrm{arccsch}(cx)bcx+\ln\left(cx+cx\sqrt{1+\frac{1}{c^2x^2}}\right)b}{c}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccsch(c*x),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*x*arccsch(c*x)+b/c*ln(c*x+c*x*(1+1/c^2/x^2)^(1/2))
```

Maxima [A]
time = 0.26, size = 49, normalized size = 1.63

$$ax+\frac{\left(2\,cx\,\mathrm{arcsch}\,(cx)+\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)-\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsch(c*x),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1
/(c^2*x^2) + 1) - 1))*b/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(28) = 56.
time = 0.47, size = 143, normalized size = 4.77

$$\frac{acx+bc\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-bc\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)-b\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx\right)+(bcx-bc)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsch(c*x),x, algorithm="fricas")
```

```
[Out] (a*c*x + b*c*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - b*c*log(c*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) - b*log(c*x*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) - c*x) + (b*c*x - b*c)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)
/(c*x)))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\,\mathrm{acsch}\,(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acsch(c*x),x)
```

```
[Out] Integral(a + b*acsch(c*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsch(c*x),x, algorithm="giac")
```

```
[Out] integrate(b*arccsch(c*x) + a, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int a + b \operatorname{asinh}\left(\frac{1}{c\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*asinh(1/(c*x)),x)
```

```
[Out] int(a + b*asinh(1/(c*x)), x)
```

3.8 $\int \frac{a+b\text{csch}^{-1}(cx)}{x} \, dx$

Optimal. Leaf size=56

$$-\frac{(a+b\text{csch}^{-1}(cx))^2}{2b}-(a+b\text{csch}^{-1}(cx))\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right)+\frac{1}{2}b\text{PolyLog}\left(2,e^{-2\text{csch}^{-1}(cx)}\right)$$

```
[Out] -1/2*(a+b*arccsch(c*x))^2/b-(a+b*arccsch(c*x))*ln(1-1/(1/c/x+(1+1/c^2/x^2)^
(1/2))^2)+1/2*b*polylog(2,1/(1/c/x+(1+1/c^2/x^2)^(1/2))^2)
```

Rubi [A]
time = 0.10, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6417, 5775, 3797, 2221, 2317, 2438}

$$-\frac{(a+b\text{csch}^{-1}(cx))^2}{2b}-\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right)(a+b\text{csch}^{-1}(cx))+\frac{1}{2}b\text{Li}_2\left(e^{-2\text{csch}^{-1}(cx)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/x,x]
```

```
[Out] -1/2*(a + b*ArcCsch[c*x])^2/b - (a + b*ArcCsch[c*x])*Log[1 - E^(-2*ArcCsch[
c*x])] + (b*PolyLog[2, E^(-2*ArcCsch[c*x])])/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
```

```
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 6417

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> -Subst[Int[(a +
b*ArcSinh[x/c])/x, x], x, 1/x] /; FreeQ[{a, b, c}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\text{csch}^{-1}(cx)}{x}\,dx &= -\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)\\
&= -\text{Subst}\left(\int (a+bx)\coth(x)\,dx, x, \text{csch}^{-1}(cx)\right)\\
&= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2b} + 2\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1-e^{2x}}\,dx, x, \text{csch}^{-1}(cx)\right)\\
&= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2b} - \left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) + b\text{Subst}\left(\int \log(1-\right.\\
&= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2b} - \left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) + \frac{1}{2}b\text{Subst}\left(\int \frac{\log(1}{x}\right.\\
&= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2b} - \left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) - \frac{1}{2}b\text{Li}_2\left(e^{2\text{csch}^{-1}(cx)}\right)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 47, normalized size = 0.84

$$a\log(x) + \frac{1}{2}b\left(-\text{csch}^{-1}(cx)\left(\text{csch}^{-1}(cx) + 2\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right)\right) + \text{PolyLog}\left(2, e^{-2\text{csch}^{-1}(cx)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/x,x]
```

```
[Out] a*Log[x] + (b*(-(ArcCsch[c*x]*(ArcCsch[c*x] + 2*Log[1 - E^(-2*ArcCsch[c*x])
])) + PolyLog[2, E^(-2*ArcCsch[c*x])]))/2
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccsch}(cx)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x,x)
```

```
[Out] int((a+b*arccsch(c*x))/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/2*(4*c^2*integrate(x^2*log(x)/(c^2*x^3 + x), x) - 2*c^2*integrate(x*log(
x)/(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x) - (log(c^2*x^2 + 1) - 2*log(x))*
log(c) + log(c^2*x^2 + 1)*log(c) - 2*log(x)*log(sqrt(c^2*x^2 + 1) + 1) + 2*
integrate(log(x)/(c^2*x^3 + x), x))*b + a*log(x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x,x)
```

```
[Out] Integral((a + b*acsch(c*x))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/x,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/x, x)
```

3.9 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^2}\,dx$

Optimal. Leaf size=30

$$bc\sqrt{1+\frac{1}{c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{x}$$

```
[Out] (-a-b*arccsch(c*x))/x+b*c*(1+1/c^2/x^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6419, 267}

$$bc\sqrt{\frac{1}{c^2x^2}+1}-\frac{a+b\text{csch}^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/x^2,x]
```

```
[Out] b*c*Sqrt[1 + 1/(c^2*x^2)] - (a + b*ArcCsch[c*x])/x
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\text{csch}^{-1}(cx)}{x^2}\,dx &= -\frac{a+b\text{csch}^{-1}(cx)}{x}-\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x^3}\,dx}{c}\\ &= bc\sqrt{1+\frac{1}{c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{x}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 1.33

$$-\frac{a}{x}+bc\sqrt{\frac{1+c^2x^2}{c^2x^2}}-\frac{b\operatorname{csch}^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/x^2,x]
```

```
[Out] -(a/x) + b*c*Sqrt[(1 + c^2*x^2)/(c^2*x^2)] - (b*ArcCsch[c*x])/x
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(30) = 60.
time = 0.24, size = 62, normalized size = 2.07

method	result	size
derivativedivides	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arccsch}(cx)}{cx}+\frac{c^2x^2+1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^2x^2}\right)\right)$	62
default	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arccsch}(cx)}{cx}+\frac{c^2x^2+1}{\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^2x^2}\right)\right)$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(-a/c/x+b*(-1/c/x*arccsch(c*x)+1/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^2/x^2*(c^2
*x^2+1)))
```

Maxima [A]
time = 0.34, size = 32, normalized size = 1.07

$$\left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsch}(cx)}{x}\right)b-\frac{a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] (c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b - a/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(28) = 56.

time = 0.52, size = 64, normalized size = 2.13

$$\frac{bcx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - b\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) - a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] (b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - b*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x
^2)) + 1)/(c*x)) - a)/x
```

Sympy [A]
time = 0.44, size = 36, normalized size = 1.20

$$\begin{cases} -\frac{a}{x} + bc\sqrt{1+\frac{1}{c^2x^2}} - \frac{b\operatorname{acsch}(cx)}{x} & \text{for } c \neq 0 \\ -\frac{a+\tilde{\infty}b}{x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**2,x)
```

```
[Out] Piecewise((-a/x + b*c*sqrt(1 + 1/(c**2*x**2)) - b*acsch(c*x)/x, Ne(c, 0)),
(-(a + zoo*b)/x, True))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/x^2, x)
```

Mupad [B]
time = 2.32, size = 35, normalized size = 1.17

$$b\,c\,\sqrt{\frac{1}{c^2\,x^2}+1} - \frac{a}{x} - \frac{b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/x^2,x)
```

```
[Out] b*c*(1/(c^2*x^2) + 1)^(1/2) - a/x - (b*asinh(1/(c*x)))/x
```

3.10 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^3} dx$

Optimal. Leaf size=50

$$\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2\text{csch}^{-1}(cx) - \frac{a+b\text{csch}^{-1}(cx)}{2x^2}$$

```
[Out] -1/4*b*c^2*arccsch(c*x)+1/2*(-a-b*arccsch(c*x))/x^2+1/4*b*c*(1+1/c^2/x^2)^(
1/2)/x
```

Rubi [A]
time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6419, 342, 327, 221}

$$-\frac{a+b\text{csch}^{-1}(cx)}{2x^2} + \frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{4x} - \frac{1}{4}bc^2\text{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/x^3,x]
```

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/(4*x) - (b*c^2*ArcCsch[c*x])/4 - (a + b*ArcCsch
[c*x])/(2*x^2)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3}\,dx = -\frac{a+b\text{csch}^{-1}(cx)}{2x^2} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x^4}\,dx}{2c}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{2x^2} + \frac{b\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2c}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{a+b\text{csch}^{-1}(cx)}{2x^2} - \frac{1}{4}(bc)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2\text{csch}^{-1}(cx) - \frac{a+b\text{csch}^{-1}(cx)}{2x^2}$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 1.32

$$-\frac{a}{2x^2} + \frac{bc\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{4x} - \frac{b\text{csch}^{-1}(cx)}{2x^2} - \frac{1}{4}bc^2\sinh^{-1}\left(\frac{1}{cx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/x^3,x]
```

```
[Out] -1/2*a/x^2 + (b*c*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(4*x) - (b*ArcCsch[c*x])/(
2*x^2) - (b*c^2*ArcSinh[1/(c*x)])/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(45) = 90$.
time = 0.22, size = 99, normalized size = 1.98

method	result
derivativedivides	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\operatorname{arccsch}(cx)}{2c^2x^2}+\frac{\sqrt{c^2x^2+1}\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^2x^2+\sqrt{c^2x^2+1}\right)}{4\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^3x^3}\right)\right)$
default	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\operatorname{arccsch}(cx)}{2c^2x^2}+\frac{\sqrt{c^2x^2+1}\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^2x^2+\sqrt{c^2x^2+1}\right)}{4\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^3x^3}\right)\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] c^2*(-1/2*a/c^2/x^2+b*(-1/2/c^2/x^2*arccsch(c*x)+1/4*(c^2*x^2+1)^(1/2)*(-ar
ctanh(1/(c^2*x^2+1)^(1/2))*c^2*x^2+(c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)
^(1/2)/c^3/x^3))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(42) = 84$.
time = 0.26, size = 105, normalized size = 2.10

$$\frac{1}{8}\,b\left(\frac{\frac{2\,c^4x\sqrt{\frac{1}{c^2x^2}+1}}{c^2x^2\left(\frac{1}{c^2x^2}+1\right)-1}-c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}+1\right)+c^3\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c}-\frac{4\,\operatorname{arcsch}\left(cx\right)}{x^2}\right)-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*b*((2*c^4*x*sqrt(1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) - c^3
*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) - 1
))/c - 4*arccsch(c*x)/x^2) - 1/2*a/x^2
```

Fricas [A]
time = 0.41, size = 76, normalized size = 1.52

$$\frac{bcx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-\left(bc^2x^2+2\,b\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-2\,a}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - (b*c^2*x^2 + 2*b)*log((c*x*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 2*a)/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/x^3, x)
```

Mupad [B]
time = 2.27, size = 51, normalized size = 1.02

$$\frac{b\,c\,\sqrt{\frac{1}{c^2\,x^2}+1}}{4\,x}-\frac{b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\,\left(\frac{c^2\,x}{4}+\frac{1}{2\,x}\right)}{x}-\frac{a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/x^3,x)
```

```
[Out] (b*c*(1/(c^2*x^2) + 1)^(1/2))/(4*x) - (b*asinh(1/(c*x))*((c^2*x)/4 + 1/(2*x
)))/x - a/(2*x^2)
```

3.11 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^4} dx$

Optimal. Leaf size=58

$$-\frac{1}{3}bc^3\sqrt{1+\frac{1}{c^2x^2}}+\frac{1}{9}bc^3\left(1+\frac{1}{c^2x^2}\right)^{3/2}-\frac{a+b\text{csch}^{-1}(cx)}{3x^3}$$

```
[Out] 1/9*b*c^3*(1+1/c^2/x^2)^(3/2)+1/3*(-a-b*arccsch(c*x))/x^3-1/3*b*c^3*(1+1/c^
2/x^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6419, 272, 45}

$$-\frac{a+b\text{csch}^{-1}(cx)}{3x^3}+\frac{1}{9}bc^3\left(\frac{1}{c^2x^2}+1\right)^{3/2}-\frac{1}{3}bc^3\sqrt{\frac{1}{c^2x^2}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/x^4,x]
```

```
[Out] -1/3*(b*c^3*Sqrt[1 + 1/(c^2*x^2)]) + (b*c^3*(1 + 1/(c^2*x^2))^(3/2))/9 - (a
 + b*ArcCsch[c*x])/(3*x^3)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^4}\,dx = -\frac{a+b\operatorname{csch}^{-1}(cx)}{3x^3} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x^5}\,dx}{3c}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{3x^3} + \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1+\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{6c}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{3x^3} + \frac{b\text{Subst}\left(\int \left(-\frac{c^2}{\sqrt{1+\frac{x}{c^2}}} + c^2\sqrt{1+\frac{x}{c^2}}\right)dx, x, \frac{1}{x^2}\right)}{6c}$$

$$= -\frac{1}{3}bc^3\sqrt{1+\frac{1}{c^2x^2}} + \frac{1}{9}bc^3\left(1+\frac{1}{c^2x^2}\right)^{3/2} - \frac{a+b\operatorname{csch}^{-1}(cx)}{3x^3}$$

Mathematica [A]
time = 0.04, size = 59, normalized size = 1.02

$$-\frac{a}{3x^3} + b\left(-\frac{2c^3}{9} + \frac{c}{9x^2}\right)\sqrt{\frac{1+c^2x^2}{c^2x^2}} - \frac{b\operatorname{csch}^{-1}(cx)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/x^4,x]
[Out] -1/3*a/x^3 + b*((-2*c^3)/9 + c/(9*x^2))*Sqrt[(1 + c^2*x^2)/(c^2*x^2)] - (b*
ArcCsch[c*x])/(3*x^3)
```

Maple [A]
time = 0.21, size = 75, normalized size = 1.29

method	result	size
derivativedivides	$c^3\left(-\frac{a}{3c^3x^3} + b\left(-\frac{\operatorname{arccsch}(cx)}{3c^3x^3} - \frac{(c^2x^2+1)(2c^2x^2-1)}{9\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^4x^4}\right)\right)$	75
default	$c^3\left(-\frac{a}{3c^3x^3} + b\left(-\frac{\operatorname{arccsch}(cx)}{3c^3x^3} - \frac{(c^2x^2+1)(2c^2x^2-1)}{9\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^4x^4}\right)\right)$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(-1/3*a/c^3/x^3+b*(-1/3/c^3/x^3*arccsch(c*x)-1/9*(c^2*x^2+1)*(2*c^2*x^2
-1)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^4/x^4))
```

Maxima [A]
time = 0.26, size = 56, normalized size = 0.97

$$\frac{1}{9}\,b\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\mathrm{arcsch}\left(cx\right)}{x^3}\right)-\frac{a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*b*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*ar
ccsch(c*x)/x^3) - 1/3*a/x^3
```

Fricas [A]
time = 0.46, size = 77, normalized size = 1.33

$$-\frac{3\,b\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(2\,bc^3x^3-bcx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}+3\,a}{9\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(3*b*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (2*b*c^3*x^3
 - b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 3*a)/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{acsch}\left(cx\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acsch(c*x))/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/x^4,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/x^4, x)
```

3.12 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^5} dx$

Optimal. Leaf size=74

$$\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16x^3}-\frac{3bc^3\sqrt{1+\frac{1}{c^2x^2}}}{32x}+\frac{3}{32}bc^4\text{csch}^{-1}(cx)-\frac{a+b\text{csch}^{-1}(cx)}{4x^4}$$

```
[Out] 3/32*b*c^4*arccsch(c*x)+1/4*(-a-b*arccsch(c*x))/x^4+1/16*b*c*(1+1/c^2/x^2)^
(1/2)/x^3-3/32*b*c^3*(1+1/c^2/x^2)^(1/2)/x
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6419, 342, 327, 221}

$$-\frac{a+b\text{csch}^{-1}(cx)}{4x^4}+\frac{3}{32}bc^4\text{csch}^{-1}(cx)+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{16x^3}-\frac{3bc^3\sqrt{\frac{1}{c^2x^2}+1}}{32x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/x^5,x]
```

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/(16*x^3) - (3*b*c^3*Sqrt[1 + 1/(c^2*x^2)])/(32*
x) + (3*b*c^4*ArcCsch[c*x])/32 - (a + b*ArcCsch[c*x])/(4*x^4)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5}\,dx = -\frac{a+b\text{csch}^{-1}(cx)}{4x^4} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x^6}\,dx}{4c}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{4x^4} + \frac{b\text{Subst}\left(\int \frac{x^4}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{4c}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16x^3} - \frac{a+b\text{csch}^{-1}(cx)}{4x^4} - \frac{1}{16}(3bc)\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16x^3} - \frac{3bc^3\sqrt{1+\frac{1}{c^2x^2}}}{32x} - \frac{a+b\text{csch}^{-1}(cx)}{4x^4} + \frac{1}{32}\left(3bc^3\right)\text{Subst}\left(\int \frac{}{\sqrt{1-}}\right.$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16x^3} - \frac{3bc^3\sqrt{1+\frac{1}{c^2x^2}}}{32x} + \frac{3}{32}bc^4\text{csch}^{-1}(cx) - \frac{a+b\text{csch}^{-1}(cx)}{4x^4}$$

Mathematica [**A**]
time = 0.03, size = 78, normalized size = 1.05

$$-\frac{a}{4x^4} + b\left(\frac{c}{16x^3} - \frac{3c^3}{32x}\right)\sqrt{\frac{1+c^2x^2}{c^2x^2}} - \frac{b\text{csch}^{-1}(cx)}{4x^4} + \frac{3}{32}bc^4\sinh^{-1}\left(\frac{1}{cx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/x^5,x]
```

```
[Out] -1/4*a/x^4 + b*(c/(16*x^3) - (3*c^3)/(32*x))*Sqrt[(1 + c^2*x^2)/(c^2*x^2)]
- (b*ArcCsch[c*x])/(4*x^4) + (3*b*c^4*ArcSinh[1/(c*x)])/32
```

Maple [**A**]
time = 0.21, size = 120, normalized size = 1.62

method	result
derivativedivides	$c^4\left(-\frac{a}{4c^4x^4}+b\left(-\frac{\operatorname{arccsch}(cx)}{4c^4x^4}-\frac{\sqrt{c^2x^2+1}\,\left(-3\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^4x^4+3\sqrt{c^2x^2+1}\,c^2x^2-\right.}{32\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^5x^5}\right.\right.$
default	$c^4\left(-\frac{a}{4c^4x^4}+b\left(-\frac{\operatorname{arccsch}(cx)}{4c^4x^4}-\frac{\sqrt{c^2x^2+1}\,\left(-3\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)c^4x^4+3\sqrt{c^2x^2+1}\,c^2x^2-\right.}{32\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^5x^5}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] c^4*(-1/4*a/c^4/x^4+b*(-1/4/c^4/x^4*arccsch(c*x)-1/32*(c^2*x^2+1)^(1/2)*(-3
*arctanh(1/(c^2*x^2+1)^(1/2))*c^4*x^4+3*(c^2*x^2+1)^(1/2)*c^2*x^2-2*(c^2*x^
2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^5/x^5))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(62) = 124$.
time = 0.26, size = 147, normalized size = 1.99

$$\frac{1}{64}\,b\left(\frac{3\,c^5\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}+1\right)-3\,c^5\log\left(cx\sqrt{\frac{1}{c^2x^2}+1}-1\right)-\frac{2\left(3\,c^8x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\,c^6x\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4x^4\left(\frac{1}{c^2x^2}+1\right)^2-2\,c^2x^2\left(\frac{1}{c^2x^2}+1\right)+1}}{c}-\frac{16\,\operatorname{arcsch}(cx)}{x^4}\right)-\frac{a}{4\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^5,x, algorithm="maxima")
```

```
[Out] 1/64*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) - 3*c^5*log(c*x*sqrt(1/(c
^2*x^2) + 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) + 1)^(3/2) - 5*c^6*x*sqrt(1/(
c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) + 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) + 1) +
 1))/c - 16*arccsch(c*x)/x^4) - 1/4*a/x^4
```

Fricas [A]
time = 0.45, size = 89, normalized size = 1.20

$$\frac{(3\,bc^4x^4-8\,b)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-(3\,bc^3x^3-2\,bcx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}-8\,a}{32\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^5,x, algorithm="fricas")
```

```
[Out] 1/32*((3*b*c^4*x^4 - 8*b)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)
) - (3*b*c^3*x^3 - 2*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 8*a)/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**5,x)
```

```
[Out] Integral((a + b*acsch(c*x))/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b \operatorname{asinh}\left(\frac{1}{c\,x}\right)}{x^5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/x^5,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/x^5, x)
```

3.13 $\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x^6}\,dx$

Optimal. Leaf size=79

$$\frac{1}{5}bc^5\sqrt{1+\frac{1}{c^2x^2}}-\frac{2}{15}bc^5\left(1+\frac{1}{c^2x^2}\right)^{3/2}+\frac{1}{25}bc^5\left(1+\frac{1}{c^2x^2}\right)^{5/2}-\frac{a+b\,\mathrm{csch}^{-1}(cx)}{5x^5}$$

```
[Out] -2/15*b*c^5*(1+1/c^2/x^2)^(3/2)+1/25*b*c^5*(1+1/c^2/x^2)^(5/2)+1/5*(-a-b*ar
ccsch(c*x))/x^5+1/5*b*c^5*(1+1/c^2/x^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6419, 272, 45}

$$-\frac{a+b\,\mathrm{csch}^{-1}(cx)}{5x^5}+\frac{1}{25}bc^5\left(\frac{1}{c^2x^2}+1\right)^{5/2}-\frac{2}{15}bc^5\left(\frac{1}{c^2x^2}+1\right)^{3/2}+\frac{1}{5}bc^5\sqrt{\frac{1}{c^2x^2}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/x^6,x]
```

```
[Out] (b*c^5*Sqrt[1 + 1/(c^2*x^2)])/5 - (2*b*c^5*(1 + 1/(c^2*x^2))^(3/2))/15 + (b
*c^5*(1 + 1/(c^2*x^2))^(5/2))/25 - (a + b*ArcCsch[c*x])/(5*x^5)
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^6}\,dx = -\frac{a+b\text{csch}^{-1}(cx)}{5x^5} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x^7}\,dx}{5c}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{5x^5} + \frac{b\text{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x}{c^2}}}\,dx, x, \frac{1}{x^2}\right)}{10c}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{5x^5} + \frac{b\text{Subst}\left(\int \left(\frac{c^4}{\sqrt{1+\frac{x}{c^2}}} - 2c^4\sqrt{1+\frac{x}{c^2}} + c^4\left(1+\frac{x}{c^2}\right)^{3/2}\right) dx$$

$$= \frac{1}{5}bc^5\sqrt{1+\frac{1}{c^2x^2}} - \frac{2}{15}bc^5\left(1+\frac{1}{c^2x^2}\right)^{3/2} + \frac{1}{25}bc^5\left(1+\frac{1}{c^2x^2}\right)^{5/2} - \frac{a+b\text{csch}^{-1}(cx}{5x^5}$$

Mathematica [**A**]
time = 0.04, size = 69, normalized size = 0.87

$$-\frac{a}{5x^5} + b\left(\frac{8c^5}{75} + \frac{c}{25x^4} - \frac{4c^3}{75x^2}\right)\sqrt{\frac{1+c^2x^2}{c^2x^2}} - \frac{b\text{csch}^{-1}(cx)}{5x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/x^6,x]
```

```
[Out] -1/5*a/x^5 + b*((8*c^5)/75 + c/(25*x^4) - (4*c^3)/(75*x^2))*Sqrt[(1 + c^2*x
^2)/(c^2*x^2)] - (b*ArcCsch[c*x])/(5*x^5)
```

Maple [**A**]
time = 0.20, size = 83, normalized size = 1.05

method	result	size
derivativedivides	$c^5\left(-\frac{a}{5c^5x^5} + b\left(-\frac{\text{arccsch}(cx)}{5c^5x^5} + \frac{(c^2x^2+1)(8c^4x^4-4c^2x^2+3)}{75\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^6x^6}\right)\right)$	83
default	$c^5\left(-\frac{a}{5c^5x^5} + b\left(-\frac{\text{arccsch}(cx)}{5c^5x^5} + \frac{(c^2x^2+1)(8c^4x^4-4c^2x^2+3)}{75\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^6x^6}\right)\right)$	83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(-1/5*a/c^5/x^5+b*(-1/5/c^5/x^5*arccsch(c*x)+1/75*(c^2*x^2+1)*(8*c^4*x^
4-4*c^2*x^2+3)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^6/x^6))
```

Maxima [A]
time = 0.25, size = 73, normalized size = 0.92

$$\frac{1}{75}\,b\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{15\,\operatorname{arcsch}\left(cx\right)}{x^5}\right)-\frac{a}{5\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] 1/75*b*((3*c^6*(1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(1/(c^2*x^2) + 1)^(3/2) + 1
5*c^6*sqrt(1/(c^2*x^2) + 1))/c - 15*arccsch(c*x)/x^5) - 1/5*a/x^5
```

Fricas [A]
time = 0.52, size = 87, normalized size = 1.10

$$-\frac{15\,b\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-\left(8\,bc^5x^5-4\,bc^3x^3+3\,bcx\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}+15\,a}{75\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/75*(15*b*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - (8*b*c^5*x
^5 - 4*b*c^3*x^3 + 3*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 15*a)/x^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\operatorname{acsch}\left(cx\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**6,x)
```

```
[Out] Integral((a + b*acsch(c*x))/x**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/x^6,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/x^6, x)
```

3.14 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^7} \, dx$

Optimal. Leaf size=98

$$\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{36x^5}-\frac{5bc^3\sqrt{1+\frac{1}{c^2x^2}}}{144x^3}+\frac{5bc^5\sqrt{1+\frac{1}{c^2x^2}}}{96x}-\frac{5}{96}bc^6\text{csch}^{-1}(cx)-\frac{a+b\text{csch}^{-1}(cx)}{6x^6}$$

```
[Out] -5/96*b*c^6*arccsch(c*x)+1/6*(-a-b*arccsch(c*x))/x^6+1/36*b*c*(1+1/c^2/x^2)
^(1/2)/x^5-5/144*b*c^3*(1+1/c^2/x^2)^(1/2)/x^3+5/96*b*c^5*(1+1/c^2/x^2)^(1/
2)/x
```

Rubi [A]

time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6419, 342, 327, 221}

$$-\frac{a+b\text{csch}^{-1}(cx)}{6x^6}-\frac{5}{96}bc^6\text{csch}^{-1}(cx)+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{36x^5}+\frac{5bc^5\sqrt{\frac{1}{c^2x^2}+1}}{96x}-\frac{5bc^3\sqrt{\frac{1}{c^2x^2}+1}}{144x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/x^7,x]
```

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/(36*x^5) - (5*b*c^3*Sqrt[1 + 1/(c^2*x^2)])/(144
*x^3) + (5*b*c^5*Sqrt[1 + 1/(c^2*x^2)])/(96*x) - (5*b*c^6*ArcCsch[c*x])/96
- (a + b*ArcCsch[c*x])/(6*x^6)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 342

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^7}\,dx = -\frac{a+b\text{csch}^{-1}(cx)}{6x^6} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x^8}\,dx}{6c}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{6x^6} + \frac{b\text{Subst}\left(\int \frac{x^6}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{6c}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{36x^5} - \frac{a+b\text{csch}^{-1}(cx)}{6x^6} - \frac{1}{36}(5bc)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^3\sqrt{1+\frac{1}{c^2x^2}}}{144x^3} - \frac{a+b\text{csch}^{-1}(cx)}{6x^6} + \frac{1}{48}\left(5bc^3\right)\text{Subst}\left(\int \frac{x}{\sqrt{1+}}\right.$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^3\sqrt{1+\frac{1}{c^2x^2}}}{144x^3} + \frac{5bc^5\sqrt{1+\frac{1}{c^2x^2}}}{96x} - \frac{a+b\text{csch}^{-1}(cx)}{6x^6} - \frac{1}{96}(5$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{36x^5} - \frac{5bc^3\sqrt{1+\frac{1}{c^2x^2}}}{144x^3} + \frac{5bc^5\sqrt{1+\frac{1}{c^2x^2}}}{96x} - \frac{5}{96}bc^6\text{csch}^{-1}(cx) - \frac{a+b}{}$$

Mathematica [A]
time = 0.05, size = 88, normalized size = 0.90

$$-\frac{a}{6x^6} + b\left(\frac{c}{36x^5} - \frac{5c^3}{144x^3} + \frac{5c^5}{96x}\right)\sqrt{\frac{1+c^2x^2}{c^2x^2}} - \frac{b\text{csch}^{-1}(cx)}{6x^6} - \frac{5}{96}bc^6\sinh^{-1}\left(\frac{1}{cx}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/x^7,x]
```

```
[Out] -1/6*a/x^6 + b*(c/(36*x^5) - (5*c^3)/(144*x^3) + (5*c^5)/(96*x))*Sqrt[(1 +
c^2*x^2)/(c^2*x^2)] - (b*ArcCsch[c*x])/(6*x^6) - (5*b*c^6*ArcSinh[1/(c*x)])
/96
```

Maple [A]
time = 0.20, size = 139, normalized size = 1.42

method	result
derivativedivides	$c^{6}\left(-\frac{a}{6c^{6}x^{6}}+b\left(-\frac{\operatorname{arccsch}(cx)}{6c^{6}x^{6}}+\frac{\sqrt{c^{2}x^{2}+1}\left(-15\operatorname{arctanh}\left(\frac{1}{\sqrt{c^{2}x^{2}+1}}\right)c^{6}x^{6}+15\sqrt{c^{2}x^{2}+1}\,c^{4}x^{4}\right.}{288\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}\,c^{7}x^{7}}\right.\right.$
default	$c^{6}\left(-\frac{a}{6c^{6}x^{6}}+b\left(-\frac{\operatorname{arccsch}(cx)}{6c^{6}x^{6}}+\frac{\sqrt{c^{2}x^{2}+1}\left(-15\operatorname{arctanh}\left(\frac{1}{\sqrt{c^{2}x^{2}+1}}\right)c^{6}x^{6}+15\sqrt{c^{2}x^{2}+1}\,c^{4}x^{4}\right.}{288\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}\,c^{7}x^{7}}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] c^6*(-1/6*a/c^6/x^6+b*(-1/6/c^6/x^6*arccsch(c*x)+1/288*(c^2*x^2+1)^(1/2)*(-
15*arctanh(1/(c^2*x^2+1)^(1/2))*c^6*x^6+15*(c^2*x^2+1)^(1/2)*c^4*x^4-10*(c^
2*x^2+1)^(1/2)*c^2*x^2+8*(c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^7
/x^7))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(82) = 164.
time = 0.27, size = 185, normalized size = 1.89

$$-\frac{1}{576}\,b\left(\frac{15\,c^{7}\log\left(cx\sqrt{\frac{1}{c^{2}x^{2}}+1}+1\right)-15\,c^{7}\log\left(cx\sqrt{\frac{1}{c^{2}x^{2}}+1}-1\right)-\frac{2\left(15\,c^{12}x^{5}\left(\frac{1}{c^{2}x^{2}}+1\right)^{\frac{5}{2}}-40\,c^{10}x^{3}\left(\frac{1}{c^{2}x^{2}}+1\right)^{\frac{3}{2}}+33\,c^{8}x\sqrt{\frac{1}{c^{2}x^{2}}+1}\right)}{c^{6}x^{6}\left(\frac{1}{c^{2}x^{2}}+1\right)^{3}-3\,c^{4}x^{4}\left(\frac{1}{c^{2}x^{2}}+1\right)^{2}+3\,c^{2}x^{2}\left(\frac{1}{c^{2}x^{2}}+1\right)-1}}{c}+\frac{96\,\operatorname{arcsch}\left(cx\right)}{x^{6}}\right)-\frac{a}{6\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^7,x, algorithm="maxima")
```

```
[Out] -1/576*b*((15*c^7*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) - 15*c^7*log(c*x*sqrt(
1/(c^2*x^2) + 1) - 1) - 2*(15*c^12*x^5*(1/(c^2*x^2) + 1)^(5/2) - 40*c^10*x^
3*(1/(c^2*x^2) + 1)^(3/2) + 33*c^8*x*sqrt(1/(c^2*x^2) + 1))/(c^6*x^6*(1/(c^
2*x^2) + 1)^3 - 3*c^4*x^4*(1/(c^2*x^2) + 1)^2 + 3*c^2*x^2*(1/(c^2*x^2) + 1)
 - 1))/c + 96*arccsch(c*x)/x^6) - 1/6*a/x^6
```

Fricas [A]
time = 0.42, size = 99, normalized size = 1.01

$$-\frac{3\left(5\,bc^{6}x^{6}+16\,b\right)\log\left(\frac{cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+1}{cx}\right)-\left(15\,bc^{5}x^{5}-10\,bc^{3}x^{3}+8\,bcx\right)\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+48\,a}{288\,x^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^7,x, algorithm="fricas")
```

```
[Out] -1/288*(3*(5*b*c^6*x^6 + 16*b)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/
(c*x)) - (15*b*c^5*x^5 - 10*b*c^3*x^3 + 8*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^
2)) + 48*a)/x^6
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\operatorname{acsch}\left(cx\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**7,x)
```

```
[Out] Integral((a + b*acsch(c*x))/x**7, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^7,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/x^7, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x^{7}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/x^7,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/x^7, x)
```

3.15 $\int x^3 \left(a+b\text{csch}^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=105

$$\frac{b^2x^2}{12c^2}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(a+b\text{csch}^{-1}(cx)\right)}{3c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^3\left(a+b\text{csch}^{-1}(cx)\right)}{6c}+\frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{b^2\log(}{3c^4}$$

```
[Out] 1/12*b^2*x^2/c^2+1/4*x^4*(a+b*arccsch(c*x))^2-1/3*b^2*ln(x)/c^4-1/3*b*x*(a+
b*arccsch(c*x))*(1+1/c^2/x^2)^(1/2)/c^3+1/6*b*x^3*(a+b*arccsch(c*x))*(1+1/c
^2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.08, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6421, 5560, 4270, 4269, 3556}

$$\frac{bx^3\sqrt{\frac{1}{c^2x^2}+1}\,\left(a+b\text{csch}^{-1}(cx)\right)}{6c}-\frac{bx\sqrt{\frac{1}{c^2x^2}+1}\,\left(a+b\text{csch}^{-1}(cx)\right)}{3c^3}+\frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{b^2\log(x)}{3c^4}+\frac{b^2x^2}{12c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (b^2*x^2)/(12*c^2) - (b*Sqrt[1 + 1/(c^2*x^2)]*x*(a + b*ArcCsch[c*x]))/(3*c^
3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^3*(a + b*ArcCsch[c*x]))/(6*c) + (x^4*(a + b
*ArcCsch[c*x])^2)/4 - (b^2*Log[x])/(3*c^4)
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int x^3\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx=-\frac{\text{Subst}\left(\int(a+bx)^2\coth(x)\text{csch}^4(x)\,dx,x,\text{csch}^{-1}(cx)\right)}{c^4}$$
$$=\frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{b\text{Subst}\left(\int(a+bx)\text{csch}^4(x)\,dx,x,\text{csch}^{-1}(cx)\right)}{2c^4}$$
$$=\frac{b^2x^2}{12c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^3\left(a+b\text{csch}^{-1}(cx)\right)}{6c}+\frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{b\text{Subs}}{}$$
$$=\frac{b^2x^2}{12c^2}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(a+b\text{csch}^{-1}(cx)\right)}{3c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^3\left(a+b\text{csch}^{-1}(cx)\right.}{6c}$$
$$=\frac{b^2x^2}{12c^2}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(a+b\text{csch}^{-1}(cx)\right)}{3c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^3\left(a+b\text{csch}^{-1}(cx)\right.}{6c}$$

Mathematica [A]
time = 0.14, size = 122, normalized size = 1.16

$$\frac{cx\left(b^2cx+3a^2c^3x^3+2ab\sqrt{1+\frac{1}{c^2x^2}}\left(-2+c^2x^2\right)\right)+2bcx\left(3ac^3x^3+b\sqrt{1+\frac{1}{c^2x^2}}\left(-2+c^2x^2\right)\right)\text{csch}^{-1}(cx)+3b^2c^4x^4\text{csch}^{-1}(cx)^2-4b^2\log(x)}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (c*x*(b^2*c*x + 3*a^2*c^3*x^3 + 2*a*b*Sqrt[1 + 1/(c^2*x^2)]*(-2 + c^2*x^2))
 + 2*b*c*x*(3*a*c^3*x^3 + b*Sqrt[1 + 1/(c^2*x^2)]*(-2 + c^2*x^2))*ArcCsch[c
*x] + 3*b^2*c^4*x^4*ArcCsch[c*x]^2 - 4*b^2*Log[x])/(12*c^4)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^3(a + b\,\mathrm{arccsch}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))^2,x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*x^4 + 1/6*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) -
3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*a*b + 1/288*(72*x^4*log(sqrt(c^2*x^2 + 1) +
 1)^2 + 1152*c^2*integrate(1/2*x^5*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*
x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 1152*c^2*integrate(1/2*x^5*log(sq
rt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 +
1) + 1), x)*log(c) + 576*c^2*integrate(1/2*sqrt(c^2*x^2 + 1)*x^5*log(x)^2/(
sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 1152*c^2
*integrate(1/2*sqrt(c^2*x^2 + 1)*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqr
t(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 576*c^2*int
egrate(1/2*x^5*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2
 + 1) + 1), x) - 1152*c^2*integrate(1/2*x^5*log(x)*log(sqrt(c^2*x^2 + 1) +
1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 1152
*integrate(1/2*x^3*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x
^2 + 1) + 1), x)*log(c) - 1152*integrate(1/2*x^3*log(sqrt(c^2*x^2 + 1) + 1)
/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) -
 24*(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2) - 12*sqrt(c^2*x^
2 + 1) + 6)*log(c)^2/c^4 - 48*(3*c^2*x^2 - 2*(c^2*x^2 + 1)^(3/2) + 6*sqrt(c
^2*x^2 + 1) - 3*log(c^2*x^2 + 1) + 3)*log(c)^2/c^4 + 144*(c^2*x^2 - 2*sqrt(
c^2*x^2 + 1) + 1)*log(c)^2/c^4 + 144*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1
))*log(c)^2/c^4 - 48*(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2)
 - 12*sqrt(c^2*x^2 + 1) + 6)*log(c)*log(x)/c^4 + 288*(c^2*x^2 - 2*sqrt(c^2*
x^2 + 1) + 1)*log(c)*log(x)/c^4 + 48*(6*c^2*x^2 - 3*(c^2*x^2 + 1)^2 + 4*(c^
2*x^2 + 1)^(3/2) - 12*sqrt(c^2*x^2 + 1) + 6)*log(c)*log(sqrt(c^2*x^2 + 1) +
 1)/c^4 - 288*(c^2*x^2 - 2*sqrt(c^2*x^2 + 1) + 1)*log(c)*log(sqrt(c^2*x^2 +
 1) + 1)/c^4 + 4*(18*c^2*x^2 - 9*(c^2*x^2 + 1)^2 + 16*(c^2*x^2 + 1)^(3/2) -
 96*sqrt(c^2*x^2 + 1) + 66*log(sqrt(c^2*x^2 + 1) + 1) - 30*log(sqrt(c^2*x^2
```

```
 + 1) - 1) + 18)*log(c)/c^4 + 4*(6*c^2*x^2 + 9*(c^2*x^2 + 1)^2 - 28*(c^2*x^
2 + 1)^(3/2) + 132*sqrt(c^2*x^2 + 1) - 132*log(sqrt(c^2*x^2 + 1) + 1) + 6)*
log(c)/c^4 - 144*(c^2*x^2 - 4*sqrt(c^2*x^2 + 1) + 3*log(sqrt(c^2*x^2 + 1) +
 1) - log(sqrt(c^2*x^2 + 1) - 1) + 1)*log(c)/c^4 + 144*(c^2*x^2 - 6*sqrt(c^
2*x^2 + 1) + 6*log(sqrt(c^2*x^2 + 1) + 1) + 1)*log(c)/c^4 + 12*(6*c^2*x^2 -
 3*(c^2*x^2 + 1)^2 + 4*(c^2*x^2 + 1)^(3/2) - 12*sqrt(c^2*x^2 + 1) + 6)*log(
sqrt(c^2*x^2 + 1) + 1)/c^4 + (6*c^2*x^2 + 9*(c^2*x^2 + 1)^2 - 28*(c^2*x^2 +
 1)^(3/2) + 132*sqrt(c^2*x^2 + 1) - 132*log(sqrt(c^2*x^2 + 1) + 1) + 6)/c^4
 + 576*integrate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*
x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 1152*integrate(1/2*sqrt(c^2*x^
2 + 1)*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c
^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 576*integrate(1/2*x^3*log(x)^2/(sqrt(
c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 1152*integrat
e(1/2*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^
2*x^2 + sqrt(c^2*x^2 + 1) + 1), x))*b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(91) = 182$.
time = 0.45, size = 272, normalized size = 2.59

$$\frac{3\,b^{2}c^{4}x^{4}\log\left(\frac{cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+1}{cx}\right)^{2}+3\,a^{2}c^{4}x^{4}+6\,abc^{4}\log\left(cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}-cx+1\right)-6\,abc^{4}\log\left(cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}-cx-1\right)+b^{2}c^{2}x^{2}-4\,b^{2}\log\left(x\right)+2\left(3\,abc^{4}x^{4}-3\,abc^{4}+\left(b^{2}c^{3}x^{3}-2\,b^{2}cx\right)\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}\right)\log\left(\frac{cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+1}{cx}\right)+2\left(abc^{3}x^{3}-2\,abcx\right)\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}}{12\,c^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(3*b^2*c^4*x^4*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 +
3*a^2*c^4*x^4 + 6*a*b*c^4*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1)
- 6*a*b*c^4*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + b^2*c^2*x^2
- 4*b^2*log(x) + 2*(3*a*b*c^4*x^4 - 3*a*b*c^4 + (b^2*c^3*x^3 - 2*b^2*c*x)*s
qrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(
c*x)) + 2*(a*b*c^3*x^3 - 2*a*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}(a+b\,\mathrm{acsch}\,(cx))^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))**2,x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^2*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(1/(c*x)))^2,x)
```

```
[Out] int(x^3*(a + b*asinh(1/(c*x)))^2, x)
```

3.16 $\int x^2\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=122

$$\frac{b^2x}{3c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2\left(a+b\text{csch}^{-1}(cx)\right)}{3c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{2b\left(a+b\text{csch}^{-1}(cx)\right)\tanh^{-1}\left(e^{\text{csch}^{-1}(cx)}\right)}{3c^3}$$

```
[Out] 1/3*b^2*x/c^2+1/3*x^3*(a+b*arccsch(c*x))^2-2/3*b*(a+b*arccsch(c*x))*arctanh
(1/c/x+(1+1/c^2/x^2)^(1/2))/c^3-1/3*b^2*polylog(2,-1/c/x-(1+1/c^2/x^2)^(1/2
))/c^3+1/3*b^2*polylog(2,1/c/x+(1+1/c^2/x^2)^(1/2))/c^3+1/3*b*x^2*(a+b*arcc
sch(c*x))*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.09, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6421, 5560, 4270, 4267, 2317, 2438}

$$-\frac{2b\tanh^{-1}\left(e^{\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)}{3c^3}+\frac{bx^2\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)}{3c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{b^2\text{Li}_2\left(-e^{\text{csch}^{-1}(cx)}\right)}{3c^3}+\frac{b^2\text{Li}_2\left(e^{\text{csch}^{-1}(cx)}\right)}{3c^3}+\frac{b^2x}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (b^2*x)/(3*c^2) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^2*(a + b*ArcCsch[c*x]))/(3*c)
+ (x^3*(a + b*ArcCsch[c*x])^2)/3 - (2*b*(a + b*ArcCsch[c*x])*ArcTanh[E^ArcC
sch[c*x]])/(3*c^3) - (b^2*PolyLog[2, -E^ArcCsch[c*x]])/(3*c^3) + (b^2*PolyL
og[2, E^ArcCsch[c*x]])/(3*c^3)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
```

```
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int x^2\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx=-\frac{\text{Subst}\left(\int(a+bx)^2\coth(x)\text{csch}^3(x)\,dx,x,\text{csch}^{-1}(cx)\right)}{c^3}$$

$$=\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{(2b)\text{Subst}\left(\int(a+bx)\text{csch}^3(x)\,dx,x,\text{csch}^{-1}(cx)\right)}{3c^3}$$

$$=\frac{b^2x}{3c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2\left(a+b\text{csch}^{-1}(cx)\right)}{3c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{b\text{Subst}\left(\right.}{}$$

$$=\frac{b^2x}{3c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2\left(a+b\text{csch}^{-1}(cx)\right)}{3c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{2b\left(a+b\right.}{}$$

$$=\frac{b^2x}{3c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2\left(a+b\text{csch}^{-1}(cx)\right)}{3c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{2b\left(a+b\right.}{}$$

$$=\frac{b^2x}{3c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2\left(a+b\text{csch}^{-1}(cx)\right)}{3c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{2b\left(a+b\right.}{}$$

Mathematica [A]
time = 0.99, size = 224, normalized size = 1.84

$$\frac{b^2cx+abc^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2+a^2c^3x^3+b^2c^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2\operatorname{csch}^{-1}(cx)+2abc^3x^3\operatorname{csch}^{-1}(cx)+b^2c^3x^3\operatorname{csch}^{-1}(cx)^2-\frac{abc\sqrt{1+\frac{1}{c^2x^2}}\,x\tanh^{-1}\left(\frac{cx}{\sqrt{1+c^2x^2}}\right)}{\sqrt{1+c^2x^2}}+b^2\operatorname{csch}^{-1}(cx)\log\left(1-e^{-\operatorname{csch}^{-1}(cx)}\right)-b^2\operatorname{csch}^{-1}(cx)\log\left(1+e^{-\operatorname{csch}^{-1}(cx)}\right)+b^2\operatorname{PolyLog}\left(2,-e^{-\operatorname{csch}^{-1}(cx)}\right)-b^2\operatorname{PolyLog}\left(2,e^{-\operatorname{csch}^{-1}(cx)}\right)}{3c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (b^2*c*x + a*b*c^2*Sqrt[1 + 1/(c^2*x^2)]*x^2 + a^2*c^3*x^3 + b^2*c^2*Sqrt[1
 + 1/(c^2*x^2)]*x^2*ArcCsch[c*x] + 2*a*b*c^3*x^3*ArcCsch[c*x] + b^2*c^3*x^3
*ArcCsch[c*x]^2 - (a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[(c*x)/Sqrt[1 + c^2
*x^2]])/Sqrt[1 + c^2*x^2] + b^2*ArcCsch[c*x]*Log[1 - E^(-ArcCsch[c*x])] - b
^2*ArcCsch[c*x]*Log[1 + E^(-ArcCsch[c*x])] + b^2*PolyLog[2, -E^(-ArcCsch[c*
x])] - b^2*PolyLog[2, E^(-ArcCsch[c*x])])/(3*c^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^2(a+b\operatorname{arccsch}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))^2,x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*x^3 + 1/6*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c
^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*
x^2) + 1) - 1)/c^2)/c)*a*b + 1/3*(x^3*log(sqrt(c^2*x^2 + 1) + 1)^2 - 3*inte
grate(-1/3*(3*c^2*x^4*log(c)^2 + 3*x^2*log(c)^2 + 3*(c^2*x^4 + x^2)*log(x)^
2 + 6*(c^2*x^4*log(c) + x^2*log(c))*log(x) - 2*(3*c^2*x^4*log(c) + 3*x^2*lo
g(c) + 3*(c^2*x^4 + x^2)*log(x) + (c^2*x^4*(3*log(c) + 1) + 3*x^2*log(c) +
3*(c^2*x^4 + x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 + 1) + 1) + 3
*(c^2*x^4*log(c)^2 + x^2*log(c)^2 + (c^2*x^4 + x^2)*log(x)^2 + 2*(c^2*x^4*l
og(c) + x^2*log(c))*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^2 + (c^2*x^2 + 1)^(3/
2) + 1), x))*b^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))^2,x, algorithm="fricas")
[Out] integral(b^2*x^2*arccsch(c*x)^2 + 2*a*b*x^2*arccsch(c*x) + a^2*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a + b\operatorname{acsch}(cx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))**2,x)
[Out] Integral(x**2*(a + b*acsch(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))^2,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^2*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(a + b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(1/(c*x)))^2,x)
[Out] int(x^2*(a + b*asinh(1/(c*x)))^2, x)
```

3.17 $\int x\left(a+b\mathbf{csch}^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=54

$$\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}$$

```
[Out] 1/2*x^2*(a+b*arccsch(c*x))^2+b^2*ln(x)/c^2+b*x*(a+b*arccsch(c*x))*(1+1/c^2/
x^2)^(1/2)/c
```

Rubi [A]
time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6421, 5560, 4269, 3556}

$$\frac{bx\sqrt{\frac{1}{c^2x^2}+1}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (b*Sqrt[1 + 1/(c^2*x^2)]*x*(a + b*ArcCsch[c*x]))/c + (x^2*(a + b*ArcCsch[c*
x])^2)/2 + (b^2*Log[x])/c^2
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
```

```
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)^2\,dx=-\frac{\text{Subst}\left(\int (a+bx)^2\coth(x)\text{csch}^2(x)\,dx,x,\text{csch}^{-1}(cx)\right)}{c^2}$$
$$=\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{b\text{Subst}\left(\int (a+bx)\text{csch}^2(x)\,dx,x,\text{csch}^{-1}(cx)\right)}{c^2}$$
$$=\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(a+b\text{csch}^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{b^2\text{Subst}\left(\int \coth(x\right.}{}$$
$$=\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(a+b\text{csch}^{-1}(cx)\right)}{c}+\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{b^2\log(x)}{c^2}$$

Mathematica [A]
time = 0.08, size = 87, normalized size = 1.61

$$\frac{acx\left(2b\sqrt{1+\frac{1}{c^2x^2}}+acx\right)+2bcx\left(b\sqrt{1+\frac{1}{c^2x^2}}+acx\right)\text{csch}^{-1}(cx)+b^2c^2x^2\text{csch}^{-1}(cx)^2+2b^2\log(cx)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCsch[c*x])^2,x]
[Out] (a*c*x*(2*b*Sqrt[1 + 1/(c^2*x^2)] + a*c*x) + 2*b*c*x*(b*Sqrt[1 + 1/(c^2*x^2
)] + a*c*x)*ArcCsch[c*x] + b^2*c^2*x^2*ArcCsch[c*x]^2 + 2*b^2*Log[c*x])/(2*
c^2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\left(a+b\,\text{arccsch}(cx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))^2,x)
[Out] int(x*(a+b*arccsch(c*x))^2,x)
```

Maxima [A]
time = 0.27, size = 82, normalized size = 1.52

$$\frac{1}{2}\,b^{2}x^{2}\operatorname{arcsch}\left(cx\right)^{2}+\frac{1}{2}\,a^{2}x^{2}+\left(x^{2}\operatorname{arcsch}\left(cx\right)+\frac{x\sqrt{\frac{1}{c^{2}x^{2}}+1}}{c}\right)ab+\left(\frac{x\sqrt{\frac{1}{c^{2}x^{2}}+1}\,\operatorname{arcsch}\left(cx\right)}{c}+\frac{\log\left(x\right)}{c^{2}}\right)b^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*x^2*arccsch(c*x)^2 + 1/2*a^2*x^2 + (x^2*arccsch(c*x) + x*sqrt(1/(c^
2*x^2) + 1)/c)*a*b + (x*sqrt(1/(c^2*x^2) + 1)*arccsch(c*x)/c + log(x)/c^2)*
b^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 234 vs. 2(50) = 100.
time = 0.58, size = 234, normalized size = 4.33

$$\frac{b^{2}c^{2}x^{2}\log\left(\frac{cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+1}{cx}\right)^{2}+a^{2}c^{2}x^{2}+2\,abc^{2}\log\left(cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}-cx+1\right)-2\,abc^{2}\log\left(cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}-cx-1\right)+2\,abcx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+2\,b^{2}\log\left(x\right)+2\left(abc^{2}x^{2}+b^{2}cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}-abc^{2}\right)\log\left(\frac{cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+1}{cx}\right)}{2\,c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*c^2*x^2*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 + a^2
*c^2*x^2 + 2*a*b*c^2*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - 2*a
*b*c^2*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 2*a*b*c*x*sqrt((c
^2*x^2 + 1)/(c^2*x^2)) + 2*b^2*log(x) + 2*(a*b*c^2*x^2 + b^2*c*x*sqrt((c^2*
x^2 + 1)/(c^2*x^2)) - a*b*c^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/
(c*x)))/c^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(a+b\operatorname{acsch}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))**2,x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^2*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(1/(c*x)))^2,x)
```

```
[Out] int(x*(a + b*asinh(1/(c*x)))^2, x)
```

3.18 $\int \left(a+b\text{csch}^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=68

$$x\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{4b\left(a+b\text{csch}^{-1}(cx)\right)\tanh^{-1}\left(e^{\text{csch}^{-1}(cx)}\right)}{c}+\frac{2b^2\text{PolyLog}\left(2,-e^{\text{csch}^{-1}(cx)}\right)}{c}-\frac{2b^2\text{PolyLog}(}{}$$

```
[Out] x*(a+b*arccsch(c*x))^2+4*b*(a+b*arccsch(c*x))*arctanh(1/c/x+(1+1/c^2/x^2)^(
1/2))/c+2*b^2*polylog(2,-1/c/x-(1+1/c^2/x^2)^(1/2))/c-2*b^2*polylog(2,1/c/x
+(1+1/c^2/x^2)^(1/2))/c
```

Rubi [A]
time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6415, 5560, 4267, 2317, 2438}

$$x\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{4b\tanh^{-1}\left(e^{\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)}{c}+\frac{2b^2\text{Li}_2\left(-e^{\text{csch}^{-1}(cx)}\right)}{c}-\frac{2b^2\text{Li}_2\left(e^{\text{csch}^{-1}(cx)}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] x*(a + b*ArcCsch[c*x])^2 + (4*b*(a + b*ArcCsch[c*x])*ArcTanh[E^ArcCsch[c*x]
])/c + (2*b^2*PolyLog[2, -E^ArcCsch[c*x]])/c - (2*b^2*PolyLog[2, E^ArcCsch[
c*x]])/c
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6415

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-c^(-1), Su
bst[Int[(a + b*x)^n*Csch[x]*Coth[x], x], x, ArcCsch[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\text{csch}^{-1}(cx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int (a+bx)^2\coth(x)\text{csch}(x)\,dx,x,\text{csch}^{-1}(cx)\right)}{c}\\ &= x\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{(2b)\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx,x,\text{csch}^{-1}(cx)\right)}{c}\\ &= x\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{4b\left(a+b\text{csch}^{-1}(cx)\right)\tanh^{-1}\left(e^{\text{csch}^{-1}(cx)}\right)}{c}+\frac{(2b^2)\,\text{Subst}(\int\\ &= x\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{4b\left(a+b\text{csch}^{-1}(cx)\right)\tanh^{-1}\left(e^{\text{csch}^{-1}(cx)}\right)}{c}+\frac{(2b^2)\,\text{Subst}\Big(\\ &= x\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{4b\left(a+b\text{csch}^{-1}(cx)\right)\tanh^{-1}\left(e^{\text{csch}^{-1}(cx)}\right)}{c}+\frac{2b^2\text{Li}_2\Big(-e^{\text{csch}}}{c}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 121, normalized size = 1.78

$$\frac{a^2cx+2abcx\text{csch}^{-1}(cx)+b^2cx\text{csch}^{-1}(cx)^2-2b^2\text{csch}^{-1}(cx)\log\left(1-e^{-\text{csch}^{-1}(cx)}\right)+2b^2\text{csch}^{-1}(cx)\log\left(1+e^{-\text{csch}^{-1}(cx)}\right)-2ab\log\left(\tanh\left(\frac{1}{2}\text{csch}^{-1}(cx)\right)\right)-2b^2\text{PolyLog}\left(2,-e^{-\text{csch}^{-1}(cx)}\right)+2b^2\text{PolyLog}\left(2,e^{-\text{csch}^{-1}(cx)}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] (a^2*c*x + 2*a*b*c*x*ArcCsch[c*x] + b^2*c*x*ArcCsch[c*x]^2 - 2*b^2*ArcCsch[
c*x]*Log[1 - E^(-ArcCsch[c*x])] + 2*b^2*ArcCsch[c*x]*Log[1 + E^(-ArcCsch[c*
x])] - 2*a*b*Log[Tanh[ArcCsch[c*x]/2]] - 2*b^2*PolyLog[2, -E^(-ArcCsch[c*x]
)] + 2*b^2*PolyLog[2, E^(-ArcCsch[c*x])])/c
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b\,\text{arccsch}(cx)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^2,x)
```

```
[Out] int((a+b*arccsch(c*x))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

```
[Out] (x*log(sqrt(c^2*x^2 + 1) + 1)^2 - integrate(-(c^2*x^2*log(c)^2 + (c^2*x^2 +
 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c))*log(x) - 2*(c^2*x^2*l
og(c) + (c^2*x^2 + 1)*log(x) + (c^2*x^2*(log(c) + 1) + (c^2*x^2 + 1)*log(x)
 + log(c))*sqrt(c^2*x^2 + 1) + log(c))*log(sqrt(c^2*x^2 + 1) + 1) + (c^2*x^
2*log(c)^2 + (c^2*x^2 + 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c)
)*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x))*b^2 +
 a^2*x + (2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/
(c^2*x^2) + 1) - 1))*a*b/c
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(b^2*arccsch(c*x)^2 + 2*a*b*arccsch(c*x) + a^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{acsch}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**2,x)
```

```
[Out] Integral((a + b*acsch(c*x))**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))^2,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))^2, x)
```

3.19 $\int \frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=81

$$\frac{\left(a+b\mathrm{csch}^{-1}(cx)\right)^3}{3b}-\left(a+b\mathrm{csch}^{-1}(cx)\right)^2\log\left(1-e^{2\mathrm{csch}^{-1}(cx)}\right)-b\left(a+b\mathrm{csch}^{-1}(cx)\right)\mathrm{PolyLog}\left(2,e^{2\mathrm{csch}^{-1}(cx)}\right)-$$

```
[Out] 1/3*(a+b*arccsch(c*x))^3/b-(a+b*arccsch(c*x))^2*ln(1-(1/c/x+(1+1/c^2/x^2)^(
1/2))^2)-b*(a+b*arccsch(c*x))*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)+1/2*
b^2*polylog(3,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)
```

Rubi [A]
time = 0.09, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6421, 3797, 2221, 2611, 2320, 6724}

$$-b\mathrm{Li}_2\left(e^{2\mathrm{csch}^{-1}(cx)}\right)\left(a+b\mathrm{csch}^{-1}(cx)\right)+\frac{\left(a+b\mathrm{csch}^{-1}(cx)\right)^3}{3b}-\log\left(1-e^{2\mathrm{csch}^{-1}(cx)}\right)\left(a+b\mathrm{csch}^{-1}(cx)\right)^2+\frac{1}{2}b^2\mathrm{Li}_3\left(e^{2\mathrm{csch}^{-1}(cx)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^2/x,x]
```

```
[Out] (a + b*ArcCsch[c*x])^3/(3*b) - (a + b*ArcCsch[c*x])^2*Log[1 - E^(2*ArcCsch[
c*x])] - b*(a + b*ArcCsch[c*x])*PolyLog[2, E^(2*ArcCsch[c*x])] + (b^2*PolyL
og[3, E^(2*ArcCsch[c*x])])/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x}\,dx = -\text{Subst}\left(\int (a+bx)^2\coth(x)\,dx, x, \text{csch}^{-1}(cx)\right)$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3b} + 2\text{Subst}\left(\int \frac{e^{2x}(a+bx)^2}{1-e^{2x}}\,dx, x, \text{csch}^{-1}(cx)\right)$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3b} - \left(a+b\text{csch}^{-1}(cx)\right)^2\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) + (2b)\text{Subst}\left(\int (\right.$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3b} - \left(a+b\text{csch}^{-1}(cx)\right)^2\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) - b\left(a+b\text{csch}^{-1}(\right.$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3b} - \left(a+b\text{csch}^{-1}(cx)\right)^2\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) - b\left(a+b\text{csch}^{-1}(\right.$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{3b} - \left(a+b\text{csch}^{-1}(cx)\right)^2\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) - b\left(a+b\text{csch}^{-1}(\right.$$

Mathematica [A]
time = 0.08, size = 115, normalized size = 1.42

$$a^2\log(cx)+ab\left(-\operatorname{csch}^{-1}(cx)\left(\operatorname{csch}^{-1}(cx)+2\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)\right)+\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(cx)}\right)\right)+b^2\left(\frac{1}{3}\operatorname{csch}^{-1}(cx)^3-\operatorname{csch}^{-1}(cx)^2\log\left(1-e^{2\operatorname{csch}^{-1}(cx)}\right)-\operatorname{csch}^{-1}(cx)\operatorname{PolyLog}\left(2,e^{2\operatorname{csch}^{-1}(cx)}\right)+\frac{1}{2}\operatorname{PolyLog}\left(3,e^{2\operatorname{csch}^{-1}(cx)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^2/x,x]
```

```
[Out] a^2*Log[c*x] + a*b*(-(ArcCsch[c*x]*(ArcCsch[c*x] + 2*Log[1 - E^(-2*ArcCsch[
c*x])])) + PolyLog[2, E^(-2*ArcCsch[c*x])]) + b^2*(ArcCsch[c*x]^3/3 - ArcCs
ch[c*x]^2*Log[1 - E^(2*ArcCsch[c*x])] - ArcCsch[c*x]*PolyLog[2, E^(2*ArcCsc
h[c*x])] + PolyLog[3, E^(2*ArcCsch[c*x])]/2)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\operatorname{arccsch}(cx)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^2/x,x)
```

```
[Out] int((a+b*arccsch(c*x))^2/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] b^2*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^2 + a^2*log(x) - integrate(-(b^2*log(
c)^2 + (b^2*c^2*log(c)^2 - 2*a*b*c^2*log(c))*x^2 - 2*a*b*log(c) + (b^2*c^2*
x^2 + b^2)*log(x)^2 + 2*((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b)
*log(x) - 2*((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b + (b^2*c^2*x
^2 + b^2)*log(x) + sqrt(c^2*x^2 + 1)*((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*
log(c) - a*b + (2*b^2*c^2*x^2 + b^2)*log(x)))*log(sqrt(c^2*x^2 + 1) + 1) +
sqrt(c^2*x^2 + 1)*(b^2*log(c)^2 + (b^2*c^2*log(c)^2 - 2*a*b*c^2*log(c))*x^2
 - 2*a*b*log(c) + (b^2*c^2*x^2 + b^2)*log(x)^2 + 2*((b^2*c^2*log(c) - a*b*c
^2)*x^2 + b^2*log(c) - a*b)*log(x)))/(c^2*x^3 + (c^2*x^3 + x)*sqrt(c^2*x^2
+ 1) + x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x,x, algorithm="fricas")
[Out] integral((b^2*arccsch(c*x)^2 + 2*a*b*arccsch(c*x) + a^2)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsch}\,(cx))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**2/x,x)
[Out] Integral((a + b*acsch(c*x))**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^2/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))^2/x,x)
[Out] int((a + b*asinh(1/(c*x)))^2/x, x)
```

3.20 $\int \frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=49

$$-\frac{2b^2}{x}+2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\mathrm{csch}^{-1}(cx)\right)-\frac{\left(a+b\mathrm{csch}^{-1}(cx)\right)^2}{x}$$

```
[Out] -2*b^2/x-(a+b*arccsch(c*x))^2/x+2*b*c*(a+b*arccsch(c*x))*(1+1/c^2/x^2)^(1/2
)
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6421, 3377, 2717}

$$2bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\mathrm{csch}^{-1}(cx)\right)-\frac{\left(a+b\mathrm{csch}^{-1}(cx)\right)^2}{x}-\frac{2b^2}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^2/x^2,x]
```

```
[Out] (-2*b^2)/x + 2*b*c*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]) - (a + b*ArcC
sch[c*x])^2/x
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{(a+b\text{csch}^{-1}(cx))^2}{x^2}\,dx = -\left(c\text{Subst}\left(\int (a+bx)^2\cosh(x)\,dx, x, \text{csch}^{-1}(cx)\right)\right)$$

$$= -\frac{(a+b\text{csch}^{-1}(cx))^2}{x} + (2bc)\text{Subst}\left(\int (a+bx)\sinh(x)\,dx, x, \text{csch}^{-1}(cx)\right)$$

$$= 2bc\sqrt{1+\frac{1}{c^2x^2}}\,(a+b\text{csch}^{-1}(cx)) - \frac{(a+b\text{csch}^{-1}(cx))^2}{x} - (2b^2c)\,\text{Subst}\left(\int \cosh$$

$$= -\frac{2b^2}{x} + 2bc\sqrt{1+\frac{1}{c^2x^2}}\,(a+b\text{csch}^{-1}(cx)) - \frac{(a+b\text{csch}^{-1}(cx))^2}{x}$$

Mathematica [A]
time = 0.10, size = 70, normalized size = 1.43

$$-\frac{a^2+2b^2-2abc\sqrt{1+\frac{1}{c^2x^2}}\,x+2b\left(a-bc\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\text{csch}^{-1}(cx)+b^2\text{csch}^{-1}(cx)^2}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^2/x^2,x]
[Out] -((a^2 + 2*b^2 - 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x + 2*b*(a - b*c*Sqrt[1 + 1/
(c^2*x^2)]*x)*ArcCsch[c*x] + b^2*ArcCsch[c*x]^2)/x)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{arccsch}(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^2/x^2,x)
[Out] int((a+b*arccsch(c*x))^2/x^2,x)
```

Maxima [A]
time = 0.26, size = 78, normalized size = 1.59

$$2\left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\text{arcsch}\,(cx)}{x}\right)ab+2\left(c\sqrt{\frac{1}{c^2x^2}+1}\;\text{arcsch}\,(cx)-\frac{1}{x}\right)b^2-\frac{b^2\,\text{arcsch}\,(cx)^2}{x}-\frac{a^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^2,x, algorithm="maxima")
```

```
[Out] 2*(c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*a*b + 2*(c*sqrt(1/(c^2*x^2) +
1)*arccsch(c*x) - 1/x)*b^2 - b^2*arccsch(c*x)^2/x - a^2/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(47) = 94$.
time = 0.40, size = 139, normalized size = 2.84

$$\frac{2\,abcx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - b^2\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 - a^2 - 2\,b^2 + 2\left(b^2cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - ab\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^2,x, algorithm="fricas")
```

```
[Out] (2*a*b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - b^2*log((c*x*sqrt((c^2*x^2 + 1)/
(c^2*x^2)) + 1)/(c*x))^2 - a^2 - 2*b^2 + 2*(b^2*c*x*sqrt((c^2*x^2 + 1)/(c^2
*x^2)) - a*b)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/x
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsch}\,(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**2/x**2,x)
```

```
[Out] Integral((a + b*acsch(c*x))**2/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^2/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))^2/x^2,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))^2/x^2, x)
```

3.21 $\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=87

$$-\frac{b^2}{4x^2}-\frac{1}{2}abc^2\operatorname{csch}^{-1}(cx)-\frac{1}{4}b^2c^2\operatorname{csch}^{-1}(cx)^2+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2x^2}$$

```
[Out] -1/4*b^2/x^2-1/2*a*b*c^2*arccsch(c*x)-1/4*b^2*c^2*arccsch(c*x)^2-1/2*(a+b*a
rccsch(c*x))^2/x^2+1/2*b*c*(a+b*arccsch(c*x))*(1+1/c^2/x^2)^(1/2)/x
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6421, 5554, 3391}

$$\frac{bc\sqrt{\frac{1}{c^2x^2}+1}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2x}-\frac{1}{2}abc^2\operatorname{csch}^{-1}(cx)-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2x^2}-\frac{1}{4}b^2c^2\operatorname{csch}^{-1}(cx)^2-\frac{b^2}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^2/x^3,x]
```

```
[Out] -1/4*b^2/x^2 - (a*b*c^2*ArcCsch[c*x])/2 - (b^2*c^2*ArcCsch[c*x]^2)/4 + (b*c
*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(2*x) - (a + b*ArcCsch[c*x])^2
/(2*x^2)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
```

```
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^3}\,dx=-\left(c^2\text{Subst}\left(\int (a+bx)^2\cosh(x)\sinh(x)\,dx,x,\text{csch}^{-1}(cx)\right)\right)$$

$$=-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2x^2}+\left(bc^2\right)\text{Subst}\left(\int (a+bx)\sinh^2(x)\,dx,x,\text{csch}^{-1}(cx)\right)$$

$$=-\frac{b^2}{4x^2}+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)}{2x}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2x^2}-\frac{1}{2}\left(bc^2\right)\text{Subs}$$

$$=-\frac{b^2}{4x^2}-\frac{1}{2}abc^2\text{csch}^{-1}(cx)-\frac{1}{4}b^2c^2\text{csch}^{-1}(cx)^2+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx\right.}{2x}$$

Mathematica [A]
time = 0.09, size = 100, normalized size = 1.15

$$-\frac{2a^2+b^2-2abc\sqrt{1+\frac{1}{c^2x^2}}\,x-2b\left(-2a+bc\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\text{csch}^{-1}(cx)+b^2\left(2+c^2x^2\right)\text{csch}^{-1}(cx)^2+2abc^2x^2\sinh^{-1}\left(\frac{1}{cx}\right)}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^2/x^3,x]
```

```
[Out] -1/4*(2*a^2 + b^2 - 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x - 2*b*(-2*a + b*c*Sqrt[
1 + 1/(c^2*x^2)]*x)*ArcCsch[c*x] + b^2*(2 + c^2*x^2)*ArcCsch[c*x]^2 + 2*a*b
*c^2*x^2*ArcSinh[1/(c*x)])/x^2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\text{arccsch}(cx)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^2/x^3,x)
```

```
[Out] int((a+b*arccsch(c*x))^2/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*b*((2*c^4*x*sqrt(1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) - c
^3*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) -
 1))/c - 4*arccsch(c*x)/x^2) - 1/2*b^2*(log(sqrt(c^2*x^2 + 1) + 1)^2/x^2 +
2*integrate(-(c^2*x^2*log(c)^2 + (c^2*x^2 + 1)*log(x)^2 + log(c)^2 + 2*(c^2
*x^2*log(c) + log(c))*log(x) - (2*c^2*x^2*log(c) + 2*(c^2*x^2 + 1)*log(x) +
 (c^2*x^2*(2*log(c) - 1) + 2*(c^2*x^2 + 1)*log(x) + 2*log(c))*sqrt(c^2*x^2
+ 1) + 2*log(c))*log(sqrt(c^2*x^2 + 1) + 1) + (c^2*x^2*log(c)^2 + (c^2*x^2
+ 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c))*log(x))*sqrt(c^2*x^2
 + 1))/(c^2*x^5 + x^3 + (c^2*x^5 + x^3)*sqrt(c^2*x^2 + 1)), x)) - 1/2*a^2/x
^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(75) = 150.
time = 0.40, size = 163, normalized size = 1.87

$$\frac{2\,abcx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - (b^2c^2x^2+2\,b^2)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 - 2\,a^2 - b^2 - 2\left(abc^2x^2 - b^2cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+2\,ab\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)}{4\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - (b^2*c^2*x^2 + 2*b^2)*log((c
*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 - 2*a^2 - b^2 - 2*(a*b*c^2*x
^2 - b^2*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*b)*log((c*x*sqrt((c^2*x^2
+ 1)/(c^2*x^2)) + 1)/(c*x)))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acsch}\,(cx))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**2/x**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^3,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^2/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))^2/x^3,x)
[Out] int((a + b*asinh(1/(c*x)))^2/x^3, x)
```

3.22 $\int \frac{\left(a+b\,\mathbf{csch}^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=100

$$-\frac{2b^2}{27x^3}+\frac{4b^2c^2}{9x}-\frac{4}{9}bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)+\frac{2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^2}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3x^3}$$

```
[Out] -2/27*b^2/x^3+4/9*b^2*c^2/x-1/3*(a+b*arccsch(c*x))^2/x^3-4/9*b*c^3*(a+b*arc
csch(c*x))*(1+1/c^2/x^2)^(1/2)+2/9*b*c*(a+b*arccsch(c*x))*(1+1/c^2/x^2)^(1/
2)/x^2
```

Rubi [A]
time = 0.08, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6421, 5554, 3391, 3377, 2717}

$$\frac{2bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^2}-\frac{4}{9}bc^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3x^3}+\frac{4b^2c^2}{9x}-\frac{2b^2}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^2/x^4,x]
```

```
[Out] (-2*b^2)/(27*x^3) + (4*b^2*c^2)/(9*x) - (4*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*(a +
 b*ArcCsch[c*x]))/9 + (2*b*c*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(9
*x^2) - (a + b*ArcCsch[c*x])^2/(3*x^3)
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

```
]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^4}\,dx &= -\left(c^3\text{Subst}\left(\int (a+bx)^2\cosh(x)\sinh^2(x)\,dx, x, \text{csch}^{-1}(cx)\right)\right)\\ &= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{3x^3}+\frac{1}{3}\left(2bc^3\right)\text{Subst}\left(\int (a+bx)\sinh^3(x)\,dx, x, \text{csch}^{-1}(cx)\right)\\ &= -\frac{2b^2}{27x^3}+\frac{2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)}{9x^2}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{3x^3}-\frac{1}{9}\left(4bc^3\right)\text{S}\\ &= -\frac{2b^2}{27x^3}-\frac{4}{9}bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx\right.}{9x^2}\\ &= -\frac{2b^2}{27x^3}+\frac{4b^2c^2}{9x}-\frac{4}{9}bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{c}\right.}{9x^2}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 106, normalized size = 1.06

$$\frac{-9a^2+6abc\sqrt{1+\frac{1}{c^2x^2}}\,x(1-2c^2x^2)+2b^2(-1+6c^2x^2)-6b\left(3a+bc\sqrt{1+\frac{1}{c^2x^2}}\,x(-1+2c^2x^2)\right)\text{csch}^{-1}(cx)-9b^2\text{csch}^{-1}(cx)^2}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^2/x^4,x]
```

```
[Out] (-9*a^2 + 6*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(1 - 2*c^2*x^2) + 2*b^2*(-1 + 6*c
^2*x^2) - 6*b*(3*a + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-1 + 2*c^2*x^2))*ArcCsch[
c*x] - 9*b^2*ArcCsch[c*x]^2)/(27*x^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccsch}(cx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^2/x^4,x)
```

```
[Out] int((a+b*arccsch(c*x))^2/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^4,x, algorithm="maxima")
```

```
[Out] 2/9*a*b*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*
arccsch(c*x)/x^3) - 1/3*b^2*(log(sqrt(c^2*x^2 + 1) + 1)^2/x^3 + 3*integrate
(-1/3*(3*c^2*x^2*log(c)^2 + 3*(c^2*x^2 + 1)*log(x)^2 + 3*log(c)^2 + 6*(c^2*
x^2*log(c) + log(c))*log(x) - 2*(3*c^2*x^2*log(c) + 3*(c^2*x^2 + 1)*log(x)
+ (c^2*x^2*(3*log(c) - 1) + 3*(c^2*x^2 + 1)*log(x) + 3*log(c))*sqrt(c^2*x^2
 + 1) + 3*log(c))*log(sqrt(c^2*x^2 + 1) + 1) + 3*(c^2*x^2*log(c)^2 + (c^2*x
^2 + 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c))*log(x))*sqrt(c^2*
x^2 + 1))/(c^2*x^6 + x^4 + (c^2*x^6 + x^4)*sqrt(c^2*x^2 + 1)), x)) - 1/3*a^
2/x^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(86) = 172.
time = 0.46, size = 178, normalized size = 1.78

$$\frac{12\,b^2c^2x^2 - 9\,b^2\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 - 9\,a^2 - 2\,b^2 - 6\left(3\,ab + (2\,b^2c^3x^3 - b^2cx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) - 6\,(2\,abc^3x^3 - abcx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(12*b^2*c^2*x^2 - 9*b^2*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c
*x))^2 - 9*a^2 - 2*b^2 - 6*(3*a*b + (2*b^2*c^3*x^3 - b^2*c*x)*sqrt((c^2*x^2
```

```
 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 6*(2
*a*b*c^3*x^3 - a*b*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/x^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsch}\,(cx))^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**2/x**4,x)
[Out] Integral((a + b*acsch(c*x))**2/x**4, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^4,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^2/x^4, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))^2/x^4,x)
[Out] int((a + b*asinh(1/(c*x)))^2/x^4, x)
```

3.23 $\int \frac{\left(a+b\,\mathbf{csch}^{-1}(cx)\right)^2}{x^5}\,dx$

Optimal. Leaf size=132

$$-\frac{b^2}{32x^4}+\frac{3b^2c^2}{32x^2}+\frac{3}{16}abc^4\text{csch}^{-1}(cx)+\frac{3}{32}b^2c^4\text{csch}^{-1}(cx)^2+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}\,\left(a+b\text{csch}^{-1}(cx)\right)}{8x^3}-\frac{3bc^3\sqrt{1+\frac{1}{c^2x^2}}\,\left(\right.}{16}$$

```
[Out] -1/32*b^2/x^4+3/32*b^2*c^2/x^2+3/16*a*b*c^4*arccsch(c*x)+3/32*b^2*c^4*arccs
ch(c*x)^2-1/4*(a+b*arccsch(c*x))^2/x^4+1/8*b*c*(a+b*arccsch(c*x))*(1+1/c^2/
x^2)^(1/2)/x^3-3/16*b*c^3*(a+b*arccsch(c*x))*(1+1/c^2/x^2)^(1/2)/x
```

Rubi [A]

time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6421, 5554, 3391}

$$\frac{3}{16}abc^4\text{csch}^{-1}(cx)+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}\,\left(a+b\text{csch}^{-1}(cx)\right)}{8x^3}-\frac{3bc^3\sqrt{\frac{1}{c^2x^2}+1}\,\left(a+b\text{csch}^{-1}(cx)\right)}{16x}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4x^4}+\frac{3}{32}b^2c^4\text{csch}^{-1}(cx)^2+\frac{3b^2c^2}{32x^2}-\frac{b^2}{32x^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^2/x^5,x]
```

```
[Out] -1/32*b^2/x^4 + (3*b^2*c^2)/(32*x^2) + (3*a*b*c^4*ArcCsch[c*x])/16 + (3*b^2
*c^4*ArcCsch[c*x]^2)/32 + (b*c*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/
(8*x^3) - (3*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x]))/(16*x) - (a
+ b*ArcCsch[c*x])^2/(4*x^4)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{x^5}\,dx = -\left(c^4\text{Subst}\left(\int (a+bx)^2\cosh(x)\sinh^3(x)\,dx, x, \text{csch}^{-1}(cx)\right)\right)$$
$$= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4x^4} + \frac{1}{2}\left(bc^4\right)\text{Subst}\left(\int (a+bx)\sinh^4(x)\,dx, x, \text{csch}^{-1}(cx)\right)$$
$$= -\frac{b^2}{32x^4} + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)}{8x^3} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{4x^4} - \frac{1}{8}\left(3bc^4\right)\text{Su}$$
$$= -\frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)}{8x^3} - \frac{3bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{cs}\right.}{16x}$$
$$= -\frac{b^2}{32x^4} + \frac{3b^2c^2}{32x^2} + \frac{3}{16}abc^4\text{csch}^{-1}(cx) + \frac{3}{32}b^2c^4\text{csch}^{-1}(cx)^2 + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+\right.}{8x^3}$$

Mathematica [A]
time = 0.11, size = 147, normalized size = 1.11

$$\frac{-8a^2-b^2+4abc\sqrt{1+\frac{1}{c^2x^2}}\,x+3b^2c^2x^2-6abc^3\sqrt{1+\frac{1}{c^2x^2}}\,x^3-2b\left(8a+bc\sqrt{1+\frac{1}{c^2x^2}}\,x\left(-2+3c^2x^2\right)\right)\text{csch}^{-1}(cx)+b^2\left(-8+3c^4x^4\right)\text{csch}^{-1}(cx)^2+6abc^4x^4\sinh^{-1}\left(\frac{1}{cx}\right)}{32x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^2/x^5,x]
```

```
[Out] (-8*a^2 - b^2 + 4*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x + 3*b^2*c^2*x^2 - 6*a*b*c^3
*Sqrt[1 + 1/(c^2*x^2)]*x^3 - 2*b*(8*a + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-2 + 3
*c^2*x^2))*ArcCsch[c*x] + b^2*(-8 + 3*c^4*x^4)*ArcCsch[c*x]^2 + 6*a*b*c^4*x
^4*ArcSinh[1/(c*x)])/(32*x^4)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\text{arccsch}(cx)\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^2/x^5,x)
```

```
[Out] int((a+b*arccsch(c*x))^2/x^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/32*a*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) - 3*c^5*log(c*x*sqrt(1/
(c^2*x^2) + 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) + 1)^(3/2) - 5*c^6*x*sqrt(1
/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) + 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) + 1)
 + 1))/c - 16*arccsch(c*x)/x^4) - 1/4*b^2*(log(sqrt(c^2*x^2 + 1) + 1)^2/x^4
 + 4*integrate(-1/2*(2*c^2*x^2*log(c)^2 + 2*(c^2*x^2 + 1)*log(x)^2 + 2*log(
c)^2 + 4*(c^2*x^2*log(c) + log(c))*log(x) - (4*c^2*x^2*log(c) + 4*(c^2*x^2
+ 1)*log(x) + (c^2*x^2*(4*log(c) - 1) + 4*(c^2*x^2 + 1)*log(x) + 4*log(c))*
sqrt(c^2*x^2 + 1) + 4*log(c))*log(sqrt(c^2*x^2 + 1) + 1) + 2*(c^2*x^2*log(c
)^2 + (c^2*x^2 + 1)*log(x)^2 + log(c)^2 + 2*(c^2*x^2*log(c) + log(c))*log(x
))*sqrt(c^2*x^2 + 1))/(c^2*x^7 + x^5 + (c^2*x^7 + x^5)*sqrt(c^2*x^2 + 1)),
x)) - 1/4*a^2/x^4
```

Fricas [A]
time = 0.45, size = 202, normalized size = 1.53

$$\frac{3\,b^2c^2x^2 + (3\,b^2c^4x^4 - 8\,b^2)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 - 8\,a^2 - b^2 + 2\left(3\,abc^4x^4 - 8\,ab - (3\,b^2c^3x^3 - 2\,b^2cx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) - 2\,(3\,abc^3x^3 - 2\,abcx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{32\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/32*(3*b^2*c^2*x^2 + (3*b^2*c^4*x^4 - 8*b^2)*log((c*x*sqrt((c^2*x^2 + 1)/(
c^2*x^2)) + 1)/(c*x))^2 - 8*a^2 - b^2 + 2*(3*a*b*c^4*x^4 - 8*a*b - (3*b^2*c
^3*x^3 - 2*b^2*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 +
 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(3*a*b*c^3*x^3 - 2*a*b*c*x)*sqrt((c^2*x^2 +
1)/(c^2*x^2)))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\,\mathrm{acsch}\,(cx)\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**2/x**5,x)
```

```
[Out] Integral((a + b*acsch(c*x))**2/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^2/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^2/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^2}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))^2/x^5,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))^2/x^5, x)
```

3.24 $\int x^3 \left(a + b\text{csch}^{-1}(cx)\right)^3 \, dx$

<u>**Optimal**</u>. Leaf size=195

$$\frac{b^3 \sqrt{1+\frac{1}{c^2x^2}}\, x}{4c^3} + \frac{b^2x^2\left(a+b\text{csch}^{-1}(cx)\right)}{4c^2} - \frac{b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^4} - \frac{b\sqrt{1+\frac{1}{c^2x^2}}\, x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^3} + \frac{b\sqrt{1+}}{}$$

```
[Out] 1/4*b^2*x^2*(a+b*arccsch(c*x))/c^2-1/2*b*(a+b*arccsch(c*x))^2/c^4+1/4*x^4*(
a+b*arccsch(c*x))^3+b^2*(a+b*arccsch(c*x))*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))
^2)/c^4+1/2*b^3*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/c^4+1/4*b^3*x*(1+1
/c^2/x^2)^(1/2)/c^3-1/2*b*x*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/c^3+1/
4*b*x^3*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/c
```

<u>**Rubi**</u> [A]
time = 0.15, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6421, 5560, 4271, 3852, 8, 4269, 3797, 2221, 2317, 2438}

$$\frac{b^2\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)}{c^4} + \frac{b^2x^2\left(a+b\text{csch}^{-1}(cx)\right)}{4c^2} - \frac{b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^4} + \frac{bx^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{4c} - \frac{bx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^3} + \frac{1}{4}x^4\left(a+b\text{csch}^{-1}(cx)\right)^3 + \frac{b^3\text{Li}_2\left(e^{2\text{csch}^{-1}(cx)}\right)}{2c^4} + \frac{b^3x\sqrt{\frac{1}{c^2x^2}+1}}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] (b^3*Sqrt[1 + 1/(c^2*x^2)]*x)/(4*c^3) + (b^2*x^2*(a + b*ArcCsch[c*x]))/(4*c
^2) - (b*(a + b*ArcCsch[c*x])^2)/(2*c^4) - (b*Sqrt[1 + 1/(c^2*x^2)]*x*(a +
b*ArcCsch[c*x])^2)/(2*c^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x^3*(a + b*ArcCsch[c*
x])^2)/(4*c) + (x^4*(a + b*ArcCsch[c*x])^3)/4 + (b^2*(a + b*ArcCsch[c*x])*L
og[1 - E^(2*ArcCsch[c*x])])/c^4 + (b^3*PolyLog[2, E^(2*ArcCsch[c*x])])/(2*c
^4)
```

<u>Rule 8</u>

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

<u>Rule 2221</u>

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

<u>Rule 2317</u>

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx=-\frac{\operatorname{Subst}\left(\int(a+bx)^3\coth(x)\operatorname{csch}^4(x)\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{c^4}$$

$$=\frac{1}{4}x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)^3-\frac{(3b)\operatorname{Subst}\left(\int(a+bx)^2\operatorname{csch}^4(x)\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{4c^4}$$

$$=\frac{b^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{4c}+\frac{1}{4}x^4\left(a+b\operatorname{csch}\right.$$

$$=\frac{b^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4c^2}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c^3}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^3}{}$$

$$=\frac{b^3\sqrt{1+\frac{1}{c^2x^2}}\,x}{4c^3}+\frac{b^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4c^2}-\frac{b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c^4}-\frac{b\sqrt{1+\frac{}{c^2}}}{}$$

$$=\frac{b^3\sqrt{1+\frac{1}{c^2x^2}}\,x}{4c^3}+\frac{b^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4c^2}-\frac{b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c^4}-\frac{b\sqrt{1+\frac{}{c^2}}}{}$$

$$=\frac{b^3\sqrt{1+\frac{1}{c^2x^2}}\,x}{4c^3}+\frac{b^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4c^2}-\frac{b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c^4}-\frac{b\sqrt{1+\frac{}{c^2}}}{}$$

$$=\frac{b^3\sqrt{1+\frac{1}{c^2x^2}}\,x}{4c^3}+\frac{b^2x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4c^2}-\frac{b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2c^4}-\frac{b\sqrt{1+\frac{}{c^2}}}{}$$

Mathematica [A]
time = 0.68, size = 271, normalized size = 1.39

$$\frac{-2a^2bc\sqrt{1+\frac{1}{c^2x^2}}\,x+b^3c\sqrt{1+\frac{1}{c^2x^2}}\,x+ab^2c^2x^2+a^2bc^3\sqrt{1+\frac{1}{c^2x^2}}\,x^3+a^3c^4x^4+b^2\left(3ac^4x^4+b\left(2-2c\sqrt{1+\frac{1}{c^2x^2}}\,x+c^3\sqrt{1+\frac{1}{c^2x^2}}\,x^3\right)\right)\operatorname{csch}^{-1}(cx)^2+b^3c^4x^4\operatorname{csch}^{-1}(cx)^3+b\operatorname{csch}^{-1}(cx)\left(cx\left(b^2cx+3a^2c^3x^3+2ab\sqrt{1+\frac{1}{c^2x^2}}\,(-2+c^2x^2)\right)+4b^2\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)\right)+4ab^2\log\left(\frac{1}{cx}\right)-2b^3\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(cx)}\right)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] (-2*a^2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x + b^3*c*Sqrt[1 + 1/(c^2*x^2)]*x + a*b^2
*c^2*x^2 + a^2*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*x^3 + a^3*c^4*x^4 + b^2*(3*a*c^4
```

```
*x^4 + b*(2 - 2*c*Sqrt[1 + 1/(c^2*x^2)]*x + c^3*Sqrt[1 + 1/(c^2*x^2)]*x^3))
*ArcCsch[c*x]^2 + b^3*c^4*x^4*ArcCsch[c*x]^3 + b*ArcCsch[c*x]*(c*x*(b^2*c*x
 + 3*a^2*c^3*x^3 + 2*a*b*Sqrt[1 + 1/(c^2*x^2)]*(-2 + c^2*x^2)) + 4*b^2*Log[
1 - E^(-2*ArcCsch[c*x])]) + 4*a*b^2*Log[1/(c*x)] - 2*b^3*PolyLog[2, E^(-2*A
rcCsch[c*x])])/(4*c^4)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x^3 (a + b\,\mathrm{arccsch}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))^3,x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*x^4*log(sqrt(c^2*x^2 + 1) + 1)^3 - 12*b^3*c^2*integrate(1/4*x^5*log
(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c
)^2 + 12*b^3*c^2*integrate(1/4*x^5*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2
 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 + 1/4*a^3*x^4
 - 12*b^3*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)^2/(sqrt(c^2*x^2 +
1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*b^3*c^2*integ
rate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*
x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 12*b^3*c^2
*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2
*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 12*b^3*c^
2*integrate(1/4*x^5*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^
2*x^2 + 1) + 1), x)*log(c) + 24*b^3*c^2*integrate(1/4*x^5*log(x)*log(sqrt(c
^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) +
 1), x)*log(c) - 12*b^3*c^2*integrate(1/4*x^5*log(sqrt(c^2*x^2 + 1) + 1)^2/
(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) +
24*a*b^2*c^2*integrate(1/4*x^5*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2
+ sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 24*a*b^2*c^2*integrate(1/4*x^5*log(sq
rt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 +
1) + 1), x)*log(c) - 4*b^3*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)^3
/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*b^3
*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)^2*log(sqrt(c^2*x^2 + 1) + 1
```

)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 12*b^
3*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)
^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 4*b^
3*c^2*integrate(1/4*x^5*log(x)^3/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqr
t(c^2*x^2 + 1) + 1), x) + 12*b^3*c^2*integrate(1/4*x^5*log(x)^2*log(sqrt(c^
2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) +
1), x) - 12*b^3*c^2*integrate(1/4*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^2/(
sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*a*b^2
*c^2*integrate(1/4*sqrt(c^2*x^2 + 1)*x^5*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^
2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*a*b^2*c^2*integrate(1/4*sqrt(
c^2*x^2 + 1)*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x
^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*a*b^2*c^2*integrate(1/4*sqrt
(c^2*x^2 + 1)*x^5*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 +
 c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 3*b^3*c^2*integrate(1/4*sqrt(c^2*x^
2 + 1)*x^5*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^
2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*a*b^2*c^2*integrate(1/4*x^5*log(x)^2/(s
qrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*a*b^2*
c^2*integrate(1/4*x^5*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*
c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*a*b^2*c^2*integrate(1/4
*x^5*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sq
rt(c^2*x^2 + 1) + 1), x) - 12*b^3*integrate(1/4*x^3*log(x)/(sqrt(c^2*x^2 +
1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 + 12*b^3*integra
te(1/4*x^3*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2
+ sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 - 12*b^3*integrate(1/4*sqrt(c^2*x^2 +
 1)*x^3*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) +
 1), x)*log(c) + 24*b^3*integrate(1/4*sqrt(c^2*x^2 + 1)*x^3*log(x)*log(sqrt
(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1)
 + 1), x)*log(c) - 12*b^3*integrate(1/4*sqrt(c^2*x^2 + 1)*x^3*log(sqrt(c^2*
x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) +
1), x)*log(c) - 12*b^3*integrate(1/4*x^3*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^
2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*b^3*integrate(1/4*x^3*
log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sq
rt(c^2*x^2 + 1) + 1), x)*log(c) - 12*b^3*integrate(1/4*x^3*log(sqrt(c^2*x^2
 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1),
 x)*log(c) + 24*a*b^2*integrate(1/4*x^3*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 +
 c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 24*a*b^2*integrate(1/4*x^3*l
og(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x
^2 + 1) + 1), x)*log(c) + 1/4*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2) +
 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*a^2*b - 4*b^3*integrate(1/4*sqr
t(c^2*x^2 + 1)*x^3*log(x)^3/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2
*x^2 + 1) + 1), x) + 12*b^3*integrate(1/4*sqrt(c^2*x^2 + 1)*x^3*log(x)^2*lo
g(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^
2 + 1) + 1), x) - 12*b^3*integrate(1/4*sqrt(c^2*x^2 + 1)*x^3*log(x)*log(sqr
t(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 +
 1) + 1), x) - 4*b^3*integrate(1/4*x^3*log(x)^3/(sqrt(c^2*x^2 + 1)*c^2*x^2

```
+ c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*b^3...
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^3*arccsch(c*x)^3 + 3*a*b^2*x^3*arccsch(c*x)^2 + 3*a^2*b*x^3*
arccsch(c*x) + a^3*x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a + b\,\mathrm{acsch}\,(cx))^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))**3,x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*asinh(1/(c*x)))^3,x)
```

```
[Out] int(x^3*(a + b*asinh(1/(c*x)))^3, x)
```

3.25 $\int x^2\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=194

$$\frac{b^2x\left(a+b\text{csch}^{-1}(cx)\right)}{c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{b\left(a+b\text{csch}^{-1}(cx)\right)^2\tan}{c^3}$$

```
[Out] b^2*x*(a+b*arccsch(c*x))/c^2+1/3*x^3*(a+b*arccsch(c*x))^3-b*(a+b*arccsch(c*
x))^2*arctanh(1/c/x+(1+1/c^2/x^2)^(1/2))/c^3+b^3*arctanh((1+1/c^2/x^2)^(1/2
))/c^3-b^2*(a+b*arccsch(c*x))*polylog(2,-1/c/x-(1+1/c^2/x^2)^(1/2))/c^3+b^2
*(a+b*arccsch(c*x))*polylog(2,1/c/x+(1+1/c^2/x^2)^(1/2))/c^3+b^3*polylog(3,
-1/c/x-(1+1/c^2/x^2)^(1/2))/c^3-b^3*polylog(3,1/c/x+(1+1/c^2/x^2)^(1/2))/c^
3+1/2*b*x^2*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.14, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.571$, Rules used = {6421, 5560, 4271, 3855, 4267, 2611, 2320, 6724}

$$-\frac{b^2\text{Li}_2\left(-e^{\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)}{c^3}+\frac{b^2\text{Li}_2\left(e^{\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)}{c^3}+\frac{b^2x\left(a+b\text{csch}^{-1}(cx)\right)}{c^2}-\frac{b\tanh^{-1}\left(e^{\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)^2}{c^3}+\frac{bx^2\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{b^3\text{Li}_3\left(-e^{\text{csch}^{-1}(cx)}\right)}{c^3}-\frac{b^3\text{Li}_3\left(e^{\text{csch}^{-1}(cx)}\right)}{c^3}+\frac{b^3\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] (b^2*x*(a + b*ArcCsch[c*x]))/c^2 + (b*Sqrt[1 + 1/(c^2*x^2)]*x^2*(a + b*ArcC
sch[c*x])^2)/(2*c) + (x^3*(a + b*ArcCsch[c*x])^3)/3 - (b*(a + b*ArcCsch[c*x
])^2*ArcTanh[E^ArcCsch[c*x]])/c^3 + (b^3*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/c^
3 - (b^2*(a + b*ArcCsch[c*x])*PolyLog[2, -E^ArcCsch[c*x]])/c^3 + (b^2*(a +
b*ArcCsch[c*x])*PolyLog[2, E^ArcCsch[c*x]])/c^3 + (b^3*PolyLog[3, -E^ArcCsc
h[c*x]])/c^3 - (b^3*PolyLog[3, E^ArcCsch[c*x]])/c^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
```

```
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx=-\frac{\text{Subst}\left(\int(a+bx)^3\coth(x)\text{csch}^3(x)\,dx,x,\text{csch}^{-1}(cx)\right)}{c^3}$$

$$=\frac{1}{3}x^3\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{b\text{Subst}\left(\int(a+bx)^2\text{csch}^3(x)\,dx,x,\text{csch}^{-1}(cx)\right)}{c^3}$$

$$=\frac{b^2x\left(a+b\text{csch}^{-1}(cx)\right)}{c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-}\right.$$

$$=\frac{b^2x\left(a+b\text{csch}^{-1}(cx)\right)}{c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-}\right.$$

$$=\frac{b^2x\left(a+b\text{csch}^{-1}(cx)\right)}{c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-}\right.$$

$$=\frac{b^2x\left(a+b\text{csch}^{-1}(cx)\right)}{c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-}\right.$$

$$=\frac{b^2x\left(a+b\text{csch}^{-1}(cx)\right)}{c^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x^2\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{3}x^3\left(a+b\text{csch}^{-}\right.$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 461 vs. 2(194) = 388.
time = 5.66, size = 461, normalized size = 2.38

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] (a^2*b*Sqrt[1 + 1/(c^2*x^2)]*x^2)/(2*c) + (a^3*x^3)/3 + a^2*b*x^3*ArcCsch[c
*x] - (a^2*b*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(2*c^3) + (a*b^2*(c*x + c^
2*Sqrt[1 + 1/(c^2*x^2)]*x^2*ArcCsch[c*x] + c^3*x^3*ArcCsch[c*x]^2 + ArcCsch
[c*x]*Log[1 - E^(-ArcCsch[c*x])] - ArcCsch[c*x]*Log[1 + E^(-ArcCsch[c*x])]
+ PolyLog[2, -E^(-ArcCsch[c*x])] - PolyLog[2, E^(-ArcCsch[c*x])]))/c^3 + (b
^3*(24*ArcCsch[c*x]*Coth[ArcCsch[c*x]/2] - 4*ArcCsch[c*x]^3*Coth[ArcCsch[c*
x]/2] + 6*ArcCsch[c*x]^2*Csch[ArcCsch[c*x]/2]^2 + (ArcCsch[c*x]^3*Csch[ArcC
sch[c*x]/2]^4)/(c*x) + 24*ArcCsch[c*x]^2*Log[1 - E^(-ArcCsch[c*x])] - 24*Ar
cCsch[c*x]^2*Log[1 + E^(-ArcCsch[c*x])] - 48*Log[Tanh[ArcCsch[c*x]/2]] + 48
*ArcCsch[c*x]*PolyLog[2, -E^(-ArcCsch[c*x])] - 48*ArcCsch[c*x]*PolyLog[2, E
```

```
^(-ArcCsch[c*x])] + 48*PolyLog[3, -E^(-ArcCsch[c*x])] - 48*PolyLog[3, E^(-A
rcCsch[c*x])] + 6*ArcCsch[c*x]^2*Sech[ArcCsch[c*x]/2]^2 + 16*c^3*x^3*ArcCsc
h[c*x]^3*Sinh[ArcCsch[c*x]/2]^4 - 24*ArcCsch[c*x]*Tanh[ArcCsch[c*x]/2] + 4*
ArcCsch[c*x]^3*Tanh[ArcCsch[c*x]/2]))/(48*c^3)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a+b\,\mathrm{arccsch}(cx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))^3,x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3*log(sqrt(c^2*x^2 + 1) + 1)^3 + 1/3*a^3*x^3 + 1/4*(4*x^3*arccsch
(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1
/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*a^2*b - i
ntegrate(((b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^4 + (b^3*c^2*x^4 + b^
3*x^2)*log(x)^3 + (b^3*log(c)^3 - 3*a*b^2*log(c)^2)*x^2 + 3*((b^3*c^2*log(c
) - a*b^2*c^2)*x^4 + (b^3*log(c) - a*b^2)*x^2)*log(x)^2 + (3*(b^3*c^2*log(c
) - a*b^2*c^2)*x^4 + 3*(b^3*log(c) - a*b^2)*x^2 + 3*(b^3*c^2*x^4 + b^3*x^2)
*log(x) + ((b^3*c^2*(3*log(c) + 1) - 3*a*b^2*c^2)*x^4 + 3*(b^3*log(c) - a*b
^2)*x^2 + 3*(b^3*c^2*x^4 + b^3*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2
*x^2 + 1) + 1)^2 + 3*((b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^4 + (b^3*lo
g(c)^2 - 2*a*b^2*log(c))*x^2)*log(x) - 3*((b^3*c^2*log(c)^2 - 2*a*b^2*c^2*l
og(c))*x^4 + (b^3*log(c)^2 - 2*a*b^2*log(c))*x^2 + (b^3*c^2*x^4 + b^3*x^2)*
log(x)^2 + 2*((b^3*c^2*log(c) - a*b^2*c^2)*x^4 + (b^3*log(c) - a*b^2)*x^2)*
log(x) + ((b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^4 + (b^3*log(c)^2 - 2*a
*b^2*log(c))*x^2 + (b^3*c^2*x^4 + b^3*x^2)*log(x)^2 + 2*((b^3*c^2*log(c) -
a*b^2*c^2)*x^4 + (b^3*log(c) - a*b^2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(s
qrt(c^2*x^2 + 1) + 1) + ((b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^4 + (b
^3*c^2*x^4 + b^3*x^2)*log(x)^3 + (b^3*log(c)^3 - 3*a*b^2*log(c)^2)*x^2 + 3*
((b^3*c^2*log(c) - a*b^2*c^2)*x^4 + (b^3*log(c) - a*b^2)*x^2)*log(x)^2 + 3*
((b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^4 + (b^3*log(c)^2 - 2*a*b^2*log(
c))*x^2)*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x^2*arccsch(c*x)^3 + 3*a*b^2*x^2*arccsch(c*x)^2 + 3*a^2*b*x^2*
arccsch(c*x) + a^3*x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a + b\,\mathrm{acsch}\,(cx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))**3,x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(1/(c*x)))^3,x)
```

```
[Out] int(x^2*(a + b*asinh(1/(c*x)))^3, x)
```

3.26 $\int x\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=117

$$\frac{3b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^2}+\frac{3b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{c^2}$$

```
[Out] 3/2*b*(a+b*arccsch(c*x))^2/c^2+1/2*x^2*(a+b*arccsch(c*x))^3-3*b^2*(a+b*arcc
sch(c*x))*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/c^2-3/2*b^3*polylog(2,(1/c/x+
(1+1/c^2/x^2)^(1/2))^2)/c^2+3/2*b*x*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2
)/c
```

Rubi [A]
time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {6421, 5560, 4269, 3797, 2221, 2317, 2438}

$$-\frac{3b^2\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)}{c^2}+\frac{3bx\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{3b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^2}+\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{3b^3\text{Li}_2\left(e^{2\text{csch}^{-1}(cx)}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] (3*b*(a + b*ArcCsch[c*x])^2)/(2*c^2) + (3*b*Sqrt[1 + 1/(c^2*x^2)]*x*(a + b*
ArcCsch[c*x])^2)/(2*c) + (x^2*(a + b*ArcCsch[c*x])^3)/2 - (3*b^2*(a + b*Arc
Csch[c*x])*Log[1 - E^(2*ArcCsch[c*x])])/c^2 - (3*b^3*PolyLog[2, E^(2*ArcCsc
h[c*x])])/(2*c^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int x\left(a+b\text{csch}^{-1}(cx)\right)^3\,dx=-\frac{\text{Subst}\left(\int(a+bx)^3\coth(x)\text{csch}^2(x)\,dx,x,\text{csch}^{-1}(cx)\right)}{c^2}$$

$$=\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{(3b)\text{Subst}\left(\int(a+bx)^2\text{csch}^2(x)\,dx,x,\text{csch}^{-1}(cx)\right)}{2c^2}$$

$$=\frac{3b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{(3b^2)\,\text{Subst}\left(\int\right.}{}$$

$$=\frac{3b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^2}+\frac{3b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\text{csch}\right.$$

$$=\frac{3b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^2}+\frac{3b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\text{csch}\right.$$

$$=\frac{3b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^2}+\frac{3b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\text{csch}\right.$$

$$=\frac{3b\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c^2}+\frac{3b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(a+b\text{csch}^{-1}(cx)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\text{csch}\right.$$

<u>**Mathematica**</u> [A]
time = 0.30, size = 171, normalized size = 1.46

$$\frac{3b^2\left(ac^2x^2+b\left(-1+c\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\right)\text{csch}^{-1}(cx)^2+b^3c^2x^2\text{csch}^{-1}(cx)^3+3b\text{csch}^{-1}(cx)\left(acx\left(2b\sqrt{1+\frac{1}{c^2x^2}}+acx\right)-2b^2\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right)\right)+a\left(acx\left(3b\sqrt{1+\frac{1}{c^2x^2}}+acx\right)-6b^2\log\left(\frac{1}{cx}\right)\right)+3b^3\text{PolyLog}\left(2,e^{-2\text{csch}^{-1}(cx)}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] (3*b^2*(a*c^2*x^2 + b*(-1 + c*Sqrt[1 + 1/(c^2*x^2)]*x))*ArcCsch[c*x]^2 + b^
3*c^2*x^2*ArcCsch[c*x]^3 + 3*b*ArcCsch[c*x]*(a*c*x*(2*b*Sqrt[1 + 1/(c^2*x^2
)] + a*c*x) - 2*b^2*Log[1 - E^(-2*ArcCsch[c*x])]) + a*(a*c*x*(3*b*Sqrt[1 +
1/(c^2*x^2)] + a*c*x) - 6*b^2*Log[1/(c*x)]) + 3*b^3*PolyLog[2, E^(-2*ArcCsc
h[c*x])])/(2*c^2)
```

<u>**Maple**</u> [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int x\left(a+b\,\text{arccsch}(cx)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))^3,x)
```

```
[Out] int(x*(a+b*arccsch(c*x))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

```
[Out] 3/2*a*b^2*x^2*arccsch(c*x)^2 + 1/2*a^3*x^2 + 3/2*(x^2*arccsch(c*x) + x*sqrt
(1/(c^2*x^2) + 1)/c)*a^2*b + 3*(x*sqrt(1/(c^2*x^2) + 1)*arccsch(c*x)/c + lo
g(x)/c^2)*a*b^2 - 1/4*(24*c^2*integrate(1/2*x^3*log(x)/(sqrt(c^2*x^2 + 1)*c
^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 - 24*c^2*integrate(1
/2*x^3*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sq
rt(c^2*x^2 + 1) + 1), x)*log(c)^2 - 2*x^2*log(sqrt(c^2*x^2 + 1) + 1)^3 + 24
*c^2*integrate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^
2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) - 48*c^2*integrate(1/2*sqrt
(c^2*x^2 + 1)*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*
x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*c^2*integrate(1/2*sq
rt(c^2*x^2 + 1)*x^3*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2
 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*c^2*integrate(1/2*x^3*l
og(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*l
og(c) - 48*c^2*integrate(1/2*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^
2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*c^2*i
ntegrate(1/2*x^3*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 +
c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 8*c^2*integrate(1/2*sqrt(c^2*
x^2 + 1)*x^3*log(x)^3/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 +
 1) + 1), x) - 24*c^2*integrate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(x)^2*log(sqrt
(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1)
 + 1), x) + 24*c^2*integrate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(x)*log(sqrt(c^2*
x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) +
1), x) + 8*c^2*integrate(1/2*x^3*log(x)^3/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*
x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*c^2*integrate(1/2*x^3*log(x)^2*log(sq
rt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 +
1) + 1), x) + 24*c^2*integrate(1/2*x^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^2/
(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 12*c^2*
integrate(1/2*sqrt(c^2*x^2 + 1)*x^3*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*
x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 24*integrate(1/2*
sqrt(c^2*x^2 + 1)*x*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*
x^2 + 1) + 1), x)*log(c)^2 + 24*integrate(1/2*x*log(x)/(sqrt(c^2*x^2 + 1)*c
^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)^2 - 24*integrate(1/2*x
*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2
*x^2 + 1) + 1), x)*log(c)^2 + 24*integrate(1/2*sqrt(c^2*x^2 + 1)*x*log(x)^2
```

```
/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) -
 48*integrate(1/2*sqrt(c^2*x^2 + 1)*x*log(x)*log(sqrt(c^2*x^2 + 1) + 1)/(sq
rt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 24*
integrate(1/2*x*log(x)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^
2 + 1) + 1), x)*log(c) - 48*integrate(1/2*x*log(x)*log(sqrt(c^2*x^2 + 1) +
1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c)
 + 24*integrate(1/2*x*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x
^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x)*log(c) + 2*(c^2*x^2 - 2*sqrt(c^2*
x^2 + 1) + 1)*log(c)^3/c^2 + 2*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1))*log
(c)^3/c^2 + 2*(log(c^2*x^2 + 1) - 2*log(sqrt(c^2*x^2 + 1) + 1))*log(c)^3/c^
2 + 6*(c^2*x^2 - 2*sqrt(c^2*x^2 + 1) + 1)*log(c)^2*log(x)/c^2 - 6*(c^2*x^2
- 2*sqrt(c^2*x^2 + 1) + 1)*log(c)^2*log(sqrt(c^2*x^2 + 1) + 1)/c^2 + 4*log(
c)^3*log(sqrt(c^2*x^2 + 1) + 1)/c^2 - 6*log(c)^2*log(sqrt(c^2*x^2 + 1) + 1)
^2/c^2 + 4*log(c)*log(sqrt(c^2*x^2 + 1) + 1)^3/c^2 - 3*(c^2*x^2 - 4*sqrt(c^
2*x^2 + 1) + 3*log(sqrt(c^2*x^2 + 1) + 1) - log(sqrt(c^2*x^2 + 1) - 1) + 1)
*log(c)^2/c^2 + 3*(c^2*x^2 - 6*sqrt(c^2*x^2 + 1) + 6*log(sqrt(c^2*x^2 + 1)
+ 1) + 1)*log(c)^2/c^2 + 8*integrate(1/2*sqrt(c^2*x^2 + 1)*x*log(x)^3/(sqrt
(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*integrate
(1/2*sqrt(c^2*x^2 + 1)*x*log(x)^2*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2
+ 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 24*integrate(1/2*sqrt
(c^2*x^2 + 1)*x*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*
x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 8*integrate(1/2*x*log(x)^3/(sq
rt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) - 24*integra
te(1/2*x*log(x)^2*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c
^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + 24*integrate(1/2*x*log(x)*log(sqrt(c^
2*x^2 + 1) + 1)^2/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqrt(c^2*x^2 + 1)
+ 1), x))*b^3
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*x*arccsch(c*x)^3 + 3*a*b^2*x*arccsch(c*x)^2 + 3*a^2*b*x*arccsc
h(c*x) + a^3*x, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b\operatorname{acsch}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))**3,x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(1/(c*x)))^3,x)
```

```
[Out] int(x*(a + b*asinh(1/(c*x)))^3, x)
```

3.27 $\int \left(a+b\text{csch}^{-1}(cx)\right)^3\,dx$

Optimal. Leaf size=120

$$x\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{6b\left(a+b\text{csch}^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\text{csch}^{-1}(cx)}\right)}{c}+\frac{6b^2\left(a+b\text{csch}^{-1}(cx)\right)\text{PolyLog}\left(2,-e^{\text{csch}^{-1}}\right.}{c}$$

```
[Out] x*(a+b*arccsch(c*x))^3+6*b*(a+b*arccsch(c*x))^2*arctanh(1/c/x+(1+1/c^2/x^2)
^(1/2))/c+6*b^2*(a+b*arccsch(c*x))*polylog(2,-1/c/x-(1+1/c^2/x^2)^(1/2))/c-
6*b^2*(a+b*arccsch(c*x))*polylog(2,1/c/x+(1+1/c^2/x^2)^(1/2))/c-6*b^3*polyl
og(3,-1/c/x-(1+1/c^2/x^2)^(1/2))/c+6*b^3*polylog(3,1/c/x+(1+1/c^2/x^2)^(1/2
))/c
```

Rubi [A]
time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6415, 5560, 4267, 2611, 2320, 6724}

$$\frac{6b^2\text{Li}_2\left(-e^{\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)}{c}-\frac{6b^2\text{Li}_2\left(e^{\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)}{c}+x\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{6b\tanh^{-1}\left(e^{\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)^2}{c}-\frac{6b^3\text{Li}_3\left(-e^{\text{csch}^{-1}(cx)}\right)}{c}+\frac{6b^3\text{Li}_3\left(e^{\text{csch}^{-1}(cx)}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] x*(a + b*ArcCsch[c*x])^3 + (6*b*(a + b*ArcCsch[c*x])^2*ArcTanh[E^ArcCsch[c*
x]])/c + (6*b^2*(a + b*ArcCsch[c*x])*PolyLog[2, -E^ArcCsch[c*x]])/c - (6*b^
2*(a + b*ArcCsch[c*x])*PolyLog[2, E^ArcCsch[c*x]])/c - (6*b^3*PolyLog[3, -E
^ArcCsch[c*x]])/c + (6*b^3*PolyLog[3, E^ArcCsch[c*x]])/c
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4267

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)]/(f*fz*I)), x]
 + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 - E^((-I)*e + f*fz*x)
], x], x] + Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1 + E^((-I)*e +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5560

```
Int[Coth[(a_.) + (b_.)*(x_)]^(p_.)*Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) +
(d_.)*(x_))^(m_.), x_Symbol] :> Simp[(-(c + d*x)^m)*(Csch[a + b*x]^n/(b*n))
, x] + Dist[d*(m/(b*n)), Int[(c + d*x)^(m - 1)*Csch[a + b*x]^n, x], x] /; F
reeQ[{a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6415

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-c^(-1), Su
bst[Int[(a + b*x)^n*Csch[x]*Coth[x], x], x, ArcCsch[c*x]], x] /; FreeQ[{a,
b, c, n}, x] && IGtQ[n, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \left(a+b\operatorname{csch}^{-1}(cx)\right)^3\,dx=-\frac{\operatorname{Subst}\left(\int (a+bx)^3\coth(x)\operatorname{csch}(x)\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{c}$$

$$=x\left(a+b\operatorname{csch}^{-1}(cx)\right)^3-\frac{(3b)\operatorname{Subst}\left(\int (a+bx)^2\operatorname{csch}(x)\,dx,x,\operatorname{csch}^{-1}(cx)\right)}{c}$$

$$=x\left(a+b\operatorname{csch}^{-1}(cx)\right)^3+\frac{6b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c}+\frac{(6b^2)\operatorname{Subst}(}{}$$

$$=x\left(a+b\operatorname{csch}^{-1}(cx)\right)^3+\frac{6b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c}+\frac{6b^2(a+b\operatorname{csc}}{}$$

$$=x\left(a+b\operatorname{csch}^{-1}(cx)\right)^3+\frac{6b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c}+\frac{6b^2(a+b\operatorname{csc}}{}$$

$$=x\left(a+b\operatorname{csch}^{-1}(cx)\right)^3+\frac{6b\left(a+b\operatorname{csch}^{-1}(cx)\right)^2\tanh^{-1}\left(e^{\operatorname{csch}^{-1}(cx)}\right)}{c}+\frac{6b^2(a+b\operatorname{csc}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 246 vs. 2(120) = 240.
time = 0.22, size = 246, normalized size = 2.05

$$a^3x+3a^2bx\operatorname{csch}^{-1}(cx)+\frac{3a^2b\log\left(cx\left(1+\sqrt{\frac{1+c^2x^2}{c^2x^2}}\right)\right)}{c}+\frac{3ab^2\left(\operatorname{csch}^{-1}(cx)\left(cx\operatorname{csch}^{-1}(cx)-2\log\left(1-e^{-\operatorname{csch}^{-1}(cx)}\right)+2\log\left(1+e^{-\operatorname{csch}^{-1}(cx)}\right)\right)-2\text{PolyLog}\left(2,-e^{-\operatorname{csch}^{-1}(cx)}\right)+2\text{PolyLog}\left(2,e^{-\operatorname{csch}^{-1}(cx)}\right)\right)}{c}+\frac{b^3\left(cx\operatorname{csch}^{-1}(cx)^3-3\operatorname{csch}^{-1}(cx)^2\log\left(1-e^{-\operatorname{csch}^{-1}(cx)}\right)+3\operatorname{csch}^{-1}(cx)^2\log\left(1+e^{-\operatorname{csch}^{-1}(cx)}\right)-6\operatorname{csch}^{-1}(cx)\text{PolyLog}\left(2,-e^{-\operatorname{csch}^{-1}(cx)}\right)+6\operatorname{csch}^{-1}(cx)\text{PolyLog}\left(2,e^{-\operatorname{csch}^{-1}(cx)}\right)-6\text{PolyLog}\left(3,-e^{-\operatorname{csch}^{-1}(cx)}\right)+6\text{PolyLog}\left(3,e^{-\operatorname{csch}^{-1}(cx)}\right)\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] a^3*x + 3*a^2*b*x*ArcCsch[c*x] + (3*a^2*b*Log[c*x*(1 + Sqrt[(1 + c^2*x^2)/(
c^2*x^2)])])/c + (3*a*b^2*(ArcCsch[c*x]*(c*x*ArcCsch[c*x] - 2*Log[1 - E^(-A
rcCsch[c*x])] + 2*Log[1 + E^(-ArcCsch[c*x])]) - 2*PolyLog[2, -E^(-ArcCsch[c
*x])] + 2*PolyLog[2, E^(-ArcCsch[c*x])]))/c + (b^3*(c*x*ArcCsch[c*x]^3 - 3*
ArcCsch[c*x]^2*Log[1 - E^(-ArcCsch[c*x])] + 3*ArcCsch[c*x]^2*Log[1 + E^(-Ar
cCsch[c*x])] - 6*ArcCsch[c*x]*PolyLog[2, -E^(-ArcCsch[c*x])] + 6*ArcCsch[c*
x]*PolyLog[2, E^(-ArcCsch[c*x])] - 6*PolyLog[3, -E^(-ArcCsch[c*x])] + 6*Pol
yLog[3, E^(-ArcCsch[c*x])]))/c
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \left(a+b\operatorname{arccsch}(cx)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^3,x)
```

```
[Out] int((a+b*arccsch(c*x))^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*log(sqrt(c^2*x^2 + 1) + 1)^3 + a^3*x + 3/2*(2*c*x*arccsch(c*x) + log(
sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*a^2*b/c - inte
grate((b^3*log(c)^3 - 3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)^3 + (b^
3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c
^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c^2*log
(c) - a*b^2*c^2)*x^2 + (b^3*c^2*x^2 + b^3)*log(x) + sqrt(c^2*x^2 + 1)*(b^3*
log(c) - a*b^2 + (b^3*c^2*(log(c) + 1) - a*b^2*c^2)*x^2 + (b^3*c^2*x^2 + b^
3)*log(x)))*log(sqrt(c^2*x^2 + 1) + 1)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c)
 + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log(x) - 3*(b^3*log(c)^2 -
```

```
2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2
 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2
)*log(x) + (b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2
*log(c))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*
c^2*log(c) - a*b^2*c^2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 +
1) + 1) + (b^3*log(c)^3 - 3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)^3 +
 (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 + (b
^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c)
 + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log(x))*sqrt(c^2*x^2 + 1))/
(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(b^3*arccsch(c*x)^3 + 3*a*b^2*arccsch(c*x)^2 + 3*a^2*b*arccsch(c*x)
 + a^3, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{acsch}\,(cx))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))^3,x)
[Out] int((a + b*asinh(1/(c*x)))^3, x)
```

3.28 $\int \frac{\left(a+b\mathbf{csch}^{-1}(cx)\right)^3}{x} \, dx$

Optimal. Leaf size=110

$$\frac{\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b}-\left(a+b\text{csch}^{-1}(cx)\right)^3\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)-\frac{3}{2}b\left(a+b\text{csch}^{-1}(cx)\right)^2\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)$$

```
[Out] 1/4*(a+b*arccsch(c*x))^4/b-(a+b*arccsch(c*x))^3*ln(1-(1/c/x+(1+1/c^2/x^2)^(
1/2))^2)-3/2*b*(a+b*arccsch(c*x))^2*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))^2
)+3/2*b^2*(a+b*arccsch(c*x))*polylog(3,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)-3/4*b
^3*polylog(4,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)
```

Rubi [A]
time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6421, 3797, 2221, 2611, 6744, 2320, 6724}

$$\frac{3}{2}b^2\text{Li}_3\left(e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)-\frac{3}{2}b\text{Li}_2\left(e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)^2+\frac{\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b}-\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)^3-\frac{3}{4}b^3\text{Li}_4\left(e^{2\text{csch}^{-1}(cx)}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^3/x,x]
```

```
[Out] (a + b*ArcCsch[c*x])^4/(4*b) - (a + b*ArcCsch[c*x])^3*Log[1 - E^(2*ArcCsch[
c*x])] - (3*b*(a + b*ArcCsch[c*x])^2*PolyLog[2, E^(2*ArcCsch[c*x])])/2 + (3
*b^2*(a + b*ArcCsch[c*x])*PolyLog[3, E^(2*ArcCsch[c*x])])/2 - (3*b^3*PolyLo
g[4, E^(2*ArcCsch[c*x])])/4
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x}\,dx = -\text{Subst}\left(\int (a+bx)^3\coth(x)\,dx, x, \text{csch}^{-1}(cx)\right)$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b} + 2\text{Subst}\left(\int \frac{e^{2x}(a+bx)^3}{1-e^{2x}}\,dx, x, \text{csch}^{-1}(cx)\right)$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b} - \left(a+b\text{csch}^{-1}(cx)\right)^3\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) + (3b)\text{Subst}\left(\int (\right.$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b} - \left(a+b\text{csch}^{-1}(cx)\right)^3\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) - \frac{3}{2}b\big(a+b\text{csch}^{-1}$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b} - \left(a+b\text{csch}^{-1}(cx)\right)^3\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) - \frac{3}{2}b\big(a+b\text{csch}^{-1}$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b} - \left(a+b\text{csch}^{-1}(cx)\right)^3\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) - \frac{3}{2}b\big(a+b\text{csch}^{-1}$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^4}{4b} - \left(a+b\text{csch}^{-1}(cx)\right)^3\log\left(1-e^{2\text{csch}^{-1}(cx)}\right) - \frac{3}{2}b\big(a+b\text{csch}^{-1}$$

Mathematica [A]
time = 0.12, size = 198, normalized size = 1.80

$$\frac{1}{4}\left(4a^3\log(cx)+6a^2b\left(-\text{csch}^{-1}(cx)\left(\text{csch}^{-1}(cx)+2\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right)\right)+\text{PolyLog}\left(2,e^{-2\text{csch}^{-1}(cx)}\right)\right)+2ab^2\left(2\text{csch}^{-1}(cx)^2\left(\text{csch}^{-1}(cx)-3\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)\right)-6\text{csch}^{-1}(cx)\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)+3\text{PolyLog}\left(3,e^{2\text{csch}^{-1}(cx)}\right)\right)+b^3\left(\text{csch}^{-1}(cx)^4-4\text{csch}^{-1}(cx)^3\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)-6\text{csch}^{-1}(cx)^2\text{PolyLog}\left(2,e^{2\text{csch}^{-1}(cx)}\right)+6\text{csch}^{-1}(cx)\text{PolyLog}\left(3,e^{2\text{csch}^{-1}(cx)}\right)-3\text{PolyLog}\left(4,e^{2\text{csch}^{-1}(cx)}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^3/x,x]
```

```
[Out] (4*a^3*Log[c*x] + 6*a^2*b*(-(ArcCsch[c*x]*(ArcCsch[c*x] + 2*Log[1 - E^(-2*A
rcCsch[c*x])])) + PolyLog[2, E^(-2*ArcCsch[c*x])]) + 2*a*b^2*(2*ArcCsch[c*x
]^2*(ArcCsch[c*x] - 3*Log[1 - E^(2*ArcCsch[c*x])]) - 6*ArcCsch[c*x]*PolyLog
[2, E^(2*ArcCsch[c*x])] + 3*PolyLog[3, E^(2*ArcCsch[c*x])]) + b^3*(ArcCsch[
c*x]^4 - 4*ArcCsch[c*x]^3*Log[1 - E^(2*ArcCsch[c*x])] - 6*ArcCsch[c*x]^2*Po
lyLog[2, E^(2*ArcCsch[c*x])] + 6*ArcCsch[c*x]*PolyLog[3, E^(2*ArcCsch[c*x])
] - 3*PolyLog[4, E^(2*ArcCsch[c*x])]))/4
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\text{arccsch}(cx)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^3/x,x)
```

```
[Out] int((a+b*arccsch(c*x))^3/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x,x, algorithm="maxima")
```

```
[Out] b^3*log(x)*log(sqrt(c^2*x^2 + 1) + 1)^3 + a^3*log(x) - integrate((b^3*log(c
)^3 - 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + (b^3*c^2*x^2 + b^3)*log(x)^3 + (b
^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2 + 3*a^2*b*c^2*log(c))*x^2 + 3*(b^3*l
og(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log(c)
- a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2 + (b^3*c^2*x^2 + b^3)*log(x) + s
qrt(c^2*x^2 + 1)*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2 + (
2*b^3*c^2*x^2 + b^3)*log(x)))*log(sqrt(c^2*x^2 + 1) + 1)^2 + 3*(b^3*log(c)^
2 - 2*a*b^2*log(c) + a^2*b + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c) + a^2*b
*c^2)*x^2)*log(x) - 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + a^2*b + (b^3*c^2*log
(c)^2 - 2*a*b^2*c^2*log(c) + a^2*b*c^2)*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2
+ 2*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x) + (b^3*l
og(c)^2 - 2*a*b^2*log(c) + a^2*b + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c) +
 a^2*b*c^2)*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b
^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2
 + 1) + 1) + (b^3*log(c)^3 - 3*a*b^2*log(c)^2 + 3*a^2*b*log(c) + (b^3*c^2*x
^2 + b^3)*log(x)^3 + (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2 + 3*a^2*b*c^2
*log(c))*x^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*lo
g(x)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + a^2*b + (b^3*c^2*log(c)^2 - 2*a
*b^2*c^2*log(c) + a^2*b*c^2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^3 + (c^
2*x^3 + x)*sqrt(c^2*x^2 + 1) + x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccsch(c*x)^3 + 3*a*b^2*arccsch(c*x)^2 + 3*a^2*b*arccsch(c*x
) + a^3)/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acsch}\left(cx\right)\right)^{3}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**3/x,x)
```

```
[Out] Integral((a + b*acsch(c*x))**3/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))^3/x,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))^3/x, x)
```

3.29 $\int \frac{\left(a+b\mathbf{csch}^{-1}(cx)\right)^3}{x^2}\,dx$

Optimal. Leaf size=78

$$6b^3c\sqrt{1+\frac{1}{c^2x^2}}-\frac{6b^2\left(a+b\text{csch}^{-1}(cx)\right)}{x}+3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x}$$

```
[Out] -6*b^2*(a+b*arccsch(c*x))/x-(a+b*arccsch(c*x))^3/x+6*b^3*c*(1+1/c^2/x^2)^(1
/2)+3*b*c*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6421, 3377, 2718}

$$-\frac{6b^2\left(a+b\text{csch}^{-1}(cx)\right)}{x}+3bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x}+6b^3c\sqrt{\frac{1}{c^2x^2}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^3/x^2,x]
```

```
[Out] 6*b^3*c*Sqrt[1 + 1/(c^2*x^2)] - (6*b^2*(a + b*ArcCsch[c*x]))/x + 3*b*c*Sqrt
[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x])^2 - (a + b*ArcCsch[c*x])^3/x
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^2}\,dx = -\left(c\text{Subst}\left(\int (a+bx)^3\cosh(x)\,dx, x, \text{csch}^{-1}(cx)\right)\right)$$
$$= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x} + (3bc)\text{Subst}\left(\int (a+bx)^2\sinh(x)\,dx, x, \text{csch}^{-1}(cx)\right)$$
$$= 3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2 - \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x} - \left(6b^2c\right)\text{Subst}\left(\int (a+$$
$$= -\frac{6b^2\left(a+b\text{csch}^{-1}(cx)\right)}{x} + 3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2 - \frac{\left(a+b\text{csch}^{-1}(cx\right.}{x}$$
$$= 6b^3c\sqrt{1+\frac{1}{c^2x^2}} - \frac{6b^2\left(a+b\text{csch}^{-1}(cx)\right)}{x} + 3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2 -$$

Mathematica [A]
time = 0.15, size = 132, normalized size = 1.69

$$-\frac{a^3+6ab^2-3a^2bc\sqrt{1+\frac{1}{c^2x^2}}\,x-6b^3c\sqrt{1+\frac{1}{c^2x^2}}\,x+3b\left(a^2+2b^2-2abc\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\text{csch}^{-1}(cx)+3b^2\left(a-bc\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\text{csch}^{-1}(cx)^2+b^3\text{csch}^{-1}(cx)^3}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^3/x^2,x]
```

```
[Out] -((a^3 + 6*a*b^2 - 3*a^2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x - 6*b^3*c*Sqrt[1 + 1/(
c^2*x^2)]*x + 3*b*(a^2 + 2*b^2 - 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x)*ArcCsch[c
*x] + 3*b^2*(a - b*c*Sqrt[1 + 1/(c^2*x^2)]*x)*ArcCsch[c*x]^2 + b^3*ArcCsch[
c*x]^3)/x)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\text{arccsch}(cx)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^3/x^2,x)
```

```
[Out] int((a+b*arccsch(c*x))^3/x^2,x)
```

Maxima [A]
time = 0.27, size = 144, normalized size = 1.85

$$-\frac{b^3\,\text{arcsch}(cx)^3}{x}+3\left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\text{arcsch}(cx)}{x}\right)a^2b+6\left(c\sqrt{\frac{1}{c^2x^2}+1}\,\text{arcsch}(cx)-\frac{1}{x}\right)ab^2+3\left(c\sqrt{\frac{1}{c^2x^2}+1}\,\text{arcsch}(cx)^2+2c\sqrt{\frac{1}{c^2x^2}+1}-\frac{2\,\text{arcsch}(cx)}{x}\right)b^3-\frac{3\,ab^2\,\text{arcsch}(cx)^2}{x}-\frac{a^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^2,x, algorithm="maxima")
```

```
[Out] -b^3*arccsch(c*x)^3/x + 3*(c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*a^2*b
+ 6*(c*sqrt(1/(c^2*x^2) + 1)*arccsch(c*x) - 1/x)*a*b^2 + 3*(c*sqrt(1/(c^2*x
^2) + 1)*arccsch(c*x)^2 + 2*c*sqrt(1/(c^2*x^2) + 1) - 2*arccsch(c*x)/x)*b^3
 - 3*a*b^2*arccsch(c*x)^2/x - a^3/x
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(74) = 148$.
time = 0.57, size = 222, normalized size = 2.85

$$-\frac{b^{3}\log\left(\frac{cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+1}{cx}\right)^{3}-3\,(a^{2}b+2\,b^{3})cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+a^{3}+6\,ab^{2}-3\left(b^{3}cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}-ab^{2}\right)\log\left(\frac{cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+1}{cx}\right)^{2}-3\left(2\,ab^{2}cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}-a^{2}b-2\,b^{3}\right)\log\left(\frac{cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+1}{cx}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^2,x, algorithm="fricas")
```

```
[Out] -(b^3*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^3 - 3*(a^2*b + 2*b
^3)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + a^3 + 6*a*b^2 - 3*(b^3*c*x*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) - a*b^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/
(c*x))^2 - 3*(2*a*b^2*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - a^2*b - 2*b^3)*lo
g((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/x
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsch}\,(cx))^{3}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**3/x**2,x)
```

```
[Out] Integral((a + b*acsch(c*x))**3/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3/x^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))^3/x^2,x)
[Out] int((a + b*asinh(1/(c*x)))^3/x^2, x)
```

3.30 $\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^3}\,dx$

Optimal. Leaf size=123

$$\frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{8x}-\frac{3}{8}b^3c^2\operatorname{csch}^{-1}(cx)-\frac{3b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4x^2}+\frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{4x}-\frac{1}{4}c^2\left(a+b\right.$$

[Out] -3/8*b^3*c^2*arccsch(c*x)-3/4*b^2*(a+b*arccsch(c*x))/x^2-1/4*c^2*(a+b*arccsch(c*x))^3-1/2*(a+b*arccsch(c*x))^3/x^2+3/8*b^3*c*(1+1/c^2/x^2)^(1/2)/x+3/4*b*c*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/x

Rubi [A]

time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6421, 5554, 3392, 32, 2715, 8}

$$-\frac{3b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4x^2}+\frac{3bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{4x}-\frac{1}{4}c^2\left(a+b\operatorname{csch}^{-1}(cx)\right)^3-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{2x^2}+\frac{3b^3c\sqrt{\frac{1}{c^2x^2}+1}}{8x}-\frac{3}{8}b^3c^2\operatorname{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcCsch[c*x])^3/x^3,x]

[Out] (3*b^3*c*Sqrt[1 + 1/(c^2*x^2)])/(8*x) - (3*b^3*c^2*ArcCsch[c*x])/8 - (3*b^2*(a + b*ArcCsch[c*x]))/(4*x^2) + (3*b*c*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x])^2)/(4*x) - (c^2*(a + b*ArcCsch[c*x])^3)/4 - (a + b*ArcCsch[c*x])^3/(2*x^2)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^3}\,dx = -\left(c^2\text{Subst}\left(\int (a+bx)^3\cosh(x)\sinh(x)\,dx, x, \text{csch}^{-1}(cx)\right)\right)$$

$$= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{2x^2} + \frac{1}{2}\left(3bc^2\right)\text{Subst}\left(\int (a+bx)^2\sinh^2(x)\,dx, x, \text{csch}^{-1}(cx)\right)$$

$$= -\frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{4x^2} + \frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2}{4x} - \frac{\left(a+b\text{csch}^{-1}(cx\right.}{2x^2}$$

$$= \frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{8x} - \frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{4x^2} + \frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2}{4x}$$

$$= \frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{8x} - \frac{3}{8}b^3c^2\text{csch}^{-1}(cx) - \frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{4x^2} + \frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\,(}{4}$$

Mathematica [A]

time = 0.20, size = 182, normalized size = 1.48

$$-\frac{4a^3+6ab^2-6a^2bc\sqrt{1+\frac{1}{c^2x^2}}\,x-3b^3c\sqrt{1+\frac{1}{c^2x^2}}\,x+6b\left(2a^2+b^2-2abc\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\text{csch}^{-1}(cx)+6b^2\left(-bc\sqrt{1+\frac{1}{c^2x^2}}\,x+a(2+c^2x^2)\right)\text{csch}^{-1}(cx)^2+2b^3(2+c^2x^2)\,\text{csch}^{-1}(cx)^3+3b(2a^2+b^2)\,c^2x^2\sinh^{-1}\left(\frac{1}{cx}\right)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^3/x^3,x]
```

```
[Out] -1/8*(4*a^3 + 6*a*b^2 - 6*a^2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x - 3*b^3*c*Sqrt[1
+ 1/(c^2*x^2)]*x + 6*b*(2*a^2 + b^2 - 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x)*ArcC
sch[c*x] + 6*b^2*(-(b*c*Sqrt[1 + 1/(c^2*x^2)]*x) + a*(2 + c^2*x^2))*ArcCsch
[c*x]^2 + 2*b^3*(2 + c^2*x^2)*ArcCsch[c*x]^3 + 3*b*(2*a^2 + b^2)*c^2*x^2*Ar
cSinh[1/(c*x)])/x^2
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{arccsch}(cx))^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^3/x^3,x)
```

```
[Out] int((a+b*arccsch(c*x))^3/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^3,x, algorithm="maxima")
```

```
[Out] 3/8*a^2*b*((2*c^4*x*sqrt(1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) -
 c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1)
 - 1))/c - 4*arccsch(c*x)/x^2) - 1/2*b^3*log(sqrt(c^2*x^2 + 1) + 1)^3/x^2 -
 1/2*a^3/x^2 - integrate(1/2*(2*b^3*log(c)^3 - 6*a*b^2*log(c)^2 + 2*(b^3*c^
2*x^2 + b^3)*log(x)^3 + 2*(b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 6
*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(2*b^
3*log(c) - 2*a*b^2 + 2*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + 2*(b^3*c^2*x^2 +
b^3)*log(x) + sqrt(c^2*x^2 + 1)*(2*b^3*log(c) - 2*a*b^2 + (b^3*c^2*(2*log(c
) - 1) - 2*a*b^2*c^2)*x^2 + 2*(b^3*c^2*x^2 + b^3)*log(x)))*log(sqrt(c^2*x^2
 + 1) + 1)^2 + 6*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b
^2*c^2*log(c))*x^2)*log(x) - 6*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*lo
g(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^3*lo
g(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x) + (b^3*log(c)^2 - 2
*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2
+ b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)
*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 + 1) + 1) + 2*(b^3*log(c)^3 -
3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)^3 + (b^3*c^2*log(c)^3 - 3*a*b
```

```
^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)
*x^2)*log(x)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a
*b^2*c^2*log(c))*x^2)*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^5 + x^3 + (c^2*x^5
+ x^3)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(107) = 214$.
time = 0.39, size = 267, normalized size = 2.17

$$-\frac{2\,(b^3c^2x^2+2\,b^3)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^3-3\,(2\,a^2b+b^3)cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+4\,a^3+6\,ab^2+6\left(ab^2c^2x^2-b^3cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+2\,ab^2\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2-3\left(4\,ab^2cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-(2\,a^2b+b^3)c^2x^2-4\,a^2b-2\,b^3\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)}{8\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(b^3*c^2*x^2 + 2*b^3)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(
c*x))^3 - 3*(2*a^2*b + b^3)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 4*a^3 + 6*a
*b^2 + 6*(a*b^2*c^2*x^2 - b^3*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*a*b^2)*
log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 - 3*(4*a*b^2*c*x*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) - (2*a^2*b + b^3)*c^2*x^2 - 4*a^2*b - 2*b^3)*log((
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acsch}\left(cx\right)\right)^{3}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**3/x**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))**3/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}^3}{x^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))^3/x^3,x)
[Out] int((a + b*asinh(1/(c*x)))^3/x^3, x)
```

3.31 $\int \frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^3}{x^4}\,dx$

Optimal. Leaf size=166

$$-\frac{14}{9}b^3c^3\sqrt{1+\frac{1}{c^2x^2}}+\frac{2}{27}b^3c^3\left(1+\frac{1}{c^2x^2}\right)^{3/2}-\frac{2b^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{9x^3}+\frac{4b^2c^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3x}-\frac{2}{3}bc^3\sqrt{1+\frac{}{c^2}}$$

```
[Out] 2/27*b^3*c^3*(1+1/c^2/x^2)^(3/2)-2/9*b^2*(a+b*arccsch(c*x))/x^3+4/3*b^2*c^2
*(a+b*arccsch(c*x))/x-1/3*(a+b*arccsch(c*x))^3/x^3-14/9*b^3*c^3*(1+1/c^2/x^
2)^(1/2)-2/3*b*c^3*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)+1/3*b*c*(a+b*ar
ccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/x^2
```

Rubi [A]
time = 0.13, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6421, 5554, 3392, 3377, 2718, 2713}

$$\frac{4b^2c^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3x}-\frac{2b^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{9x^3}+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^2}{3x^2}-\frac{2}{3}bc^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^2-\frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^3}{3x^3}+\frac{2}{27}b^3c^3\left(\frac{1}{c^2x^2}+1\right)^{3/2}-\frac{14}{9}b^3c^3\sqrt{\frac{1}{c^2x^2}+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^3/x^4,x]
```

```
[Out] (-14*b^3*c^3*Sqrt[1 + 1/(c^2*x^2)])/9 + (2*b^3*c^3*(1 + 1/(c^2*x^2))^(3/2))
/27 - (2*b^2*(a + b*ArcCsch[c*x]))/(9*x^3) + (4*b^2*c^2*(a + b*ArcCsch[c*x]
))/(3*x) - (2*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x])^2)/3 + (b*c*
Sqrt[1 + 1/(c^2*x^2)]*(a + b*ArcCsch[c*x])^2)/(3*x^2) - (a + b*ArcCsch[c*x]
)^3/(3*x^3)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
```

```
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{x^4}\,dx = -\left(c^3\text{Subst}\left(\int (a+bx)^3\cosh(x)\sinh^2(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)\right)$$

$$= -\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^3}{3x^3} + \left(bc^3\right)\text{Subst}\left(\int (a+bx)^2\sinh^3(x)\,dx, x, \operatorname{csch}^{-1}(cx)\right)$$

$$= -\frac{2b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^3} + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{3x^2} - \frac{\left(a+b\operatorname{csch}^{-1}(cx\right.}{3x^3}$$

$$= -\frac{2b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^3} - \frac{2}{3}bc^3\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\operatorname{csch}^{-1}(cx)\right)^2 + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\,(}{}$$

$$= -\frac{2}{9}b^3c^3\sqrt{1+\frac{1}{c^2x^2}} + \frac{2}{27}b^3c^3\left(1+\frac{1}{c^2x^2}\right)^{3/2} - \frac{2b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^3} + \frac{4b^2c^2(a}{}$$

$$= -\frac{14}{9}b^3c^3\sqrt{1+\frac{1}{c^2x^2}} + \frac{2}{27}b^3c^3\left(1+\frac{1}{c^2x^2}\right)^{3/2} - \frac{2b^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{9x^3} + \frac{4b^2c^2(a}{}$$

Mathematica [A]
time = 0.19, size = 200, normalized size = 1.20

$$\frac{-9a^3+2b^3c\sqrt{1+\frac{1}{c^2x^2}}\,x(1-20c^2x^2)+9a^2bc\sqrt{1+\frac{1}{c^2x^2}}\,x(1-2c^2x^2)+6ab^2(-1+6c^2x^2)+3b\left(-9a^2+6abc\sqrt{1+\frac{1}{c^2x^2}}\,x(1-2c^2x^2)+2b^2(-1+6c^2x^2)\right)\operatorname{csch}^{-1}(cx)-9b^2\left(3a+bc\sqrt{1+\frac{1}{c^2x^2}}\,x(-1+2c^2x^2)\right)\operatorname{csch}^{-1}(cx)^2-9b^3\operatorname{csch}^{-1}(cx)^3}{27x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^3/x^4,x]
```

```
[Out] (-9*a^3 + 2*b^3*c*Sqrt[1 + 1/(c^2*x^2)]*x*(1 - 20*c^2*x^2) + 9*a^2*b*c*Sqrt
[1 + 1/(c^2*x^2)]*x*(1 - 2*c^2*x^2) + 6*a*b^2*(-1 + 6*c^2*x^2) + 3*b*(-9*a^
2 + 6*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(1 - 2*c^2*x^2) + 2*b^2*(-1 + 6*c^2*x^2
))*ArcCsch[c*x] - 9*b^2*(3*a + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-1 + 2*c^2*x^2)
)*ArcCsch[c*x]^2 - 9*b^3*ArcCsch[c*x]^3)/(27*x^3)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccsch}(cx)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^3/x^4,x)
```

```
[Out] int((a+b*arccsch(c*x))^3/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*b*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c -
3*arccsch(c*x)/x^3) - 1/3*b^3*log(sqrt(c^2*x^2 + 1) + 1)^3/x^3 - 1/3*a^3/x^
3 - integrate((b^3*log(c)^3 - 3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)
^3 + (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2
+ (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + (3*b^3*log(c) - 3*a*b^2 + 3*
(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + 3*(b^3*c^2*x^2 + b^3)*log(x) + sqrt(c^2*
x^2 + 1)*(3*b^3*log(c) - 3*a*b^2 + (b^3*c^2*(3*log(c) - 1) - 3*a*b^2*c^2)*x
^2 + 3*(b^3*c^2*x^2 + b^3)*log(x)))*log(sqrt(c^2*x^2 + 1) + 1)^2 + 3*(b^3*l
og(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log
(x) - 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*lo
g(c))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2
*log(c) - a*b^2*c^2)*x^2)*log(x) + (b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^
2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*(b^
3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x))*sqrt(c^2*x^2 +
 1))*log(sqrt(c^2*x^2 + 1) + 1) + (b^3*log(c)^3 - 3*a*b^2*log(c)^2 + (b^3*c
^2*x^2 + b^3)*log(x)^3 + (b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2 + 3*
(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*(b^3*l
og(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2)*log
(x))*sqrt(c^2*x^2 + 1))/(c^2*x^6 + x^4 + (c^2*x^6 + x^4)*sqrt(c^2*x^2 + 1))
, x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(144) = 288$.
time = 0.44, size = 301, normalized size = 1.81

$$\frac{36\,ab^2c^2x^2 - 9\,b^3\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^3 - 9\,a^3 - 6\,ab^2 - 9\left(3\,ab^2 + (2\,b^3c^3x^3 - b^3cx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2 + 3\left(12\,b^3c^2x^2 - 9\,a^2b - 2\,b^3 - 6\,(2\,ab^2c^3x^3 - ab^2cx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) - (2\,(9\,a^2b + 20\,b^3)c^3x^3 - (9\,a^2b + 2\,b^3)cx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{27\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^4,x, algorithm="fricas")
```

```
[Out] 1/27*(36*a*b^2*c^2*x^2 - 9*b^3*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/
(c*x))^3 - 9*a^3 - 6*a*b^2 - 9*(3*a*b^2 + (2*b^3*c^3*x^3 - b^3*c*x)*sqrt((c
^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^
2 + 3*(12*b^3*c^2*x^2 - 9*a^2*b - 2*b^3 - 6*(2*a*b^2*c^3*x^3 - a*b^2*c*x)*s
qrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(
```

```
c*x)) - (2*(9*a^2*b + 20*b^3)*c^3*x^3 - (9*a^2*b + 2*b^3)*c*x)*sqrt((c^2*x^
2 + 1)/(c^2*x^2)))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsch}(cx))^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**3/x**4,x)
[Out] Integral((a + b*acsch(c*x))**3/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^4,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^3/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))^3/x^4,x)
[Out] int((a + b*asinh(1/(c*x)))^3/x^4, x)
```

3.32 $\int \frac{\left(a+b\,\text{csch}^{-1}(cx)\right)^3}{x^5}\,dx$

Optimal. Leaf size=204

$$\frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{128x^3}-\frac{45b^3c^3\sqrt{1+\frac{1}{c^2x^2}}}{256x}+\frac{45}{256}b^3c^4\text{csch}^{-1}(cx)-\frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{32x^4}+\frac{9b^2c^2\left(a+b\text{csch}^{-1}(cx)\right)}{32x^2}$$

```
[Out] 45/256*b^3*c^4*arccsch(c*x)-3/32*b^2*(a+b*arccsch(c*x))/x^4+9/32*b^2*c^2*(a
+b*arccsch(c*x))/x^2+3/32*c^4*(a+b*arccsch(c*x))^3-1/4*(a+b*arccsch(c*x))^3
/x^4+3/128*b^3*c*(1+1/c^2/x^2)^(1/2)/x^3-45/256*b^3*c^3*(1+1/c^2/x^2)^(1/2)
/x+3/16*b*c*(a+b*arccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/x^3-9/32*b*c^3*(a+b*ar
ccsch(c*x))^2*(1+1/c^2/x^2)^(1/2)/x
```

Rubi [A]
time = 0.13, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6421, 5554, 3392, 32, 2715, 8}

$$\frac{9b^2c^2\left(a+b\text{csch}^{-1}(cx)\right)}{32x^2}-\frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{32x^4}+\frac{3}{32}c^4\left(a+b\text{csch}^{-1}(cx)\right)^3+\frac{3bc\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{16x^3}-\frac{9bc^3\sqrt{\frac{1}{c^2x^2}+1}\left(a+b\text{csch}^{-1}(cx)\right)^2}{32x}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4x^4}+\frac{45}{256}b^3c^4\text{csch}^{-1}(cx)+\frac{3b^3c\sqrt{\frac{1}{c^2x^2}+1}}{128x^3}-\frac{45b^3c^3\sqrt{\frac{1}{c^2x^2}+1}}{256x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])^3/x^5,x]
```

```
[Out] (3*b^3*c*Sqrt[1 + 1/(c^2*x^2)])/(128*x^3) - (45*b^3*c^3*Sqrt[1 + 1/(c^2*x^2
)])/(256*x) + (45*b^3*c^4*ArcCsch[c*x])/256 - (3*b^2*(a + b*ArcCsch[c*x]))/
(32*x^4) + (9*b^2*c^2*(a + b*ArcCsch[c*x]))/(32*x^2) + (3*b*c*Sqrt[1 + 1/(c
^2*x^2)]*(a + b*ArcCsch[c*x])^2)/(16*x^3) - (9*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*
(a + b*ArcCsch[c*x])^2)/(32*x) + (3*c^4*(a + b*ArcCsch[c*x])^3)/32 - (a + b
*ArcCsch[c*x])^3/(4*x^4)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 5554

```
Int[Cosh[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*
(x_)]^(n_.), x_Symbol] :> Simp[(c + d*x)^m*(Sinh[a + b*x]^(n + 1)/(b*(n + 1
))), x] - Dist[d*(m/(b*(n + 1))), Int[(c + d*x)^(m - 1)*Sinh[a + b*x]^(n +
1), x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{x^5}\,dx = -\left(c^4\text{Subst}\left(\int (a+bx)^3\cosh(x)\sinh^3(x)\,dx, x, \text{csch}^{-1}(cx)\right)\right)$$

$$= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)^3}{4x^4} + \frac{1}{4}\left(3bc^4\right)\text{Subst}\left(\int (a+bx)^2\sinh^4(x)\,dx, x, \text{csch}^{-1}(cx)\right)$$

$$= -\frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{32x^4} + \frac{3bc\sqrt{1+\frac{1}{c^2x^2}}\left(a+b\text{csch}^{-1}(cx)\right)^2}{16x^3} - \frac{\left(a+b\text{csch}^{-1}(\right.}{4x^4}$$

$$= \frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{128x^3} - \frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{32x^4} + \frac{9b^2c^2\left(a+b\text{csch}^{-1}(cx)\right)}{32x^2} + \frac{3bc\sqrt{1}}{}$$

$$= \frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{128x^3} - \frac{45b^3c^3\sqrt{1+\frac{1}{c^2x^2}}}{256x} - \frac{3b^2\left(a+b\text{csch}^{-1}(cx)\right)}{32x^4} + \frac{9b^2c^2\left(a+bc\right.}{32x}$$

$$= \frac{3b^3c\sqrt{1+\frac{1}{c^2x^2}}}{128x^3} - \frac{45b^3c^3\sqrt{1+\frac{1}{c^2x^2}}}{256x} + \frac{45}{256}b^3c^4\text{csch}^{-1}(cx) - \frac{3b^2\left(a+b\text{csch}^{-}\right.}{32x^4}$$

Mathematica [A]
time = 0.23, size = 277, normalized size = 1.36

$$\frac{-64a^3-24ab^2+48a^2bc\sqrt{1+\frac{1}{c^2x^2}}\,x+6b^3c\sqrt{1+\frac{1}{c^2x^2}}\,x+72ab^2c^2x^2-72a^2bc^3\sqrt{1+\frac{1}{c^2x^2}}\,x^3-45b^3c^3\sqrt{1+\frac{1}{c^2x^2}}\,x^3-24b\left(8a^2+b^2(1-3c^2x^2)+2abc\sqrt{1+\frac{1}{c^2x^2}}\,x(-2+3c^2x^2)\right)\text{csch}^{-1}(cx)+24b^2\left(bc\sqrt{1+\frac{1}{c^2x^2}}\,x(2-3c^2x^2)+a(-8+3c^4x^4)\right)\text{csch}^{-1}(cx)^2+8b^3(-8+3c^4x^4)\text{csch}^{-1}(cx)^3+9b(8a^2+5b^2)c^4x^4\sinh^{-1}\left(\frac{1}{cx}\right)}{256x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])^3/x^5,x]
```

```
[Out] (-64*a^3 - 24*a*b^2 + 48*a^2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x + 6*b^3*c*Sqrt[1 +
 1/(c^2*x^2)]*x + 72*a*b^2*c^2*x^2 - 72*a^2*b*c^3*Sqrt[1 + 1/(c^2*x^2)]*x^3
 - 45*b^3*c^3*Sqrt[1 + 1/(c^2*x^2)]*x^3 - 24*b*(8*a^2 + b^2*(1 - 3*c^2*x^2)
 + 2*a*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-2 + 3*c^2*x^2))*ArcCsch[c*x] + 24*b^2*
(b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(2 - 3*c^2*x^2) + a*(-8 + 3*c^4*x^4))*ArcCsch[
c*x]^2 + 8*b^3*(-8 + 3*c^4*x^4)*ArcCsch[c*x]^3 + 9*b*(8*a^2 + 5*b^2)*c^4*x^
4*ArcSinh[1/(c*x)])/(256*x^4)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\text{arccsch}(cx)\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))^3/x^5,x)
```

```
[Out] int((a+b*arccsch(c*x))^3/x^5,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^5,x, algorithm="maxima")
```

```
[Out] 3/64*a^2*b*((3*c^5*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) - 3*c^5*log(c*x*sqrt(
1/(c^2*x^2) + 1) - 1) - 2*(3*c^8*x^3*(1/(c^2*x^2) + 1)^(3/2) - 5*c^6*x*sqrt
(1/(c^2*x^2) + 1))/(c^4*x^4*(1/(c^2*x^2) + 1)^2 - 2*c^2*x^2*(1/(c^2*x^2) +
1) + 1))/c - 16*arccsch(c*x)/x^4) - 1/4*b^3*log(sqrt(c^2*x^2 + 1) + 1)^3/x^
4 - 1/4*a^3/x^4 - integrate(1/4*(4*b^3*log(c)^3 - 12*a*b^2*log(c)^2 + 4*(b^
3*c^2*x^2 + b^3)*log(x)^3 + 4*(b^3*c^2*log(c)^3 - 3*a*b^2*c^2*log(c)^2)*x^2
 + 12*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x)^2 + 3*
(4*b^3*log(c) - 4*a*b^2 + 4*(b^3*c^2*log(c) - a*b^2*c^2)*x^2 + 4*(b^3*c^2*x
^2 + b^3)*log(x) + sqrt(c^2*x^2 + 1)*(4*b^3*log(c) - 4*a*b^2 + (b^3*c^2*(4*
log(c) - 1) - 4*a*b^2*c^2)*x^2 + 4*(b^3*c^2*x^2 + b^3)*log(x)))*log(sqrt(c^
2*x^2 + 1) + 1)^2 + 12*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 -
 2*a*b^2*c^2*log(c))*x^2)*log(x) - 12*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3
*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c^2*x^2 + b^3)*log(x)^2 + 2*
(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^2)*x^2)*log(x) + (b^3*log(c
)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^2 - 2*a*b^2*c^2*log(c))*x^2 + (b^3*c
^2*x^2 + b^3)*log(x)^2 + 2*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b^2*c^
2)*x^2)*log(x))*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 + 1) + 1) + 4*(b^3*log(
c)^3 - 3*a*b^2*log(c)^2 + (b^3*c^2*x^2 + b^3)*log(x)^3 + (b^3*c^2*log(c)^3
- 3*a*b^2*c^2*log(c)^2)*x^2 + 3*(b^3*log(c) - a*b^2 + (b^3*c^2*log(c) - a*b
^2*c^2)*x^2)*log(x)^2 + 3*(b^3*log(c)^2 - 2*a*b^2*log(c) + (b^3*c^2*log(c)^
2 - 2*a*b^2*c^2*log(c))*x^2)*log(x))*sqrt(c^2*x^2 + 1))/(c^2*x^7 + x^5 + (c
^2*x^7 + x^5)*sqrt(c^2*x^2 + 1)), x)
```

Fricas [A]
time = 0.49, size = 346, normalized size = 1.70

$$\frac{72\,ab^2c^2x^2+8\,(3\,b^3c^4x^4-8\,b^3)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^3-64\,a^3-24\,ab^2+24\left(3\,ab^2c^4x^4-8\,ab^2-(3\,b^3c^3x^3-2\,b^3cx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)^2+3\left(3\,(8\,a^2b+5\,b^3)c^4x^4+24\,b^3c^2x^2-64\,a^2b-8\,b^3-16\,(3\,ab^2c^3x^3-2\,ab^2cx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-3\,(3\,(8\,a^2b+5\,b^3)c^3x^3-2\,(8\,a^2b+b^3)cx)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{256\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^5,x, algorithm="fricas")
```

```
[Out] 1/256*(72*a*b^2*c^2*x^2 + 8*(3*b^3*c^4*x^4 - 8*b^3)*log((c*x*sqrt((c^2*x^2
+ 1)/(c^2*x^2)) + 1)/(c*x))^3 - 64*a^3 - 24*a*b^2 + 24*(3*a*b^2*c^4*x^4 - 8
*a*b^2 - (3*b^3*c^3*x^3 - 2*b^3*c*x)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*
```

```
x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))^2 + 3*(3*(8*a^2*b + 5*b^3)*c^4*
x^4 + 24*b^3*c^2*x^2 - 64*a^2*b - 8*b^3 - 16*(3*a*b^2*c^3*x^3 - 2*a*b^2*c*x
)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1
)/(c*x)) - 3*(3*(8*a^2*b + 5*b^3)*c^3*x^3 - 2*(8*a^2*b + b^3)*c*x)*sqrt((c^
2*x^2 + 1)/(c^2*x^2)))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsch}(cx))^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))**3/x**5,x)
```

```
[Out] Integral((a + b*acsch(c*x))**3/x**5, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))^3/x^5,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3/x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)^3}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))^3/x^5,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))^3/x^5, x)
```

3.33 $\int \frac{x}{a+b\text{csch}^{-1}(cx)}\,dx$

Optimal. Leaf size=15

$$\text{Int}\left(\frac{x}{a+b\text{csch}^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(x/(a+b*arccsch(c*x)),x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x}{a+b\text{csch}^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[x/(a + b*ArcCsch[c*x]),x]
[Out] Defer[Int][x/(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\int \frac{x}{a+b\text{csch}^{-1}(cx)}\,dx = \int \frac{x}{a+b\text{csch}^{-1}(cx)}\,dx$$

Mathematica [A]
time = 2.64, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b\text{csch}^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x/(a + b*ArcCsch[c*x]),x]
[Out] Integrate[x/(a + b*ArcCsch[c*x]), x]
```

Maple [A]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{x}{a+b\,\text{arccsch}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccsch(c*x)),x)
[Out] int(x/(a+b*arccsch(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] integrate(x/(b*arccsch(c*x) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral(x/(b*arccsch(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{a + b \operatorname{acsch}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acsch(c*x)),x)
[Out] Integral(x/(a + b*acsch(c*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate(x/(b*arccsch(c*x) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{x}{a + b \operatorname{asinh}\left(\frac{1}{c\,x}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + b*asinh(1/(c*x))),x)
[Out] int(x/(a + b*asinh(1/(c*x))), x)
```

3.34 $\int \frac{1}{a+b\mathbf{csch}^{-1}(cx)}\,dx$

Optimal. Leaf size=13

$$\operatorname{Int}\left(\frac{1}{a+b\operatorname{csch}^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(1/(a+b*arccsch(c*x)),x)
```

Rubi [A]
time = 0.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{a+b\operatorname{csch}^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])^(-1),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])^(-1), x]
```

Rubi steps

$$\int \frac{1}{a+b\operatorname{csch}^{-1}(cx)}\,dx=\int \frac{1}{a+b\operatorname{csch}^{-1}(cx)}\,dx$$

Mathematica [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\operatorname{csch}^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])^(-1),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])^(-1), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\,\mathrm{arccsch}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccsch(c*x)),x)
```

```
[Out] int(1/(a+b*arccsch(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arccsch(c*x) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arccsch(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \operatorname{acsch}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acsch(c*x)),x)
```

```
[Out] Integral(1/(a + b*acsch(c*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arccsch(c*x) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{a + b \operatorname{asinh}\left(\frac{1}{c\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*asinh(1/(c*x))),x)
[Out] int(1/(a + b*asinh(1/(c*x))), x)
```

3.35 $\int \frac{1}{x\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left(\frac{1}{x\left(a+b\mathrm{csch}^{-1}(cx)\right)},x\right)$$

[Out] Unintegrable(1/x/(a+b*arccsch(c*x)),x)

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\left(a+b\mathrm{csch}^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

[In] Int[1/(x*(a + b*ArcCsch[c*x])),x]

[Out] Defer[Int][1/(x*(a + b*ArcCsch[c*x])), x]

Rubi steps

$$\int \frac{1}{x\left(a+b\mathrm{csch}^{-1}(cx)\right)}\,dx=\int \frac{1}{x\left(a+b\mathrm{csch}^{-1}(cx)\right)}\,dx$$

Mathematica [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\mathrm{csch}^{-1}(cx)\right)}\,dx$$

Verification is not applicable to the result.

[In] Integrate[1/(x*(a + b*ArcCsch[c*x])),x]

[Out] Integrate[1/(x*(a + b*ArcCsch[c*x])), x]

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{arccsch}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arccsch(c*x)),x)
```

```
[Out] int(1/x/(a+b*arccsch(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccsch(c*x) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x*arccsch(c*x) + a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,(a + b\,\mathrm{acsch}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*acsch(c*x)),x)
```

```
[Out] Integral(1/(x*(a + b*acsch(c*x))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*arccsch(c*x) + a)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*asinh(1/(c*x)))),x)
[Out] int(1/(x*(a + b*asinh(1/(c*x)))), x)
```

3.36 $\int \frac{1}{x^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=46

$$-\frac{c\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{b}+\frac{c\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{b}$$

```
[Out] -c*Chi(a/b+arccsch(c*x))*cosh(a/b)/b+c*Shi(a/b+arccsch(c*x))*sinh(a/b)/b
```

Rubi [A]
time = 0.08, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6421, 3384, 3379, 3382}

$$\frac{c\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{b}-\frac{c\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*ArcCsch[c*x])),x]
```

```
[Out] -((c*Cosh[a/b]*CoshIntegral[a/b + ArcCsch[c*x]])/b) + (c*Sinh[a/b]*SinhInte
gral[a/b + ArcCsch[c*x]])/b
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
```

```
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\int \frac{1}{x^2 \left(a+b\text{csch}^{-1}(cx)\right)} \, dx = -\left(c\text{Subst}\left(\int \frac{\cosh(x)}{a+bx} \, dx, x, \text{csch}^{-1}(cx)\right)\right)$$
$$= -\left(\left(c\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx} \, dx, x, \text{csch}^{-1}(cx)\right)\right)+\left(c\sinh\left(\frac{a}{b}\right)\right)$$
$$= -\frac{c\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{b}+\frac{c\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{b}$$

Mathematica [**A**]
time = 0.06, size = 44, normalized size = 0.96

$$-\frac{c\left(\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*ArcCsch[c*x])),x]
[Out] -((c*(Cosh[a/b]*CoshIntegral[a/b + ArcCsch[c*x]] - Sinh[a/b]*SinhIntegral[a
/b + ArcCsch[c*x]]))/b)
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a+b\,\text{arccsch}\,(cx)\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arccsch(c*x)),x)
[Out] int(1/x^2/(a+b*arccsch(c*x)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccsch(c*x) + a)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x^2*arccsch(c*x) + a*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a + b \operatorname{acsch}\left(cx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*acsch(c*x)),x)
[Out] Integral(1/(x**2*(a + b*acsch(c*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate(1/((b*arccsch(c*x) + a)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*asinh(1/(c*x)))),x)
[Out] int(1/(x^2*(a + b*asinh(1/(c*x)))), x)
```

3.37 $\int \frac{1}{x^3\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=63

$$\frac{c^2\mathrm{Chi}\left(\frac{2a}{b}+2\mathrm{csch}^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2b}-\frac{c^2\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\mathrm{csch}^{-1}(cx)\right)}{2b}$$

```
[Out] -1/2*c^2*cosh(2*a/b)*Shi(2*a/b+2*arccsch(c*x))/b+1/2*c^2*Chi(2*a/b+2*arccsc
h(c*x))*sinh(2*a/b)/b
```

Rubi [A]
time = 0.11, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6421, 5556, 12, 3384, 3379, 3382}

$$\frac{c^2\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\mathrm{csch}^{-1}(cx)\right)}{2b}-\frac{c^2\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\mathrm{csch}^{-1}(cx)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b*ArcCsch[c*x])),x]
```

```
[Out] (c^2*CoshIntegral[(2*a)/b + 2*ArcCsch[c*x]]*Sinh[(2*a)/b])/(2*b) - (c^2*Cos
h[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCsch[c*x]])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3\left(a+b\text{csch}^{-1}(cx)\right)}\,dx &= -\left(c^2\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx,x,\text{csch}^{-1}(cx)\right)\right)\\ &= -\left(c^2\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx,x,\text{csch}^{-1}(cx)\right)\right)\\ &= -\left(\frac{1}{2}c^2\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx,x,\text{csch}^{-1}(cx)\right)\right)\\ &= -\left(\frac{1}{2}\left(c^2\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,x,\text{csch}^{-1}(cx)\right)\right)+\frac{1}{2}\Big(c\\ &= \frac{c^2\text{Chi}\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{2b}-\frac{c^2\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.05, size = 56, normalized size = 0.89

$$\frac{c^2\left(\text{Chi}\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)-\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\text{csch}^{-1}(cx)\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(a + b*ArcCsch[c*x])),x]
```

```
[Out] (c^2*(CoshIntegral[(2*a)/b + 2*ArcCsch[c*x]]*Sinh[(2*a)/b] - Cosh[(2*a)/b]*
SinhIntegral[(2*a)/b + 2*ArcCsch[c*x]]))/(2*b)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(a+b \,\mathrm{arccsch}\left(c x\right)\right)}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+b*arccsch(c*x)),x)
[Out] int(1/x^3/(a+b*arccsch(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arccsch(c*x) + a)*x^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x^3*arccsch(c*x) + a*x^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{3} \left(a + b \operatorname{acsch}{\left(c x \right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(a+b*acsch(c*x)),x)
[Out] Integral(1/(x**3*(a + b*acsch(c*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate(1/((b*arccsch(c*x) + a)*x^3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x^3\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + b*asinh(1/(c*x)))),x)
[Out] int(1/(x^3*(a + b*asinh(1/(c*x)))), x)
```

3.38 $\int \frac{1}{x^4\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=117

$$\frac{c^3\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\mathrm{csch}^{-1}(cx)\right)}{4b}-\frac{c^3\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\mathrm{csch}^{-1}(cx)\right)}{4b}-\frac{c^3\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{csch}^{-1}(cx)\right)}{4b}+\frac{c^3 s}{}$$

```
[Out] 1/4*c^3*Chi(a/b+arccsch(c*x))*cosh(a/b)/b-1/4*c^3*Chi(3*a/b+3*arccsch(c*x))
*cosh(3*a/b)/b-1/4*c^3*Shi(a/b+arccsch(c*x))*sinh(a/b)/b+1/4*c^3*Shi(3*a/b+
3*arccsch(c*x))*sinh(3*a/b)/b
```

Rubi [A]
time = 0.19, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6421, 5556, 3384, 3379, 3382}

$$\frac{c^3\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\mathrm{csch}^{-1}(cx)\right)}{4b}-\frac{c^3\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\mathrm{csch}^{-1}(cx)\right)}{4b}-\frac{c^3\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\mathrm{csch}^{-1}(cx)\right)}{4b}+\frac{c^3\sinh\left(\frac{3a}{b}\right)\mathrm{Shi}\left(\frac{3a}{b}+3\mathrm{csch}^{-1}(cx)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(a + b*ArcCsch[c*x])),x]
```

```
[Out] (c^3*Cosh[a/b]*CoshIntegral[a/b + ArcCsch[c*x]])/(4*b) - (c^3*Cosh[(3*a)/b]
*CoshIntegral[(3*a)/b + 3*ArcCsch[c*x]])/(4*b) - (c^3*Sinh[a/b]*SinhIntegra
l[a/b + ArcCsch[c*x]])/(4*b) + (c^3*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*
ArcCsch[c*x]])/(4*b)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5556

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 6421

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
-(c^(m + 1))^(-1), Subst[Int[(a + b*x)^n*Csch[x]^(m + 1)*Coth[x], x], x, Ar
cCsch[c*x]], x] /; FreeQ[{a, b, c}, x] && IntegerQ[n] && IntegerQ[m] && (Gt
Q[n, 0] || LtQ[m, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{x^4\left(a+b\text{csch}^{-1}(cx)\right)}\,dx &= -\left(c^3\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx, x, \text{csch}^{-1}(cx)\right)\right)\\
&= -\left(c^3\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)\,dx, x, \text{csch}^{-1}(cx)\right)\right)\\
&= \frac{1}{4}c^3\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \text{csch}^{-1}(cx)\right)-\frac{1}{4}c^3\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx, x, \text{csc}\right.\\
&= \frac{1}{4}\left(c^3\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \text{csch}^{-1}(cx)\right)-\frac{1}{4}\left(c^3\cosh\left(\frac{3a}{b}\right.\right.\\
&= \frac{c^3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)}{4b}-\frac{c^3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\text{csch}^{-1}(cx)\right)}{4b}-\frac{c^3\text{s}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 91, normalized size = 0.78

$$-\frac{c^3\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)-\sinh\left(\frac{3a}{b}\right)\text{Shi}\left(3\left(\frac{a}{b}+\text{csch}^{-1}(cx)\right)\right)\right)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(a + b*ArcCsch[c*x])),x]
[Out] -1/4*(c^3*(-(Cosh[a/b]*CoshIntegral[a/b + ArcCsch[c*x]]) + Cosh[(3*a)/b]*Co
shIntegral[3*(a/b + ArcCsch[c*x])] + Sinh[a/b]*SinhIntegral[a/b + ArcCsch[c
*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCsch[c*x])]))/b
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\left(a+b\,\text{arccsch}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(a+b*arccsch(c*x)),x)
```

```
[Out] int(1/x^4/(a+b*arccsch(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccsch(c*x) + a)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*x^4*arccsch(c*x) + a*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \left(a + b \operatorname{acsch}\left(cx\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(a+b*acsch(c*x)),x)
```

```
[Out] Integral(1/(x**4*(a + b*acsch(c*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*arccsch(c*x) + a)*x^4), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^4\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + b*asinh(1/(c*x)))),x)
```

```
[Out] int(1/(x^4*(a + b*asinh(1/(c*x)))), x)
```

3.39 $\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx$

Optimal. Leaf size=19

$$\text{Int}\left((dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^3, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arccsch(c*x))^3,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] Defer[Int][(d*x)^m*(a + b*ArcCsch[c*x])^3, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx=\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx$$

Mathematica [A]
time = 4.58, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^3 \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcCsch[c*x])^3,x]
```

```
[Out] Integrate[(d*x)^m*(a + b*ArcCsch[c*x])^3, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx)^m (a+b\,\text{arccsch}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arccsch(c*x))^3,x)
```

```
[Out] int((d*x)^m*(a+b*arccsch(c*x))^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x))^3,x, algorithm="maxima")
```

```
[Out] b^3*d^m*x*x^m*log(sqrt(c^2*x^2 + 1) + 1)^3/(m + 1) + (d*x)^(m + 1)*a^3/(d*(
m + 1)) - integrate((3*((b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m + 1) - (a*b^
2*c^2*d^m*(m + 1) - (d^m*(m + 1)*log(c) + d^m)*b^3*c^2)*x^2 + (b^3*c^2*d^m*
(m + 1)*x^2 + b^3*d^m*(m + 1))*log(x))*sqrt(c^2*x^2 + 1)*x^m + (b^3*d^m*(m
+ 1)*log(c) - a*b^2*d^m*(m + 1) + (b^3*c^2*d^m*(m + 1)*log(c) - a*b^2*c^2*d
^m*(m + 1))*x^2 + (b^3*c^2*d^m*(m + 1)*x^2 + b^3*d^m*(m + 1))*log(x))*x^m)*
log(sqrt(c^2*x^2 + 1) + 1)^2 + (b^3*d^m*(m + 1)*log(c)^3 - 3*a*b^2*d^m*(m +
 1)*log(c)^2 + 3*a^2*b*d^m*(m + 1)*log(c) + (b^3*c^2*d^m*(m + 1)*x^2 + b^3*
d^m*(m + 1))*log(x)^3 + (b^3*c^2*d^m*(m + 1)*log(c)^3 - 3*a*b^2*c^2*d^m*(m
+ 1)*log(c)^2 + 3*a^2*b*c^2*d^m*(m + 1)*log(c))*x^2 + 3*(b^3*d^m*(m + 1)*lo
g(c) - a*b^2*d^m*(m + 1) + (b^3*c^2*d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m +
 1))*x^2)*log(x)^2 + 3*(b^3*d^m*(m + 1)*log(c)^2 - 2*a*b^2*d^m*(m + 1)*log(
c) + a^2*b*d^m*(m + 1) + (b^3*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b^2*c^2*d^m*(m
 + 1)*log(c) + a^2*b*c^2*d^m*(m + 1))*x^2)*log(x))*sqrt(c^2*x^2 + 1)*x^m +
(b^3*d^m*(m + 1)*log(c)^3 - 3*a*b^2*d^m*(m + 1)*log(c)^2 + 3*a^2*b*d^m*(m +
 1)*log(c) + (b^3*c^2*d^m*(m + 1)*x^2 + b^3*d^m*(m + 1))*log(x)^3 + (b^3*c^
2*d^m*(m + 1)*log(c)^3 - 3*a*b^2*c^2*d^m*(m + 1)*log(c)^2 + 3*a^2*b*c^2*d^m
*(m + 1)*log(c))*x^2 + 3*(b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m + 1) + (b^3
*c^2*d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*x^2)*log(x)^2 + 3*(b^3*d^m
*(m + 1)*log(c)^2 - 2*a*b^2*d^m*(m + 1)*log(c) + a^2*b*d^m*(m + 1) + (b^3*c
^2*d^m*(m + 1)*log(c)^2 - 2*a*b^2*c^2*d^m*(m + 1)*log(c) + a^2*b*c^2*d^m*(m
 + 1))*x^2)*log(x))*x^m - 3*((b^3*d^m*(m + 1)*log(c)^2 - 2*a*b^2*d^m*(m + 1
)*log(c) + a^2*b*d^m*(m + 1) + (b^3*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b^2*c^2*
d^m*(m + 1)*log(c) + a^2*b*c^2*d^m*(m + 1))*x^2 + (b^3*c^2*d^m*(m + 1)*x^2
+ b^3*d^m*(m + 1))*log(x)^2 + 2*(b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m + 1)
 + (b^3*c^2*d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*x^2)*log(x))*sqrt(c
^2*x^2 + 1)*x^m + (b^3*d^m*(m + 1)*log(c)^2 - 2*a*b^2*d^m*(m + 1)*log(c) +
a^2*b*d^m*(m + 1) + (b^3*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b^2*c^2*d^m*(m + 1)
*log(c) + a^2*b*c^2*d^m*(m + 1))*x^2 + (b^3*c^2*d^m*(m + 1)*x^2 + b^3*d^m*(
m + 1))*log(x)^2 + 2*(b^3*d^m*(m + 1)*log(c) - a*b^2*d^m*(m + 1) + (b^3*c^2
*d^m*(m + 1)*log(c) - a*b^2*c^2*d^m*(m + 1))*x^2)*log(x))*x^m)*log(sqrt(c^2
*x^2 + 1) + 1))/(c^2*(m + 1)*x^2 + (c^2*(m + 1)*x^2 + m + 1)*sqrt(c^2*x^2 +
 1) + m + 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral((b^3*arccsch(c*x)^3 + 3*a*b^2*arccsch(c*x)^2 + 3*a^2*b*arccsch(c*x
) + a^3)*(d*x)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b \operatorname{acsch}(cx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*acsch(c*x))**3,x)
```

```
[Out] Integral((d*x)**m*(a + b*acsch(c*x))**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x))^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)^3*(d*x)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*asinh(1/(c*x)))^3,x)
```

```
[Out] int((d*x)^m*(a + b*asinh(1/(c*x)))^3, x)
```

3.40 $\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=19

$$\text{Int}\left((dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^2, x\right)$$

```
[Out] Unintegrable((d*x)^m*(a+b*arccsch(c*x))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m*(a + b*ArcCsch[c*x])^2,x]
[Out] Defer[Int][(d*x)^m*(a + b*ArcCsch[c*x])^2, x]
```

Rubi steps

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^2 dx = \int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^2 dx$$

Mathematica [A]
time = 3.03, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)^2 dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m*(a + b*ArcCsch[c*x])^2,x]
[Out] Integrate[(d*x)^m*(a + b*ArcCsch[c*x])^2, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx)^m (a+b\,\text{arccsch}(cx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arccsch(c*x))^2,x)
```

```
[Out] int((d*x)^m*(a+b*arccsch(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*d^m*x*x^m*log(sqrt(c^2*x^2 + 1) + 1)^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*(
m + 1)) - integrate(-((b^2*d^m*(m + 1)*log(c)^2 - 2*a*b*d^m*(m + 1)*log(c)
+ (b^2*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b*c^2*d^m*(m + 1)*log(c))*x^2 + (b^2*
c^2*d^m*(m + 1)*x^2 + b^2*d^m*(m + 1))*log(x)^2 + 2*(b^2*d^m*(m + 1)*log(c)
 - a*b*d^m*(m + 1) + (b^2*c^2*d^m*(m + 1)*log(c) - a*b*c^2*d^m*(m + 1))*x^2
)*log(x))*sqrt(c^2*x^2 + 1)*x^m + (b^2*d^m*(m + 1)*log(c)^2 - 2*a*b*d^m*(m
+ 1)*log(c) + (b^2*c^2*d^m*(m + 1)*log(c)^2 - 2*a*b*c^2*d^m*(m + 1)*log(c))
*x^2 + (b^2*c^2*d^m*(m + 1)*x^2 + b^2*d^m*(m + 1))*log(x)^2 + 2*(b^2*d^m*(m
 + 1)*log(c) - a*b*d^m*(m + 1) + (b^2*c^2*d^m*(m + 1)*log(c) - a*b*c^2*d^m*
(m + 1))*x^2)*log(x))*x^m - 2*((b^2*d^m*(m + 1)*log(c) - a*b*d^m*(m + 1) -
(a*b*c^2*d^m*(m + 1) - (d^m*(m + 1)*log(c) + d^m)*b^2*c^2)*x^2 + (b^2*c^2*d
^m*(m + 1)*x^2 + b^2*d^m*(m + 1))*log(x))*sqrt(c^2*x^2 + 1)*x^m + (b^2*d^m*
(m + 1)*log(c) - a*b*d^m*(m + 1) + (b^2*c^2*d^m*(m + 1)*log(c) - a*b*c^2*d^
m*(m + 1))*x^2 + (b^2*c^2*d^m*(m + 1)*x^2 + b^2*d^m*(m + 1))*log(x))*x^m)*l
og(sqrt(c^2*x^2 + 1) + 1))/(c^2*(m + 1)*x^2 + (c^2*(m + 1)*x^2 + m + 1)*sqr
t(c^2*x^2 + 1) + m + 1), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*arccsch(c*x)^2 + 2*a*b*arccsch(c*x) + a^2)*(d*x)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b\operatorname{acsch}(cx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*acsch(c*x))**2,x)
```

```
[Out] Integral((d*x)**m*(a + b*acsch(c*x))**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x))^2,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)^2*(d*x)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (d\,x)^m\,{\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*asinh(1/(c*x)))^2,x)
[Out] int((d*x)^m*(a + b*asinh(1/(c*x)))^2, x)
```

3.41 $\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

Optimal. Leaf size=67

$$\frac{(dx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{d(1+m)}+\frac{b(dx)^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};-\frac{1}{c^2x^2}\right)}{cm(1+m)}$$

```
[Out] (d*x)^(1+m)*(a+b*arccsch(c*x))/d/(1+m)+b*(d*x)^m*hypergeom([1/2, -1/2*m],[1
-1/2*m],-1/c^2/x^2)/c/m/(1+m)
```

Rubi [A]

time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6419, 346, 371}

$$\frac{(dx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{d(m+1)}+\frac{b(dx)^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};-\frac{1}{c^2x^2}\right)}{cm(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(a + b*ArcCsch[c*x]),x]
```

```
[Out] ((d*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(d*(1 + m)) + (b*(d*x)^m*Hypergeometri
c2F1[1/2, -1/2*m, 1 - m/2, -(1/(c^2*x^2))])/(c*m*(1 + m))
```

Rule 346

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1), Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x
, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 6419

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[(d*x)^(m + 1)*((a + b*ArcCsch[c*x])/(d*(m + 1))), x] + Dist[b*(d/(c*(m +
 1))), Int[(d*x)^(m - 1)/Sqrt[1 + 1/(c^2*x^2)], x], x] /; FreeQ[{a, b, c, d
, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (dx)^m \left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{(dx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{d(1+m)} + \frac{(bd)\int \frac{(dx)^{-1+m}}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{c(1+m)}$$

$$= \frac{(dx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{d(1+m)} - \frac{\left(b\left(\frac{1}{x}\right)^m (dx)^m\right)\text{Subst}\left(\int \frac{x^{-1-m}}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right.}{c(1+m)}$$

$$= \frac{(dx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{d(1+m)} + \frac{b(dx)^m\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};1-\frac{m}{2};-\frac{1}{c^2x^2}\right)}{cm(1+m)}$$

Mathematica [**A**]
time = 0.16, size = 81, normalized size = 1.21

$$\frac{x(dx)^m\left((1+m)\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}\,x\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};-c^2x^2\right)}{\sqrt{1+c^2x^2}}\right)}{(1+m)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (x*(d*x)^m*((1 + m)*(a + b*ArcCsch[c*x]) + (b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Hyp
ergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/Sqrt[1 + c^2*x^2]))/
(1 + m)^2
```

Maple [**F**]
time = 0.02, size = 0, normalized size = 0.00

$$\int (dx)^m \left(a+b\,\text{arccsch}(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a+b*arccsch(c*x)),x)
```

```
[Out] int((d*x)^m*(a+b*arccsch(c*x)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] (c^2*d^m*integrate(x^2*x^m/(c^2*(m + 1)*x^2 + (c^2*(m + 1)*x^2 + m + 1)*sqr
t(c^2*x^2 + 1) + m + 1), x) - (d^m*x*x^m*log(x) - d^m*x*x^m*log(sqrt(c^2*x^
2 + 1) + 1))/(m + 1) - integrate((c^2*d^m*(m + 1)*x^2*log(c) + d^m*(m + 1)*
log(c) - d^m)*x^m/(c^2*(m + 1)*x^2 + m + 1), x))*b + (d*x)^(m + 1)*a/(d*(m
+ 1))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)*(d*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (dx)^m (a + b\operatorname{acsch}(cx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(a+b*acsch(c*x)),x)
```

```
[Out] Integral((d*x)**m*(a + b*acsch(c*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*(d*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int (d\,x)^m \left(a + b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int((d*x)^m*(a + b*asinh(1/(c*x))), x)
```

3.42 $\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{(dx)^m}{a+b\text{csch}^{-1}(cx)},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arccsch(c*x)),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcCsch[c*x]),x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}\,dx=\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}\,dx$$

Mathematica [A]
time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\text{csch}^{-1}(cx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcCsch[c*x]),x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcCsch[c*x]), x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a+b\,\mathrm{arccsch}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arccsch(c*x)),x)
[Out] int((d*x)^m/(a+b*arccsch(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] integrate((d*x)^m/(b*arccsch(c*x) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral((d*x)^m/(b*arccsch(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{a + b\,\mathrm{acsch}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*acsch(c*x)),x)
[Out] Integral((d*x)**m/(a + b*acsch(c*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((d*x)^m/(b*arccsch(c*x) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*asinh(1/(c*x))),x)
[Out] int((d*x)^m/(a + b*asinh(1/(c*x))), x)
```

3.43 $\int \frac{(dx)^m}{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(\frac{(dx)^m}{\left(a+b\mathrm{csch}^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((d*x)^m/(a+b*arccsch(c*x))^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(dx)^m}{\left(a+b\mathrm{csch}^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(d*x)^m/(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] Defer[Int][(d*x)^m/(a + b*ArcCsch[c*x])^2, x]
```

Rubi steps

$$\int \frac{(dx)^m}{\left(a+b\mathrm{csch}^{-1}(cx)\right)^2}\,dx = \int \frac{(dx)^m}{\left(a+b\mathrm{csch}^{-1}(cx)\right)^2}\,dx$$

Mathematica [A]
time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\mathrm{csch}^{-1}(cx)\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d*x)^m/(a + b*ArcCsch[c*x])^2,x]
```

```
[Out] Integrate[(d*x)^m/(a + b*ArcCsch[c*x])^2, x]
```

Maple [A]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{\left(a+b\,\mathrm{arccsch}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a+b*arccsch(c*x))^2,x)
```

```
[Out] int((d*x)^m/(a+b*arccsch(c*x))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsch(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*d^m*x^3 + d^m*x)*sqrt(c^2*x^2 + 1)*x^m + (c^2*d^m*x^3 + d^m*x)*x^m)/
((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b + (b^2*c^2*x^2 + b^2)*lo
g(x) - (b^2*c^2*x^2 + sqrt(c^2*x^2 + 1)*b^2 + b^2)*log(sqrt(c^2*x^2 + 1) +
1) + sqrt(c^2*x^2 + 1)*(b^2*log(c) + b^2*log(x) - a*b)) - integrate(-((c^2*
d^m*(m + 3)*x^2 + d^m*(m + 1))*(c^2*x^2 + 1)*x^m + (c^4*d^m*(m + 2)*x^4 + c
^2*d^m*(3*m + 5)*x^2 + 2*d^m*(m + 1))*sqrt(c^2*x^2 + 1)*x^m + (c^4*d^m*(m +
 1)*x^4 + 2*c^2*d^m*(m + 1)*x^2 + d^m*(m + 1))*x^m)/((b^2*c^4*log(c) - a*b*
c^4)*x^4 + 2*(b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) + (c^2*x^2 + 1)*(b
^2*log(c) + b^2*log(x) - a*b) - a*b + (b^2*c^4*x^4 + 2*b^2*c^2*x^2 + b^2)*l
og(x) - (b^2*c^4*x^4 + 2*b^2*c^2*x^2 + (c^2*x^2 + 1)*b^2 + b^2 + 2*(b^2*c^2
*x^2 + b^2)*sqrt(c^2*x^2 + 1))*log(sqrt(c^2*x^2 + 1) + 1) + 2*sqrt(c^2*x^2
+ 1)*((b^2*c^2*log(c) - a*b*c^2)*x^2 + b^2*log(c) - a*b + (b^2*c^2*x^2 + b^
2)*log(x))), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsch(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((d*x)^m/(b^2*arccsch(c*x)^2 + 2*a*b*arccsch(c*x) + a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m}{(a + b\,\mathrm{acsch}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m/(a+b*acsch(c*x))**2,x)
```

```
[Out] Integral((d*x)**m/(a + b*acsch(c*x))**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m/(a+b*arccsch(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x)^m/(b*arccsch(c*x) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(d\,x)^m}{{\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m/(a + b*asinh(1/(c*x)))^2,x)
```

```
[Out] int((d*x)^m/(a + b*asinh(1/(c*x)))^2, x)
```

3.44 $\int (d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=167

$$\frac{be(9c^2d^2-e^2)\sqrt{1+\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{2c}+\frac{be^3\sqrt{1+\frac{1}{c^2x^2}}\,x^3}{12c}-\frac{bd^4\text{csch}^{-1}(cx)}{4e}+\frac{(d+ex)^4\left(a+b\text{csc}\right.}{4e}$$

```
[Out] -1/4*b*d^4*arccsch(c*x)/e+1/4*(e*x+d)^4*(a+b*arccsch(c*x))/e+1/2*b*d*(2*c^2
*d^2-e^2)*arctanh((1+1/c^2/x^2)^(1/2))/c^3+1/6*b*e*(9*c^2*d^2-e^2)*x*(1+1/c
^2/x^2)^(1/2)/c^3+1/2*b*d*e^2*x^2*(1+1/c^2/x^2)^(1/2)/c+1/12*b*e^3*x^3*(1+1
/c^2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.27, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {6425, 1582, 1489, 1821, 858, 221, 272, 65, 214}

$$\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}+\frac{bde^2x^2\sqrt{\frac{1}{c^2x^2}+1}}{2c}+\frac{be^3x^3\sqrt{\frac{1}{c^2x^2}+1}}{12c}+\frac{bex\sqrt{\frac{1}{c^2x^2}+1}\,(9c^2d^2-e^2)}{6c^3}+\frac{bd(2c^2d^2-e^2)\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{2c^3}-\frac{bd^4\text{csch}^{-1}(cx)}{4e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^3*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*e*(9*c^2*d^2 - e^2)*Sqrt[1 + 1/(c^2*x^2)]*x)/(6*c^3) + (b*d*e^2*Sqrt[1 +
 1/(c^2*x^2)]*x^2)/(2*c) + (b*e^3*Sqrt[1 + 1/(c^2*x^2)]*x^3)/(12*c) - (b*d^
4*ArcCsch[c*x])/(4*e) + ((d + e*x)^4*(a + b*ArcCsch[c*x]))/(4*e) + (b*d*(2*
c^2*d^2 - e^2)*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(2*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}+\frac{b\int \frac{(d+ex)^4}{\sqrt{1+\frac{1}{c^2x^2}}\,x^2}dx}{4ce}$$

$$= \frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}+\frac{b\int \frac{\left(e+\frac{d}{x}\right)^4x^2}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{4ce}$$

$$= \frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{b\text{Subst}\left(\int \frac{(e+dx)^4}{x^4\sqrt{1+\frac{x^2}{c^2}}}dx,x,\frac{1}{x}\right)}{4ce}$$

$$= \frac{be^3\sqrt{1+\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}+\frac{b\text{Subst}\left(\int \frac{-12de^3-}{}\right.}{}$$

$$= \frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{2c}+\frac{be^3\sqrt{1+\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{(d+ex)^4\left(a+b\text{csch}^{-1}(cx)\right)}{4e}$$

$$= \frac{be(9c^2d^2-e^2)\sqrt{1+\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{2c}+\frac{be^3\sqrt{1+\frac{1}{c^2x^2}}\,x}{12c}$$

$$= \frac{be(9c^2d^2-e^2)\sqrt{1+\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{2c}+\frac{be^3\sqrt{1+\frac{1}{c^2x^2}}\,x}{12c}$$

$$= \frac{be(9c^2d^2-e^2)\sqrt{1+\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{2c}+\frac{be^3\sqrt{1+\frac{1}{c^2x^2}}\,x}{12c}$$

$$= \frac{be(9c^2d^2-e^2)\sqrt{1+\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{2c}+\frac{be^3\sqrt{1+\frac{1}{c^2x^2}}\,x}{12c}$$

$$= \frac{be(9c^2d^2-e^2)\sqrt{1+\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{bde^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{2c}+\frac{be^3\sqrt{1+\frac{1}{c^2x^2}}\,x}{12c}$$

Mathematica [A]

time = 0.16, size = 165, normalized size = 0.99

$$\frac{3ac^3x(4d^3+6d^2ex+4de^2x^2+e^3x^3)+be\sqrt{1+\frac{1}{c^2x^2}}\,x(-2e^2+c^2(18d^2+6dex+e^2x^2))+3bc^3x(4d^3+6d^2ex+4de^2x^2+e^3x^3)\operatorname{csch}^{-1}(cx)+6bd(2c^2d^2-e^2)\log\left(\left(1+\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^3*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (3*a*c^3*x*(4*d^3 + 6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3) + b*e*Sqrt[1 + 1/(c^
2*x^2)]*x*(-2*e^2 + c^2*(18*d^2 + 6*d*e*x + e^2*x^2)) + 3*b*c^3*x*(4*d^3 +
6*d^2*e*x + 4*d*e^2*x^2 + e^3*x^3)*ArcCsch[c*x] + 6*b*d*(2*c^2*d^2 - e^2)*L
og[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(12*c^3)
```

Maple [A]

time = 0.30, size = 269, normalized size = 1.61

method	result
derivativedivides	$\frac{\frac{(ecx+cd)^4 a}{4c^3e}+\frac{b\left(\frac{\operatorname{arccsch}(cx)c^4d^4}{4e}+\operatorname{arccsch}(cx)c^4d^3x+\frac{3e\operatorname{arccsch}(cx)c^4d^2x^2}{2}+e^2\operatorname{arccsch}(cx)c^4d\,x^3+\frac{e^3\operatorname{arccsch}(cx)c^4x^4}{4}+\frac{\sqrt{c^2x^2+\cdots}}{\cdots}\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{(ecx+cd)^4 a}{4c^3e}+\frac{b\left(\frac{\operatorname{arccsch}(cx)c^4d^4}{4e}+\operatorname{arccsch}(cx)c^4d^3x+\frac{3e\operatorname{arccsch}(cx)c^4d^2x^2}{2}+e^2\operatorname{arccsch}(cx)c^4d\,x^3+\frac{e^3\operatorname{arccsch}(cx)c^4x^4}{4}+\frac{\sqrt{c^2x^2+\cdots}}{\cdots}\right.}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/4*(c*e*x+c*d)^4*a/c^3/e+b/c^3*(1/4/e*arccsch(c*x)*c^4*d^4+arccsch(c*
x)*c^4*d^3*x+3/2*e*arccsch(c*x)*c^4*d^2*x^2+e^2*arccsch(c*x)*c^4*d*x^3+1/4*
e^3*arccsch(c*x)*c^4*x^4+1/12/e*(c^2*x^2+1)^(1/2)*(-3*c^4*d^4*arctanh(1/(c^
2*x^2+1)^(1/2))+12*c^3*d^3*e*arcsinh(c*x)+18*c^2*d^2*e^2*(c^2*x^2+1)^(1/2)+
6*c^2*d*e^3*x*(c^2*x^2+1)^(1/2)+e^4*c^2*x^2*(c^2*x^2+1)^(1/2)-6*c*d*e^3*arc
sinh(c*x)-2*e^4*(c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A]

time = 0.27, size = 259, normalized size = 1.55

$$\frac{1}{4}ax^4e^3+adx^3e^2+\frac{3}{2}ad^2x^2e+ad^3x+\frac{3}{2}\left(x^2\operatorname{arcsch}(cx)+\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)bd^2e+\frac{\left(2\,cx\operatorname{arcsch}(cx)+\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)-\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)bd^3}{2\,c}+\frac{1}{4}\left(4\,x^3\operatorname{arcsch}(cx)+\frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}-\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bde^2+\frac{1}{12}\left(3\,x^4\operatorname{arcsch}(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)be^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4*e^3 + a*d*x^3*e^2 + 3/2*a*d^2*x^2*e + a*d^3*x + 3/2*(x^2*arccsch(
c*x) + x*sqrt(1/(c^2*x^2) + 1)/c)*b*d^2*e + 1/2*(2*c*x*arccsch(c*x) + log(s
qrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*b*d^3/c + 1/4*(
4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2)
 - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)
/c)*b*d*e^2 + 1/12*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) -
 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b*e^3
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(143) = 286$.
time = 0.53, size = 879, normalized size = 5.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^3*x^4*cosh(1)^3 + 3*a*c^3*x^4*sinh(1)^3 + 12*a*c^3*d*x^3*cosh(1
)^2 + 18*a*c^3*d^2*x^2*cosh(1) + 12*a*c^3*d^3*x + 3*(3*a*c^3*x^4*cosh(1) +
4*a*c^3*d*x^3)*sinh(1)^2 + 3*(4*b*c^3*d^3 + 6*b*c^3*d^2*cosh(1) + 4*b*c^3*d
*cosh(1)^2 + b*c^3*cosh(1)^3 + b*c^3*sinh(1)^3 + (4*b*c^3*d + 3*b*c^3*cosh(
1))*sinh(1)^2 + (6*b*c^3*d^2 + 8*b*c^3*d*cosh(1) + 3*b*c^3*cosh(1)^2)*sinh(
1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - 6*(2*b*c^2*d^3 - b*d
*cosh(1)^2 - 2*b*d*cosh(1)*sinh(1) - b*d*sinh(1)^2)*log(c*x*sqrt((c^2*x^2 +
 1)/(c^2*x^2)) - c*x) - 3*(4*b*c^3*d^3 + 6*b*c^3*d^2*cosh(1) + 4*b*c^3*d*co
sh(1)^2 + b*c^3*cosh(1)^3 + b*c^3*sinh(1)^3 + (4*b*c^3*d + 3*b*c^3*cosh(1))
*sinh(1)^2 + (6*b*c^3*d^2 + 8*b*c^3*d*cosh(1) + 3*b*c^3*cosh(1)^2)*sinh(1))
*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 3*(4*b*c^3*d^3*x - 4*b*
c^3*d^3 + (b*c^3*x^4 - b*c^3)*cosh(1)^3 + (b*c^3*x^4 - b*c^3)*sinh(1)^3 + 4
*(b*c^3*d*x^3 - b*c^3*d)*cosh(1)^2 + (4*b*c^3*d*x^3 - 4*b*c^3*d + 3*(b*c^3*
x^4 - b*c^3)*cosh(1))*sinh(1)^2 + 6*(b*c^3*d^2*x^2 - b*c^3*d^2)*cosh(1) + (
6*b*c^3*d^2*x^2 - 6*b*c^3*d^2 + 3*(b*c^3*x^4 - b*c^3)*cosh(1)^2 + 8*(b*c^3*
d*x^3 - b*c^3*d)*cosh(1))*sinh(1))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c*x)) + 3*(3*a*c^3*x^4*cosh(1)^2 + 8*a*c^3*d*x^3*cosh(1) + 6*a*c^3*d^2
*x^2)*sinh(1) + (6*b*c^2*d*x^2*cosh(1)^2 + 18*b*c^2*d^2*x*cosh(1) + (b*c^2*
x^3 - 2*b*x)*cosh(1)^3 + (b*c^2*x^3 - 2*b*x)*sinh(1)^3 + 3*(2*b*c^2*d*x^2 +
 (b*c^2*x^3 - 2*b*x)*cosh(1))*sinh(1)^2 + 3*(4*b*c^2*d*x^2*cosh(1) + 6*b*c^
2*d^2*x + (b*c^2*x^3 - 2*b*x)*cosh(1)^2)*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x
^2)))/c^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{acsch}\,(cx))\,(d + ex)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(a+b*acsch(c*x)),x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^3*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(b*arccsch(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,{\left(d+e\,x\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))*(d + e*x)^3,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))*(d + e*x)^3, x)
```

3.45 $\int (d+ex)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

Optimal. Leaf size=122

$$\frac{bde\sqrt{1+\frac{1}{c^2x^2}}\,x}{c}+\frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{6c}-\frac{bd^3\text{csch}^{-1}(cx)}{3e}+\frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e}+\frac{b(6c^2d^2-e^2)\tanh^{-1}\left(\cdots\right.}{6c^3}$$

```
[Out] -1/3*b*d^3*arccsch(c*x)/e+1/3*(e*x+d)^3*(a+b*arccsch(c*x))/e+1/6*b*(6*c^2*d
^2-e^2)*arctanh((1+1/c^2/x^2)^(1/2))/c^3+b*d*e*x*(1+1/c^2/x^2)^(1/2)/c+1/6*
b*e^2*x^2*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.18, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {6425, 1582, 1489, 1821, 858, 221, 272, 65, 214}

$$\frac{(d+ex)^3\left(a+b\text{csch}^{-1}(cx)\right)}{3e}+\frac{bdex\sqrt{\frac{1}{c^2x^2}+1}}{c}+\frac{be^2x^2\sqrt{\frac{1}{c^2x^2}+1}}{6c}+\frac{b(6c^2d^2-e^2)\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{6c^3}-\frac{bd^3\text{csch}^{-1}(cx)}{3e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*d*e*Sqrt[1 + 1/(c^2*x^2)]*x)/c + (b*e^2*Sqrt[1 + 1/(c^2*x^2)]*x^2)/(6*c)
 - (b*d^3*ArcCsch[c*x])/(3*e) + ((d + e*x)^3*(a + b*ArcCsch[c*x]))/(3*e) +
(b*(6*c^2*d^2 - e^2)*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(6*c^3)
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^2 \left(a+b\text{csch}^{-1}(cx)\right) dx = \frac{(d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{b\int \frac{(d+ex)^3}{\sqrt{1+\frac{1}{c^2x^2}}\,x^2}dx}{3ce}$$

$$= \frac{(d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{b\int \frac{\left(e+\frac{d}{x}\right)^3 x}{\sqrt{1+\frac{1}{c^2x^2}}}dx}{3ce}$$

$$= \frac{(d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)}{3e} - \frac{b\text{Subst}\left(\int \frac{(e+dx)^3}{x^3\sqrt{1+\frac{x^2}{c^2}}}dx, x, \frac{1}{x}\right)}{3ce}$$

$$= \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{6c} + \frac{(d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{b\text{Subst}\left(\int \frac{-6de^2-e \cdots}{x^2 \cdots}\right.}{\cdots}$$

$$= \frac{bde\sqrt{1+\frac{1}{c^2x^2}}\,x}{c} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{6c} + \frac{(d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)}{3e} \cdots$$

$$= \frac{bde\sqrt{1+\frac{1}{c^2x^2}}\,x}{c} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{6c} + \frac{(d+ex)^3 \left(a+b\text{csch}^{-1}(cx)\right)}{3e} \cdots$$

$$= \frac{bde\sqrt{1+\frac{1}{c^2x^2}}\,x}{c} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{6c} - \frac{bd^3\text{csch}^{-1}(cx)}{3e} + \frac{(d+ex)^3 \left(a \cdots\right.}{\cdots}$$

$$= \frac{bde\sqrt{1+\frac{1}{c^2x^2}}\,x}{c} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{6c} - \frac{bd^3\text{csch}^{-1}(cx)}{3e} + \frac{(d+ex)^3 \left(a \cdots\right.}{\cdots}$$

$$= \frac{bde\sqrt{1+\frac{1}{c^2x^2}}\,x}{c} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2}{6c} - \frac{bd^3\text{csch}^{-1}(cx)}{3e} + \frac{(d+ex)^3 \left(a \cdots\right.}{\cdots}$$

Mathematica [A]
time = 0.11, size = 122, normalized size = 1.00

$$\frac{c^2x\left(be\sqrt{1+\frac{1}{c^2x^2}}\,(6d+ex)+2ac(3d^2+3dex+e^2x^2)\right)+2bc^3x(3d^2+3dex+e^2x^2)\operatorname{csch}^{-1}(cx)+b(6c^2d^2-e^2)\log\left(\left(1+\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (c^2*x*(b*e*Sqrt[1 + 1/(c^2*x^2)]*(6*d + e*x) + 2*a*c*(3*d^2 + 3*d*e*x + e^
2*x^2)) + 2*b*c^3*x*(3*d^2 + 3*d*e*x + e^2*x^2)*ArcCsch[c*x] + b*(6*c^2*d^2
 - e^2)*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(6*c^3)
```

Maple [A]
time = 0.26, size = 204, normalized size = 1.67

method	result
derivativedivides	$\frac{\frac{(ecx+cd)^3 a}{3c^2e}+\frac{b\left(\frac{\operatorname{arccsch}(cx)c^3d^3}{3e}+\operatorname{arccsch}(cx)c^3d^2x+e\operatorname{arccsch}(cx)c^3dx^2+\frac{e^2\operatorname{arccsch}(cx)c^3x^3}{3}+\frac{\sqrt{c^2x^2+1}\left(-2c^3d^3\operatorname{arctanh}\right(}{}\right.}{c^2}}{c}$
default	$\frac{\frac{(ecx+cd)^3 a}{3c^2e}+\frac{b\left(\frac{\operatorname{arccsch}(cx)c^3d^3}{3e}+\operatorname{arccsch}(cx)c^3d^2x+e\operatorname{arccsch}(cx)c^3dx^2+\frac{e^2\operatorname{arccsch}(cx)c^3x^3}{3}+\frac{\sqrt{c^2x^2+1}\left(-2c^3d^3\operatorname{arctanh}\right(}{}\right.}{c^2}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/3*(c*e*x+c*d)^3*a/c^2/e+b/c^2*(1/3/e*arccsch(c*x)*c^3*d^3+arccsch(c*
x)*c^3*d^2*x+e*arccsch(c*x)*c^3*d*x^2+1/3*e^2*arccsch(c*x)*c^3*x^3+1/6/e*(c
^2*x^2+1)^(1/2)*(-2*c^3*d^3*arctanh(1/(c^2*x^2+1)^(1/2))+6*c^2*d^2*e*arcsin
h(c*x)+6*c*d*e^2*(c^2*x^2+1)^(1/2)+e^3*c*x*(c^2*x^2+1)^(1/2)-e^3*arcsinh(c*
x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A]
time = 0.26, size = 192, normalized size = 1.57

$$\frac{1}{3}ax^3e^2+adx^2e+ad^2x+\left(x^2\operatorname{arcsch}(cx)+\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)bde+\frac{\left(2cx\operatorname{arcsch}(cx)+\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)-\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)bd^2}{2c}+\frac{1}{12}\left(4x^3\operatorname{arcsch}(cx)+\frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}-\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3*e^2 + a*d*x^2*e + a*d^2*x + (x^2*arccsch(c*x) + x*sqrt(1/(c^2*x^2
) + 1)/c)*b*d*e + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1)
- log(sqrt(1/(c^2*x^2) + 1) - 1))*b*d^2/c + 1/12*(4*x^3*arccsch(c*x) + (2*s
qrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) +
 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*e^2
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(106) = 212$.
time = 0.51, size = 543, normalized size = 4.45

$$\frac{2\,ac^3x^3\cosh(1)^2+2\,ac^3x^3\sinh(1)^2+6\,ac^3dx^2\cosh(1)+6\,ac^3d^2x+2\left(3\,bc^3d^2+3\,bc^3d\cosh(1)+bc^3\cosh(1)^2+bc^3\sinh(1)^2+(3\,bc^3d+2\,bc^3\cosh(1))\sinh(1)\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-\left(6\,bc^2d^2-b\cosh(1)^2-2\,b\cosh(1)\sinh(1)-b\sinh(1)^2\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx\right)-2\left(3\,bc^3d^2+3\,bc^3d\cosh(1)+bc^3\cosh(1)^2+bc^3\sinh(1)^2+(3\,bc^3d+2\,bc^3\cosh(1))\sinh(1)\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)+2\left(3\,bc^3d^2x-3\,bc^3d^2+(bc^3x^3-bc^3)\cosh(1)^2+(bc^3x^3-bc^3)\sinh(1)^2+3\,(bc^3dx^2-bc^3d)\cosh(1)+(3\,bc^3dx^2-3\,bc^3d+2\,(bc^3x^3-bc^3)\cosh(1))\sinh(1)\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+2\left(2\,ac^3x^3\cosh(1)+3\,ac^3dx^2\right)\sinh(1)+\left(bc^2x^2\cosh(1)^2+bc^2x^2\sinh(1)^2+6\,bc^2dx\cosh(1)+2\,(bc^2x^2\cosh(1)+3\,bc^2dx)\sinh(1)\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{6\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*x^3*cosh(1)^2 + 2*a*c^3*x^3*sinh(1)^2 + 6*a*c^3*d*x^2*cosh(1)
+ 6*a*c^3*d^2*x + 2*(3*b*c^3*d^2 + 3*b*c^3*d*cosh(1) + b*c^3*cosh(1)^2 + b*
c^3*sinh(1)^2 + (3*b*c^3*d + 2*b*c^3*cosh(1))*sinh(1))*log(c*x*sqrt((c^2*x^
2 + 1)/(c^2*x^2)) - c*x + 1) - (6*b*c^2*d^2 - b*cosh(1)^2 - 2*b*cosh(1)*sin
h(1) - b*sinh(1)^2)*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 2*(3*b*c
^3*d^2 + 3*b*c^3*d*cosh(1) + b*c^3*cosh(1)^2 + b*c^3*sinh(1)^2 + (3*b*c^3*d
 + 2*b*c^3*cosh(1))*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x -
1) + 2*(3*b*c^3*d^2*x - 3*b*c^3*d^2 + (b*c^3*x^3 - b*c^3)*cosh(1)^2 + (b*c^
3*x^3 - b*c^3)*sinh(1)^2 + 3*(b*c^3*d*x^2 - b*c^3*d)*cosh(1) + (3*b*c^3*d*x
^2 - 3*b*c^3*d + 2*(b*c^3*x^3 - b*c^3)*cosh(1))*sinh(1))*log((c*x*sqrt((c^2
*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(2*a*c^3*x^3*cosh(1) + 3*a*c^3*d*x^2)*
sinh(1) + (b*c^2*x^2*cosh(1)^2 + b*c^2*x^2*sinh(1)^2 + 6*b*c^2*d*x*cosh(1)
+ 2*(b*c^2*x^2*cosh(1) + 3*b*c^2*d*x)*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)
))/c^3
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{acsch}\,(cx))\,(d + ex)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*arccsch(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,{\left(d+e\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))*(d + e*x)^2,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))*(d + e*x)^2, x)
```

3.46 $\int (d+ex)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=81

$$\frac{be\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c}-\frac{bd^2\text{csch}^{-1}(cx)}{2e}+\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}+\frac{bd\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}$$

```
[Out] -1/2*b*d^2*arccsch(c*x)/e+1/2*(e*x+d)^2*(a+b*arccsch(c*x))/e+b*d*arctanh((1
+1/c^2/x^2)^(1/2))/c+1/2*b*e*x*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6425, 1582, 1410, 1821, 858, 221, 272, 65, 214}

$$\frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e}+\frac{bd\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c}+\frac{bex\sqrt{\frac{1}{c^2x^2}+1}}{2c}-\frac{bd^2\text{csch}^{-1}(cx)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*e*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*c) - (b*d^2*ArcCsch[c*x])/(2*e) + ((d + e*
x)^2*(a + b*ArcCsch[c*x]))/(2*e) + (b*d*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/c
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1410

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_.)*((a_) + (c_.)*(x_)^(n2_))^(p_), x_Symbol
] :> -Subst[Int[(d + e/x^n)^q*((a + c/x^(2*n))^p/x^2), x], x, 1/x] /; FreeQ
[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && ILtQ[n, 0]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1821

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e} + \frac{b\int \frac{(d+ex)^2}{\sqrt{1+\frac{1}{c^2x^2}}\,x^2}\,dx}{2ce}$$

$$= \frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e} + \frac{b\int \frac{\left(e+\frac{d}{x}\right)^2}{\sqrt{1+\frac{1}{c^2x^2}}}\,dx}{2ce}$$

$$= \frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e} - \frac{b\text{Subst}\left(\int \frac{(e+dx)^2}{x^2\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2ce}$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} + \frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e} + \frac{b\text{Subst}\left(\int \frac{-2de-d^2x}{x\sqrt{1+\frac{x^2}{c^2}}}\right.}{2ce}$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} + \frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e} - \frac{(bd)\text{Subst}\left(\int \frac{1}{x\sqrt{1+}}\right.}{c}$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} - \frac{bd^2\text{csch}^{-1}(cx)}{2e} + \frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e} - \frac{(bd)}{}$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} - \frac{bd^2\text{csch}^{-1}(cx)}{2e} + \frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e} - (bcd$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} - \frac{bd^2\text{csch}^{-1}(cx)}{2e} + \frac{(d+ex)^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e} + \frac{bd\,\text{t}}{}$$

Mathematica [A]

time = 0.27, size = 133, normalized size = 1.64

$$adx + \frac{1}{2}aex^2 + \frac{bex\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{2c} + bdx\operatorname{csch}^{-1}(cx) + \frac{1}{2}bex^2\operatorname{csch}^{-1}(cx) - \frac{bd\sqrt{1+c^2x^2}\log\left(-\sqrt{c^2}\,x+\sqrt{1+c^2x^2}\right)}{c\sqrt{c^2}\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^2)/2 + (b*e*x*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(2*c) + b*d*x*A
rcCsch[c*x] + (b*e*x^2*ArcCsch[c*x])/2 - (b*d*Sqrt[1 + c^2*x^2]*Log[-(Sqrt[
c^2]*x) + Sqrt[1 + c^2*x^2]])/(c*Sqrt[c^2]*Sqrt[1 + 1/(c^2*x^2)]*x)
```

Maple [A]
time = 0.20, size = 115, normalized size = 1.42

method	result	size
derivativedivides	$\frac{\frac{a\left(d\,c^2x+\frac{1}{2}e\,c^2x^2\right)}{c}+\frac{b\left(\operatorname{arccsch}(cx)d\,c^2x+\frac{\operatorname{arccsch}(cx)e\,c^2x^2}{2}+\frac{\sqrt{c^2x^2+1}\left(2dc\operatorname{arcsinh}(cx)+e\sqrt{c^2x^2+1}\right)}{2\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c}}{c}$	115
default	$\frac{\frac{a\left(d\,c^2x+\frac{1}{2}e\,c^2x^2\right)}{c}+\frac{b\left(\operatorname{arccsch}(cx)d\,c^2x+\frac{\operatorname{arccsch}(cx)e\,c^2x^2}{2}+\frac{\sqrt{c^2x^2+1}\left(2dc\operatorname{arcsinh}(cx)+e\sqrt{c^2x^2+1}\right)}{2\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right)}{c}}{c}$	115

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/c*(d*c^2*x+1/2*e*c^2*x^2)+b/c*(arccsch(c*x)*d*c^2*x+1/2*arccsch(c*x)
*e*c^2*x^2+1/2/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x*(c^2*x^2+1)^(1/2)*(2*d*c*arc
sinh(c*x)+e*(c^2*x^2+1)^(1/2))))
```

Maxima [A]
time = 0.26, size = 89, normalized size = 1.10

$$\frac{1}{2}\,ax^2e + adx + \frac{1}{2}\left(x^2\operatorname{arcsch}\left(cx\right) + \frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)be + \frac{\left(2\,cx\operatorname{arcsch}\left(cx\right) + \log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right) - \log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)bd}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2*e + a*d*x + 1/2*(x^2*arccsch(c*x) + x*sqrt(1/(c^2*x^2) + 1)/c)*b*
e + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(
c^2*x^2) + 1) - 1))*b*d/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(71) = 142$.
time = 0.43, size = 252, normalized size = 3.11

$$\frac{acx^2\cosh(1) + acx^2\sinh(1) + 2\,acdx - 2\,bd\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx\right) + (2\,bcd + bc\cosh(1) + bc\sinh(1))\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx + 1\right) - (2\,bcd + bc\cosh(1) + bc\sinh(1))\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}} - cx - 1\right) + (2\,bcdx - 2\,bcd + (bcx^2 - bc)\cosh(1) + (bcx^2 - bc)\sinh(1))\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + (bx\cosh(1) + bx\sinh(1))\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*c*x^2*cosh(1) + a*c*x^2*sinh(1) + 2*a*c*d*x - 2*b*d*log(c*x*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) - c*x) + (2*b*c*d + b*c*cosh(1) + b*c*sinh(1))*log(c*
x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - (2*b*c*d + b*c*cosh(1) + b*c*s
inh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + (2*b*c*d*x - 2*b
*c*d + (b*c*x^2 - b*c)*cosh(1) + (b*c*x^2 - b*c)*sinh(1))*log((c*x*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (b*x*cosh(1) + b*x*sinh(1))*sqrt((c^2*x
^2 + 1)/(c^2*x^2)))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\,\mathrm{acsch}\,(cx))\,(d + ex)\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*acsch(c*x)),x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*arccsch(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,\left(d + e\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))*(d + e*x),x)
[Out] int((a + b*asinh(1/(c*x)))*(d + e*x), x)
```

3.47 $\int \left(a+b\,\mathrm{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=30

$$ax+bx\,\mathrm{csch}^{-1}(cx)+\frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}$$

```
[Out] a*x+b*x*arccsch(c*x)+b*arctanh((1+1/c^2/x^2)^(1/2))/c
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6413, 272, 65, 214}

$$ax+\frac{b\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{c}+bx\,\mathrm{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*ArcCsch[c*x],x]
```

```
[Out] a*x + b*x*ArcCsch[c*x] + (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/c
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6413

```
Int[ArcCsch[(c_.)*(x_)], x_Symbol] :> Simp[x*ArcCsch[c*x], x] + Dist[1/c, I
nt[1/(x*Sqrt[1 + 1/(c^2*x^2)]), x], x] /; FreeQ[c, x]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b\text{csch}^{-1}(cx)\right) dx &= ax+b\int \text{csch}^{-1}(cx)\,dx \\
&= ax+bx\text{csch}^{-1}(cx)+\frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x}\,dx}{c} \\
&= ax+bx\text{csch}^{-1}(cx)-\frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x}{c^2}}}\,dx,x,\frac{1}{x^2}\right)}{2c} \\
&= ax+bx\text{csch}^{-1}(cx)-(bc)\text{Subst}\left(\int \frac{1}{-c^2+c^2x^2}\,dx,x,\sqrt{1+\frac{1}{c^2x^2}}\right) \\
&= ax+bx\text{csch}^{-1}(cx)+\frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 78 vs. 2(30) = 60.
time = 0.05, size = 78, normalized size = 2.60

$$ax+bx\text{csch}^{-1}(cx)-\frac{b\sqrt{1+c^2x^2}\,\log\left(-\sqrt{c^2}\,x+\sqrt{1+c^2x^2}\right)}{c\sqrt{c^2}\,\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*ArcCsch[c*x],x]
```

```
[Out] a*x + b*x*ArcCsch[c*x] - (b*Sqrt[1 + c^2*x^2]*Log[-(Sqrt[c^2]*x) + Sqrt[1 +
 c^2*x^2]])/(c*Sqrt[c^2]*Sqrt[1 + 1/(c^2*x^2)]*x)
```

Maple [A]
time = 0.10, size = 36, normalized size = 1.20

method	result	size

default	$ax+bx\,\mathrm{arccsch}(cx)+\frac{b\ln\left(cx+cx\sqrt{1+\frac{1}{c^2x^2}}\right)}{c}$	36
derivativedivides	$\frac{acx+\mathrm{arccsch}(cx)bcx+\ln\left(cx+cx\sqrt{1+\frac{1}{c^2x^2}}\right)b}{c}$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*arccsch(c*x),x,method=_RETURNVERBOSE)
```

```
[Out] a*x+b*x*arccsch(c*x)+b/c*ln(c*x+c*x*(1+1/c^2/x^2)^(1/2))
```

Maxima [A]
time = 0.25, size = 49, normalized size = 1.63

$$ax+\frac{\left(2\,cx\,\mathrm{arcsch}\,(cx)+\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)-\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)b}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsch(c*x),x, algorithm="maxima")
```

```
[Out] a*x + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1
/(c^2*x^2) + 1) - 1))*b/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(28) = 56.
time = 0.38, size = 143, normalized size = 4.77

$$\frac{acx+bc\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-bc\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)-b\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx\right)+(bcx-bc)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsch(c*x),x, algorithm="fricas")
```

```
[Out] (a*c*x + b*c*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - b*c*log(c*x
*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) - b*log(c*x*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) - c*x) + (b*c*x - b*c)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)
/(c*x)))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{acsch}\,(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*acsch(c*x),x)
```

```
[Out] Integral(a + b*acsch(c*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*arccsch(c*x),x, algorithm="giac")
```

```
[Out] integrate(b*arccsch(c*x) + a, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*asinh(1/(c*x)),x)
```

```
[Out] int(a + b*asinh(1/(c*x)), x)
```

3.48 $\int \frac{a+b\text{csch}^{-1}(cx)}{d+ex}\,dx$

Optimal. Leaf size=215

$$\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right.}{e}$$

```
[Out] -(a+b*arccsch(c*x))*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e+(a+b*arccsch(c*x)
)*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))*(e-(c^2*d^2+e^2)^(1/2))/c/d)/e+(a+b*arcc
sch(c*x))*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))*(e+(c^2*d^2+e^2)^(1/2))/c/d)/e-1
/2*b*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e+b*polylog(2,(1/c/x+(1+1/c^2
/x^2)^(1/2))*(e-(c^2*d^2+e^2)^(1/2))/c/d)/e+b*polylog(2,(1/c/x+(1+1/c^2/x^2
)^(1/2))*(e+(c^2*d^2+e^2)^(1/2))/c/d)/e
```

Rubi [A]
time = 0.26, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = $\{6424, 2598\}$

$$\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(\sqrt{c^2d^2+e^2}+e\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e}-\frac{\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)}{e}+\frac{b\text{Li}_2\left(\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e}+\frac{b\text{Li}_2\left(\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e}-\frac{b\text{Li}_2\left(e^{2\text{csch}^{-1}(cx)}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x),x]
```

```
[Out] ((a + b*ArcCsch[c*x])*Log[1 - ((e - Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c
*d)])/e + ((a + b*ArcCsch[c*x])*Log[1 - ((e + Sqrt[c^2*d^2 + e^2])*E^ArcCsc
h[c*x])/(c*d)])/e - ((a + b*ArcCsch[c*x])*Log[1 - E^(2*ArcCsch[c*x])])/e +
(b*PolyLog[2, ((e - Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)])/e + (b*Pol
yLog[2, ((e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)])/e - (b*PolyLog[2
, E^(2*ArcCsch[c*x])])/(2*e)
```

Rule 2598

```
Int[Log[v_]*(u_), x_Symbol] :> With[{w = DerivativeDivides[v, u*(1 - v), x]
}, Simp[w*PolyLog[2, 1 - v], x] /;  !FalseQ[w]]
```

Rule 6424

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))/((d_.) + (e_.)*(x_)), x_Symbol] :>
Simp[(a + b*ArcCsch[c*x])*(Log[1 - (e - Sqrt[c^2*d^2 + e^2])*(E^ArcCsch[c*x
]/(c*d))]/e), x] + (Dist[b/(c*e), Int[Log[1 - (e - Sqrt[c^2*d^2 + e^2])*(E^
ArcCsch[c*x]/(c*d))]/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x] + Dist[b/(c*e), In
t[Log[1 - (e + Sqrt[c^2*d^2 + e^2])*(E^ArcCsch[c*x]/(c*d))]/(x^2*Sqrt[1 + 1
/(c^2*x^2)]), x], x] - Dist[b/(c*e), Int[Log[1 - E^(2*ArcCsch[c*x])]/(x^2*S
```

```
qrt[1 + 1/(c^2*x^2)]), x], x] + Simp[(a + b*ArcCsch[c*x])*(Log[1 - (e + Sqr
t[c^2*d^2 + e^2])*(E^ArcCsch[c*x]/(c*d))]/e), x] - Simp[(a + b*ArcCsch[c*x]
)*(Log[1 - E^(2*ArcCsch[c*x])]/e), x]) /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{d+ex}\,dx = \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e} + \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\right.}{}$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)}{e} + \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.45, size = 506, normalized size = 2.35

$$\frac{a\log(d+ex)}{e}+\frac{b\left(\pi^2-4i\pi\text{csch}^{-1}(cx)-8\text{csch}^{-1}(cx)^2-32\text{ArcSin}\left(\frac{\sqrt{1+\frac{ie}{cd}}}{\sqrt{2}}\right)\text{ArcTan}\left(\frac{(icd+e)\cot\left(\frac{1}{4}\left(\pi+2i\text{csch}^{-1}(cx)\right)\right)}{\sqrt{c^2d^2+e^2}}\right)-8\text{csch}^{-1}(cx)\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right)+4i\pi\log\left(1+\frac{\left(-e+\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)+8\text{csch}^{-1}(cx)\log\left(1+\frac{\left(-e+\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)+16i\text{ArcSin}\left(\frac{\sqrt{1+\frac{ie}{cd}}}{\sqrt{2}}\right)\log\left(1+\frac{\left(-e+\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)+4i\pi\log\left(1-\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)+8\text{csch}^{-1}(cx)\log\left(1-\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)-16i\text{ArcSin}\left(\frac{\sqrt{1+\frac{ie}{cd}}}{\sqrt{2}}\right)\log\left(1-\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)-4i\pi\log\left(e+\frac{d}{x}\right)+4\text{PolyLog}\left(2,e^{-2\text{csch}^{-1}(cx)}\right)+8\text{PolyLog}\left(2,\frac{\left(e-\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)+8\text{PolyLog}\left(2,\frac{\left(e+\sqrt{c^2d^2+e^2}\right)e^{\text{csch}^{-1}(cx)}}{cd}\right)\right)}{8e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x),x]
```

```
[Out] (a*Log[d + e*x])/e + (b*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 -
32*ArcSin[Sqrt[1 + (I*e)/(c*d)]/Sqrt[2]]*ArcTan[((I*c*d + e)*Cot[(Pi + (2*I
)*ArcCsch[c*x])/4])/Sqrt[c^2*d^2 + e^2]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*Arc
Csch[c*x])] + (4*I)*Pi*Log[1 + ((-e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/
(c*d)] + 8*ArcCsch[c*x]*Log[1 + ((-e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])
/(c*d)] + (16*I)*ArcSin[Sqrt[1 + (I*e)/(c*d)]/Sqrt[2]]*Log[1 + ((-e + Sqrt[
c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)] + (4*I)*Pi*Log[1 - ((e + Sqrt[c^2*d^
2 + e^2])*E^ArcCsch[c*x])/(c*d)] + 8*ArcCsch[c*x]*Log[1 - ((e + Sqrt[c^2*d^
2 + e^2])*E^ArcCsch[c*x])/(c*d)] - (16*I)*ArcSin[Sqrt[1 + (I*e)/(c*d)]/Sqrt
[2]]*Log[1 - ((e + Sqrt[c^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)] - (4*I)*Pi*L
og[e + d/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((e - Sqrt[c
^2*d^2 + e^2])*E^ArcCsch[c*x])/(c*d)] + 8*PolyLog[2, ((e + Sqrt[c^2*d^2 + e
^2])*E^ArcCsch[c*x])/(c*d)]))/(8*e)
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsch}(cx)}{ex+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x+d),x)
[Out] int((a+b*arccsch(c*x))/(e*x+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d),x, algorithm="maxima")
[Out] a*e^(-1)*log(x*e + d) + b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(x
*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d),x, algorithm="fricas")
[Out] integral((b*arccsch(c*x) + a)/(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x+d),x)
[Out] Integral((a + b*acsch(c*x))/(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{d+e\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x),x)
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x), x)
```

3.49 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^2}\,dx$

Optimal. Leaf size=98

$$\frac{b\operatorname{csch}^{-1}(cx)}{de}-\frac{a+b\operatorname{csch}^{-1}(cx)}{e(d+ex)}+\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{c^2d^2+e^2}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{d\sqrt{c^2d^2+e^2}}$$

```
[Out] b*arccsch(c*x)/d/e+(-a-b*arccsch(c*x))/e/(e*x+d)+b*arctanh((c^2*d-e/x)/c/(c
^2*d^2+e^2)^(1/2)/(1+1/c^2/x^2)^(1/2))/d/(c^2*d^2+e^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6425, 1582, 1489, 858, 221, 739, 212}

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{e(d+ex)}+\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d^2+e^2}}\right)}{d\sqrt{c^2d^2+e^2}}+\frac{b\operatorname{csch}^{-1}(cx)}{de}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x)^2,x]
```

```
[Out] (b*ArcCsch[c*x])/(d*e) - (a + b*ArcCsch[c*x])/(e*(d + e*x)) + (b*ArcTanh[(c
^2*d - e/x)/(c*Sqrt[c^2*d^2 + e^2]*Sqrt[1 + 1/(c^2*x^2)])])/(d*Sqrt[c^2*d^2
 + e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
```

```
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^2}\,dx = -\frac{a+b\text{csch}^{-1}(cx)}{e(d+ex)} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x^2(d+ex)}\,dx}{ce}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{e(d+ex)} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\left(e+\frac{d}{x}\right)x^3}\,dx}{ce}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{e(d+ex)} + \frac{b\text{Subst}\left(\int \frac{x}{(e+dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{ce}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{e(d+ex)} - \frac{b\text{Subst}\left(\int \frac{1}{(e+dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cd} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\right.}{cde}$$

$$= \frac{b\text{csch}^{-1}(cx)}{de} - \frac{a+b\text{csch}^{-1}(cx)}{e(d+ex)} + \frac{b\text{Subst}\left(\int \frac{1}{d^2+\frac{e^2}{c^2}-x^2}\,dx, x, \frac{d-\frac{e}{c^2x}}{\sqrt{1+\frac{1}{c^2x^2}}}\right)}{cd}$$

$$= \frac{b\text{csch}^{-1}(cx)}{de} - \frac{a+b\text{csch}^{-1}(cx)}{e(d+ex)} + \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{c^2d^2+e^2}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{d\sqrt{c^2d^2+e^2}}$$

Mathematica [A]
time = 0.15, size = 134, normalized size = 1.37

$$-\frac{a}{e(d+ex)} - \frac{b\text{csch}^{-1}(cx)}{e(d+ex)} + \frac{b\sinh^{-1}\left(\frac{1}{cx}\right)}{de} + \frac{b\log(d+ex)}{d\sqrt{c^2d^2+e^2}} - \frac{b\log\left(e+c\left(-cd+\sqrt{c^2d^2+e^2}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{d\sqrt{c^2d^2+e^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^2,x]
```

```
[Out] -(a/(e*(d + e*x))) - (b*ArcCsch[c*x])/(e*(d + e*x)) + (b*ArcSinh[1/(c*x)])/
(d*e) + (b*Log[d + e*x])/(d*Sqrt[c^2*d^2 + e^2]) - (b*Log[e + c*(-(c*d) + S
qrt[c^2*d^2 + e^2]*Sqrt[1 + 1/(c^2*x^2)])*x])/(d*Sqrt[c^2*d^2 + e^2])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(94) = 188$.
time = 1.81, size = 210, normalized size = 2.14

method	result
derivativedivides	$\frac{-\frac{a c^2}{(ecx+cd)e}-\frac{b c^2 \operatorname{arccsch}(cx)}{(ecx+cd)e}+\frac{b\sqrt{c^2x^2+1}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)}{e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,xd}-\frac{b\sqrt{c^2x^2+1}\,\ln\left(\frac{2\sqrt{\frac{c^2d^2+e^2}{e^2}}\sqrt{c^2x^2}}{ecx+cd}\right)}{e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,xd\sqrt{\frac{c^2d^2+e}{e^2}}}}{c}$
default	$\frac{-\frac{a c^2}{(ecx+cd)e}-\frac{b c^2 \operatorname{arccsch}(cx)}{(ecx+cd)e}+\frac{b\sqrt{c^2x^2+1}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)}{e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,xd}-\frac{b\sqrt{c^2x^2+1}\,\ln\left(\frac{2\sqrt{\frac{c^2d^2+e^2}{e^2}}\sqrt{c^2x^2}}{ecx+cd}\right)}{e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,xd\sqrt{\frac{c^2d^2+e}{e^2}}}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-a*c^2/(c*e*x+c*d)/e-b*c^2/(c*e*x+c*d)/e*arccsch(c*x)+b/e*(c^2*x^2+1)^
(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d*arctanh(1/(c^2*x^2+1)^(1/2))-b/e*(c^2
*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/((c^2*d^2+e^2)/e^2)^(1/2)*ln(
2*(((c^2*d^2+e^2)/e^2)^(1/2)*(c^2*x^2+1)^(1/2)*e-d*c^2*x+e)/(c*e*x+c*d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^2*integrate(x/(c^2*x^3*e^2 + c^2*d*x^2*e + x*e^2 + d*e + (c^2*x^3
*e^2 + c^2*d*x^2*e + x*e^2 + d*e)*sqrt(c^2*x^2 + 1)), x) + I*c*(log(I*c*x +
 1) - log(-I*c*x + 1))/(c^2*d^2 + e^2) - 2*e*log(x*e + d)/(c^2*d^3 + d*e^2)
 - (2*c^2*d^3*log(c) + 2*d*e^2*log(c) - 2*(c^2*d^2*e + e^3)*x*log(x) + (c^2
*d^2*x*e + c^2*d^3)*log(c^2*x^2 + 1) - 2*(c^2*d^3 + d*e^2)*log(sqrt(c^2*x^2
 + 1) + 1))/(c^2*d^4*e + d^2*e^3 + (c^2*d^3*e^2 + d*e^4)*x))*b - a/(x*e^2 +
 d*e)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 687 vs. $2(90) = 180$.
time = 0.41, size = 687, normalized size = 7.01

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^2,x, algorithm="fricas")
```

```
[Out] -(a*c^2*d^3 + a*d*cosh(1)^2 + 2*a*d*cosh(1)*sinh(1) + a*d*sinh(1)^2 - (b*x*
cosh(1)^2 + b*x*sinh(1)^2 + b*d*cosh(1) + (2*b*x*cosh(1) + b*d)*sinh(1))*sq
rt(((c^2*d^2 + 1)*cosh(1) - (c^2*d^2 - 1)*sinh(1))/(cosh(1) - sinh(1)))*log
(-(c^3*d^2*x - c*d*cosh(1) - c*d*sinh(1) + (c^2*d*x*sqrt((c^2*x^2 + 1)/(c^2
*x^2)) + c^2*d*x - cosh(1) - sinh(1))*sqrt(((c^2*d^2 + 1)*cosh(1) - (c^2*d^
2 - 1)*sinh(1))/(cosh(1) - sinh(1))) + (c^3*d^2*x + c*x*cosh(1)^2 + 2*c*x*c
osh(1)*sinh(1) + c*x*sinh(1)^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(x*cosh(1) +
 x*sinh(1) + d)) - (b*c^2*d^2*x*cosh(1) + b*c^2*d^3 + b*x*cosh(1)^3 + b*x*s
inh(1)^3 + b*d*cosh(1)^2 + (3*b*x*cosh(1) + b*d)*sinh(1)^2 + (b*c^2*d^2*x +
 3*b*x*cosh(1)^2 + 2*b*d*cosh(1))*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*
x^2)) - c*x + 1) + (b*c^2*d^2*x*cosh(1) + b*c^2*d^3 + b*x*cosh(1)^3 + b*x*s
inh(1)^3 + b*d*cosh(1)^2 + (3*b*x*cosh(1) + b*d)*sinh(1)^2 + (b*c^2*d^2*x +
 3*b*x*cosh(1)^2 + 2*b*d*cosh(1))*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*
x^2)) - c*x - 1) + (b*c^2*d^3 + b*d*cosh(1)^2 + 2*b*d*cosh(1)*sinh(1) + b*d
*sinh(1)^2)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^3*x*
cosh(1)^2 + c^2*d^4*cosh(1) + d*x*cosh(1)^4 + d*x*sinh(1)^4 + d^2*cosh(1)^3
 + (4*d*x*cosh(1) + d^2)*sinh(1)^3 + (c^2*d^3*x + 6*d*x*cosh(1)^2 + 3*d^2*c
osh(1))*sinh(1)^2 + (2*c^2*d^3*x*cosh(1) + c^2*d^4 + 4*d*x*cosh(1)^3 + 3*d^
2*cosh(1)^2)*sinh(1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acsch}\,(cx)}{(d + ex)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**2,x)
```

```
[Out] Integral((a + b*acsch(c*x))/(d + e*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x)^2,x)
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x)^2, x)
```

3.50 $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^3} dx$

Optimal. Leaf size=163

$$-\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{2d\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)}+\frac{b\text{csch}^{-1}(cx)}{2d^2e}-\frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2}+\frac{b(2c^2d^2+e^2)\tanh^{-1}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{c^2d^2+e^2}\sqrt{1+\frac{1}{c^2x^2}}}\right)}{2d^2\left(c^2d^2+e^2\right)^{3/2}}$$

```
[Out] 1/2*b*arccsch(c*x)/d^2/e+1/2*(-a-b*arccsch(c*x))/e/(e*x+d)^2+1/2*b*(2*c^2*d
^2+e^2)*arctanh((c^2*d-e/x)/c/(c^2*d^2+e^2)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^2/
(c^2*d^2+e^2)^(3/2)-1/2*b*c*e*(1+1/c^2/x^2)^(1/2)/d/(c^2*d^2+e^2)/(e+d/x)
```

Rubi [A]
time = 0.20, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6425, 1582, 1489, 1665, 858, 221, 739, 212}

$$-\frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2}-\frac{bce\sqrt{\frac{1}{c^2x^2}+1}}{2d\left(c^2d^2+e^2\right)\left(\frac{d}{x}+e\right)}+\frac{b(2c^2d^2+e^2)\tanh^{-1}\left(\frac{c^2d-\frac{e}{x}}{c\sqrt{\frac{1}{c^2x^2}+1}\sqrt{c^2d^2+e^2}}\right)}{2d^2\left(c^2d^2+e^2\right)^{3/2}}+\frac{b\text{csch}^{-1}(cx)}{2d^2e}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x)^3,x]
```

```
[Out] -1/2*(b*c*e*Sqrt[1 + 1/(c^2*x^2)])/(d*(c^2*d^2 + e^2)*(e + d/x)) + (b*ArcCs
ch[c*x])/(2*d^2*e) - (a + b*ArcCsch[c*x])/(2*e*(d + e*x)^2) + (b*(2*c^2*d^2
 + e^2)*ArcTanh[(c^2*d - e/x)/(c*Sqrt[c^2*d^2 + e^2]*Sqrt[1 + 1/(c^2*x^2)])
])/(2*d^2*(c^2*d^2 + e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1489

```
Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q
_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)
^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] &&
EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1582

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^
(p_.), x_Symbol] :> Int[x^(m + mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; F
reeQ[{a, c, d, e, m, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n
2] ||  !IntegerQ[p])
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^3}\,dx = -\frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x^2(d+ex)^2}\,dx}{2ce}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2} - \frac{b\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\left(e+\frac{d}{x}\right)^2 x^4}\,dx}{2ce}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2} + \frac{b\text{Subst}\left(\int \frac{x^2}{(e+dx)^2\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2ce}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{2d\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)} - \frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2} - \frac{(bc)\text{Subst}\left(\int \frac{e-\left(d+\frac{e^2}{c^2d}\right)x}{(e+dx)\sqrt{1+\frac{x^2}{c^2}}}\,dx, x,\right.}{2e\left(c^2d^2+e^2\right)}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{2d\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)} - \frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2} + \frac{b\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{2cd^2e} - $$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{2d\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)} + \frac{b\text{csch}^{-1}(cx)}{2d^2e} - \frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2} + \frac{\left(bc\left(2+\frac{e^2}{c^2d^2}\right)\right)\text{Subs}}{}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{2d\left(c^2d^2+e^2\right)\left(e+\frac{d}{x}\right)} + \frac{b\text{csch}^{-1}(cx)}{2d^2e} - \frac{a+b\text{csch}^{-1}(cx)}{2e(d+ex)^2} + \frac{b\left(2c^2d^2+e^2\right)\tanh^{-1}}{2d^2}$$

Mathematica [A]
time = 0.32, size = 204, normalized size = 1.25

$$\frac{1}{2}\left(-\frac{a}{e(d+ex)^2} - \frac{bce\sqrt{1+\frac{1}{c^2x^2}}\,x}{d\left(c^2d^2+e^2\right)(d+ex)} - \frac{b\text{csch}^{-1}(cx)}{e(d+ex)^2} + \frac{b\sinh^{-1}\left(\frac{1}{cx}\right)}{d^2e} + \frac{b\left(2c^2d^2+e^2\right)\log(d+ex)}{d^2\left(c^2d^2+e^2\right)^{3/2}} - \frac{b\left(2c^2d^2+e^2\right)\log\left(e+c\left(-cd+\sqrt{c^2d^2+e^2}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{d^2\left(c^2d^2+e^2\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^3,x]
```

```
[Out] (-(a/(e*(d + e*x)^2)) - (b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x)/(d*(c^2*d^2 + e^2)*
(d + e*x)) - (b*ArcCsch[c*x])/(e*(d + e*x)^2) + (b*ArcSinh[1/(c*x)])/(d^2*e
) + (b*(2*c^2*d^2 + e^2)*Log[d + e*x])/(d^2*(c^2*d^2 + e^2)^(3/2)) - (b*(2*
c^2*d^2 + e^2)*Log[e + c*(-(c*d) + Sqrt[c^2*d^2 + e^2]*Sqrt[1 + 1/(c^2*x^2)
])*x])/(d^2*(c^2*d^2 + e^2)^(3/2)))/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(150) = 300$.

time = 2.32, size = 929, normalized size = 5.70

method	result
derivativedivides	$\frac{-\frac{a c^3}{2(ecx+cd)^2 e}-\frac{b c^3 \operatorname{arccsch}(cx)}{2(ecx+cd)^2 e}+\frac{b c^3 \sqrt{c^2x^2+1}\, d \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)}{2e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\, x\left(c^2d^2+e^2\right)(ecx+cd)}+\frac{b c^3 \sqrt{c^2x^2+1} \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+}}\right.}{2\sqrt{\frac{c^2x^2+1}{c^2x^2}}\left(c^2d^2+e^2\right)(ecx+cd)}}{}$
default	$\frac{-\frac{a c^3}{2(ecx+cd)^2 e}-\frac{b c^3 \operatorname{arccsch}(cx)}{2(ecx+cd)^2 e}+\frac{b c^3 \sqrt{c^2x^2+1}\, d \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)}{2e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\, x\left(c^2d^2+e^2\right)(ecx+cd)}+\frac{b c^3 \sqrt{c^2x^2+1} \operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+}}\right.}{2\sqrt{\frac{c^2x^2+1}{c^2x^2}}\left(c^2d^2+e^2\right)(ecx+cd)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-1/2*a*c^3/(c*e*x+c*d)^2/e-1/2*b*c^3/(c*e*x+c*d)^2/e*arccsch(c*x)+1/2*
b*c^3/e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x*d/(c^2*d^2+e^2)/(c*
e*x+c*d)*arctanh(1/(c^2*x^2+1)^(1/2))+1/2*b*c^3*(c^2*x^2+1)^(1/2)/((c^2*x^2
+1)/c^2/x^2)^(1/2)/(c^2*d^2+e^2)/(c*e*x+c*d)*arctanh(1/(c^2*x^2+1)^(1/2))-b
*c^3/e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x*d/((c^2*d^2+e^2)/e^2
)^(1/2)/(c^2*d^2+e^2)/(c*e*x+c*d)*ln(2*(((c^2*d^2+e^2)/e^2)^(1/2)*(c^2*x^2+
1)^(1/2)*e-d*c^2*x+e)/(c*e*x+c*d))-b*c^3*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2
/x^2)^(1/2)/((c^2*d^2+e^2)/e^2)^(1/2)/(c^2*d^2+e^2)/(c*e*x+c*d)*ln(2*(((c^2
*d^2+e^2)/e^2)^(1/2)*(c^2*x^2+1)^(1/2)*e-d*c^2*x+e)/(c*e*x+c*d))-1/2*b*c*e*
(c^2*x^2+1)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/(c^2*d^2+e^2)/(c*e*x+c*d)+1/2*b
*c*e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/(c^2*d^2+e^2)/(c*e*x
+c*d)*arctanh(1/(c^2*x^2+1)^(1/2))+1/2*b*c*e^2*(c^2*x^2+1)^(1/2)/((c^2*x^2+
1)/c^2/x^2)^(1/2)/d^2/(c^2*d^2+e^2)/(c*e*x+c*d)*arctanh(1/(c^2*x^2+1)^(1/2)
)-1/2*b*c*e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/((c^2*d^2+e^2
)/e^2)^(1/2)/(c^2*d^2+e^2)/(c*e*x+c*d)*ln(2*(((c^2*d^2+e^2)/e^2)^(1/2)*(c^2
*x^2+1)^(1/2)*e-d*c^2*x+e)/(c*e*x+c*d))-1/2*b*c*e^2*(c^2*x^2+1)^(1/2)/((c^2
```

```
*x^2+1)/c^2/x^2)^(1/2)/d^2/((c^2*d^2+e^2)/e^2)^(1/2)/(c^2*d^2+e^2)/(c*e*x+c
*d)*ln(2*(((c^2*d^2+e^2)/e^2)^(1/2)*(c^2*x^2+1)^(1/2)*e-d*c^2*x+e)/(c*e*x+c
*d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*I*c^3*d*(log(I*c*x + 1) - log(-I*c*x + 1))/(c^4*d^4 + 2*c^2*d^2*e^2
 + e^4) + 4*c^2*integrate(1/2*x/(c^2*x^4*e^3 + 2*c^2*d*x^3*e^2 + (c^2*d^2*e
 + e^3)*x^2 + 2*d*x*e^2 + d^2*e + (c^2*x^4*e^3 + 2*c^2*d*x^3*e^2 + (c^2*d^2
*e + e^3)*x^2 + 2*d*x*e^2 + d^2*e)*sqrt(c^2*x^2 + 1)), x) - 2*(3*c^2*d^2*e
+ e^3)*log(x*e + d)/(c^4*d^6 + 2*c^2*d^4*e^2 + d^2*e^4) - (2*c^4*d^6*log(c)
 + 2*(2*d^4*log(c) - d^4)*c^2*e^2 - 2*(c^2*d^3*e^3 + d*e^5)*x + 2*(d^2*log(
c) - d^2)*e^4 + (c^4*d^6 - c^2*d^4*e^2 + (c^4*d^4*e^2 - c^2*d^2*e^4)*x^2 +
2*(c^4*d^5*e - c^2*d^3*e^3)*x)*log(c^2*x^2 + 1) - 2*((c^4*d^4*e^2 + 2*c^2*d
^2*e^4 + e^6)*x^2 + 2*(c^4*d^5*e + 2*c^2*d^3*e^3 + d*e^5)*x)*log(x) - 2*(c^
4*d^6 + 2*c^2*d^4*e^2 + d^2*e^4)*log(sqrt(c^2*x^2 + 1) + 1))/(c^4*d^8*e + 2
*c^2*d^6*e^3 + d^4*e^5 + (c^4*d^6*e^3 + 2*c^2*d^4*e^5 + d^2*e^7)*x^2 + 2*(c
^4*d^7*e^2 + 2*c^2*d^5*e^4 + d^3*e^6)*x))*b - 1/2*a/(x^2*e^3 + 2*d*x*e^2 +
d^2*e)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2579 vs. $2(145) = 290$.
time = 1.02, size = 2579, normalized size = 15.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(a*c^4*d^6 + b*c^3*d^5*cosh(1) + b*c*d*x^2*cosh(1)^5 + b*c*d*x^2*sinh(
1)^5 + (2*b*c*d^2*x + a*d^2)*cosh(1)^4 + (5*b*c*d*x^2*cosh(1) + 2*b*c*d^2*x
 + a*d^2)*sinh(1)^4 + (b*c^3*d^3*x^2 + b*c*d^3)*cosh(1)^3 + (b*c^3*d^3*x^2
+ 10*b*c*d*x^2*cosh(1)^2 + b*c*d^3 + 4*(2*b*c*d^2*x + a*d^2)*cosh(1))*sinh(
1)^3 + 2*(b*c^3*d^4*x + a*c^2*d^4)*cosh(1)^2 + (2*b*c^3*d^4*x + 10*b*c*d*x^
2*cosh(1)^3 + 2*a*c^2*d^4 + 6*(2*b*c*d^2*x + a*d^2)*cosh(1)^2 + 3*(b*c^3*d^
3*x^2 + b*c*d^3)*cosh(1))*sinh(1)^2 - (4*b*c^2*d^3*x*cosh(1)^2 + 2*b*c^2*d^
4*cosh(1) + b*x^2*cosh(1)^5 + b*x^2*sinh(1)^5 + 2*b*d*x*cosh(1)^4 + (5*b*x^
2*cosh(1) + 2*b*d*x)*sinh(1)^4 + (2*b*c^2*d^2*x^2 + b*d^2)*cosh(1)^3 + (2*b
*c^2*d^2*x^2 + 10*b*x^2*cosh(1)^2 + 8*b*d*x*cosh(1) + b*d^2)*sinh(1)^3 + (4
*b*c^2*d^3*x + 10*b*x^2*cosh(1)^3 + 12*b*d*x*cosh(1)^2 + 3*(2*b*c^2*d^2*x^2
```

```
 + b*d^2)*cosh(1))*sinh(1)^2 + (8*b*c^2*d^3*x*cosh(1) + 2*b*c^2*d^4 + 5*b*x
^2*cosh(1)^4 + 8*b*d*x*cosh(1)^3 + 3*(2*b*c^2*d^2*x^2 + b*d^2)*cosh(1)^2)*s
inh(1))*sqrt(((c^2*d^2 + 1)*cosh(1) - (c^2*d^2 - 1)*sinh(1))/(cosh(1) - sin
h(1)))*log(-(c^3*d^2*x - c*d*cosh(1) - c*d*sinh(1) + (c^2*d*x*sqrt((c^2*x^2
 + 1)/(c^2*x^2)) + c^2*d*x - cosh(1) - sinh(1))*sqrt(((c^2*d^2 + 1)*cosh(1)
 - (c^2*d^2 - 1)*sinh(1))/(cosh(1) - sinh(1))) + (c^3*d^2*x + c*x*cosh(1)^2
 + 2*c*x*cosh(1)*sinh(1) + c*x*sinh(1)^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(x
*cosh(1) + x*sinh(1) + d)) - (2*b*c^4*d^5*x*cosh(1) + b*c^4*d^6 + 4*b*c^2*d
^3*x*cosh(1)^3 + b*x^2*cosh(1)^6 + b*x^2*sinh(1)^6 + 2*b*d*x*cosh(1)^5 + 2*
(3*b*x^2*cosh(1) + b*d*x)*sinh(1)^5 + (2*b*c^2*d^2*x^2 + b*d^2)*cosh(1)^4 +
 (2*b*c^2*d^2*x^2 + 15*b*x^2*cosh(1)^2 + 10*b*d*x*cosh(1) + b*d^2)*sinh(1)^
4 + 4*(b*c^2*d^3*x + 5*b*x^2*cosh(1)^3 + 5*b*d*x*cosh(1)^2 + (2*b*c^2*d^2*x
^2 + b*d^2)*cosh(1))*sinh(1)^3 + (b*c^4*d^4*x^2 + 2*b*c^2*d^4)*cosh(1)^2 +
(b*c^4*d^4*x^2 + 12*b*c^2*d^3*x*cosh(1) + 2*b*c^2*d^4 + 15*b*x^2*cosh(1)^4
+ 20*b*d*x*cosh(1)^3 + 6*(2*b*c^2*d^2*x^2 + b*d^2)*cosh(1)^2)*sinh(1)^2 + 2
*(b*c^4*d^5*x + 6*b*c^2*d^3*x*cosh(1)^2 + 3*b*x^2*cosh(1)^5 + 5*b*d*x*cosh(
1)^4 + 2*(2*b*c^2*d^2*x^2 + b*d^2)*cosh(1)^3 + (b*c^4*d^4*x^2 + 2*b*c^2*d^4
)*cosh(1))*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + (2*b
*c^4*d^5*x*cosh(1) + b*c^4*d^6 + 4*b*c^2*d^3*x*cosh(1)^3 + b*x^2*cosh(1)^6
+ b*x^2*sinh(1)^6 + 2*b*d*x*cosh(1)^5 + 2*(3*b*x^2*cosh(1) + b*d*x)*sinh(1)
^5 + (2*b*c^2*d^2*x^2 + b*d^2)*cosh(1)^4 + (2*b*c^2*d^2*x^2 + 15*b*x^2*cosh
(1)^2 + 10*b*d*x*cosh(1) + b*d^2)*sinh(1)^4 + 4*(b*c^2*d^3*x + 5*b*x^2*cosh
(1)^3 + 5*b*d*x*cosh(1)^2 + (2*b*c^2*d^2*x^2 + b*d^2)*cosh(1))*sinh(1)^3 +
(b*c^4*d^4*x^2 + 2*b*c^2*d^4)*cosh(1)^2 + (b*c^4*d^4*x^2 + 12*b*c^2*d^3*x*c
osh(1) + 2*b*c^2*d^4 + 15*b*x^2*cosh(1)^4 + 20*b*d*x*cosh(1)^3 + 6*(2*b*c^2
*d^2*x^2 + b*d^2)*cosh(1)^2)*sinh(1)^2 + 2*(b*c^4*d^5*x + 6*b*c^2*d^3*x*cos
h(1)^2 + 3*b*x^2*cosh(1)^5 + 5*b*d*x*cosh(1)^4 + 2*(2*b*c^2*d^2*x^2 + b*d^2
)*cosh(1)^3 + (b*c^4*d^4*x^2 + 2*b*c^2*d^4)*cosh(1))*sinh(1))*log(c*x*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + (b*c^4*d^6 + 2*b*c^2*d^4*cosh(1)^2 +
b*d^2*cosh(1)^4 + 4*b*d^2*cosh(1)*sinh(1)^3 + b*d^2*sinh(1)^4 + 2*(b*c^2*d^
4 + 3*b*d^2*cosh(1)^2)*sinh(1)^2 + 4*(b*c^2*d^4*cosh(1) + b*d^2*cosh(1)^3)*
sinh(1))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (b*c^3*d^5 +
5*b*c*d*x^2*cosh(1)^4 + 4*(2*b*c*d^2*x + a*d^2)*cosh(1)^3 + 3*(b*c^3*d^3*x^
2 + b*c*d^3)*cosh(1)^2 + 4*(b*c^3*d^4*x + a*c^2*d^4)*cosh(1))*sinh(1) + (b*
c^3*d^3*x^2*cosh(1)^3 + b*c^3*d^4*x*cosh(1)^2 + b*c*d*x^2*cosh(1)^5 + b*c*d
*x^2*sinh(1)^5 + b*c*d^2*x*cosh(1)^4 + (5*b*c*d*x^2*cosh(1) + b*c*d^2*x)*si
nh(1)^4 + (b*c^3*d^3*x^2 + 10*b*c*d*x^2*cosh(1)^2 + 4*b*c*d^2*x*cosh(1))*si
nh(1)^3 + (3*b*c^3*d^3*x^2*cosh(1) + b*c^3*d^4*x + 10*b*c*d*x^2*cosh(1)^3 +
 6*b*c*d^2*x*cosh(1)^2)*sinh(1)^2 + (3*b*c^3*d^3*x^2*cosh(1)^2 + 2*b*c^3*d^
4*x*cosh(1) + 5*b*c*d*x^2*cosh(1)^4 + 4*b*c*d^2*x*cosh(1)^3)*sinh(1))*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)))/(2*c^4*d^7*x*cosh(1)^2 + c^4*d^8*cosh(1) + 4*c^2*
d^5*x*cosh(1)^4 + d^2*x^2*cosh(1)^7 + d^2*x^2*sinh(1)^7 + 2*d^3*x*cosh(1)^6
 + (7*d^2*x^2*cosh(1) + 2*d^3*x)*sinh(1)^6 + (2*c^2*d^4*x^2 + d^4)*cosh(1)^
5 + (2*c^2*d^4*x^2 + 21*d^2*x^2*cosh(1)^2 + 12*d^3*x*cosh(1) + d^4)*sinh(1)
^5 + (4*c^2*d^5*x + 35*d^2*x^2*cosh(1)^3 + 30*d^3*x*cosh(1)^2 + 5*(2*c^2*d^
```

```
4*x^2 + d^4)*cosh(1))*sinh(1)^4 + (c^4*d^6*x^2 + 2*c^2*d^6)*cosh(1)^3 + (c^
4*d^6*x^2 + 16*c^2*d^5*x*cosh(1) + 2*c^2*d^6 + 35*d^2*x^2*cosh(1)^4 + 40*d^
3*x*cosh(1)^3 + 10*(2*c^2*d^4*x^2 + d^4)*cosh(1)^2)*sinh(1)^3 + (2*c^4*d^7*
x + 24*c^2*d^5*x*cosh(1)^2 + 21*d^2*x^2*cosh(1)^5 + 30*d^3*x*cosh(1)^4 + 10
*(2*c^2*d^4*x^2 + d^4)*cosh(1)^3 + 3*(c^4*d^6*x^2 + 2*c^2*d^6)*cosh(1))*sin
h(1)^2 + (4*c^4*d^7*x*cosh(1) + c^4*d^8 + 16*c^2*d^5*x*cosh(1)^3 + 7*d^2*x^
2*cosh(1)^6 + 12*d^3*x*cosh(1)^5 + 5*(2*c^2*d^4*x^2 + d^4)*cosh(1)^4 + 3*(c
^4*d^6*x^2 + 2*c^2*d^6)*cosh(1)^2)*sinh(1))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acsch}(cx)}{(d + ex)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))/(d + e*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\operatorname{asinh}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x)^3,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x)^3, x)
```

3.51 $\int x^2 \sqrt{d+ex}\,\left(a+b\,\mathrm{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=918

$$-\frac{4bd\sqrt{d+ex}\,(1+c^2x^2)}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{4b(d+ex)^{3/2}\,(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{2d^2(d+ex)^{3/2}\,\left(a+b\mathrm{csch}^{-1}(cx)\right)}{3e^3}-\frac{4d(d+ex)^{5/2}\,(a+b\cdots}{5e^3}$$

```
[Out] 2/3*d^2*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^3-4/5*d*(e*x+d)^(5/2)*(a+b*arccs
ch(c*x))/e^3+2/7*(e*x+d)^(7/2)*(a+b*arccsch(c*x))/e^3+4/35*b*(c^2*x^2+1)*(e
*x+d)^(1/2)/c^3/(1+1/c^2/x^2)^(1/2)+8/105*b*d*(c^2*x^2+1)*(e*x+d)^(1/2)/c^3
/e/x/(1+1/c^2/x^2)^(1/2)-32/105*b*d^4*EllipticPi(1/2*(1-(-c^2)^(1/2)*x)^(1/
2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+
d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e^3/x/(1+1/c^2/x^2)^(1/2)/(e*x+
d)^(1/2)-4/35*b*c*d^2*EllipticE(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),(-2*e*
(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)
/(-c^2)^(3/2)/e^2/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2))
)^(1/2)+4/105*b*c*(2*c^2*d^2+9*e^2)*EllipticE(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*
2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^
2*x^2+1)^(1/2)/(-c^2)^(5/2)/e^2/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e
*(-c^2)^(1/2)))^(1/2)+32/105*b*c*d^3*EllipticF(1/2*(1-(-c^2)^(1/2)*x)^(1/2)
*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2
)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/e^2/x/(1+1/c^2/x^
2)^(1/2)/(e*x+d)^(1/2)-4/105*b*c*d*(c^2*d^2+e^2)*EllipticF(1/2*(1-(-c^2)^(1
/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2
*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(5/2)/e^2/x
/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]
time = 2.17, antiderivative size = 918, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 16, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.762$, Rules used = {45, 6445, 12, 6853, 6874, 757, 858, 733, 435, 430, 972, 946, 174, 552, 551, 847}

$$-\frac{32b\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\sqrt{c^2x^2+1}\,\Pi\left(2;\mathrm{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}d+e}\right)d^4}{105ce^3\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}+\frac{32bc\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{c^2x^2+1}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)d^3}{105\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}+\frac{2(d+ex)^{3/2}\left(a+b\mathrm{csch}^{-1}(cx)\right)d^2}{3e^3}-\frac{32bc\sqrt{d+ex}\sqrt{c^2x^2+1}\,E\left(\mathrm{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)d^2}{105\left(-c^2\right)^{3/2}e^2\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}-\frac{4b\sqrt{d+ex}\,(c^2x^2+1)\,d}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}\,x}-\frac{4(d+ex)^{5/2}\left(a+b\mathrm{csch}^{-1}(cx)\right)d}{5e^3}-\frac{4bc(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\sqrt{c^2x^2+1}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)d}{105\left(-c^2\right)^{5/2}e^2\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}+\frac{4b(d+ex)^{3/2}\,(c^2x^2+1)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{2(d+ex)^{7/2}\left(a+b\mathrm{csch}^{-1}(cx)\right)}{7e^3}-\frac{4bc(c^2d^2-3e^2)\sqrt{d+ex}\sqrt{c^2x^2+1}\,E\left(\mathrm{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{35\left(-c^2\right)^{5/2}e^2\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (-4*b*d*Sqrt[d + e*x]*(1 + c^2*x^2))/(105*c^3*e*Sqrt[1 + 1/(c^2*x^2)]*x) +
(4*b*(d + e*x)^(3/2)*(1 + c^2*x^2))/(35*c^3*e*Sqrt[1 + 1/(c^2*x^2)]*x) + (2
*d^2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) - (4*d*(d + e*x)^(5/2)*(
```

```
a + b*ArcCsch[c*x]))/(5*e^3) + (2*(d + e*x)^(7/2)*(a + b*ArcCsch[c*x]))/(7*
e^3) - (32*b*c*d^2*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1
- Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(
-c^2)^(3/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[
-c^2]*e)]) - (4*b*c*(c^2*d^2 - 3*e^2)*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*Ellip
ticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqr
t[-c^2]*e)])/(35*(-c^2)^(5/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*
x))/(c^2*d - Sqrt[-c^2]*e)]) + (32*b*c*d^3*Sqrt[(c^2*(d + e*x))/(c^2*d - Sq
rt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt
[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(-c^2)^(3/2)*e^2*Sqrt
[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*c*d*(c^2*d^2 + e^2)*Sqrt[(c^2*(d
+ e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 -
 Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(-
c^2)^(5/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (32*b*d^4*Sqrt[(Sqr
t[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcS
in[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(105*c*e^3*S
qrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$=\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$=\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$=\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$=\frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}-\frac{4d(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

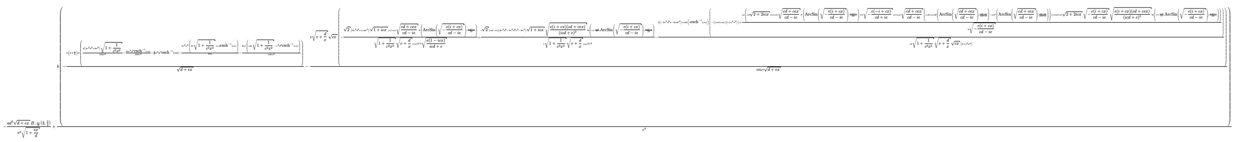
$$=-\frac{16bd\sqrt{d+ex}\,(1+c^2x^2)}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{4b(d+ex)^{3/2}\,(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{2d^2(d+ex)^{3}}{}$$

$$=-\frac{4bd\sqrt{d+ex}\,(1+c^2x^2)}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{4b(d+ex)^{3/2}\,(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{2d^2(d+ex)^{3/}}{}$$

$$=-\frac{4bd\sqrt{d+ex}\,(1+c^2x^2)}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{4b(d+ex)^{3/2}\,(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{2d^2(d+ex)^{3/}}{}$$

$$=-\frac{4bd\sqrt{d+ex}\,(1+c^2x^2)}{105c^3e\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{4b(d+ex)^{3/2}\,(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{2d^2(d+ex)^{3/}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 39.58, size = 1094, normalized size = 1.19



Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -((a*d^3*Sqrt[d + e*x]*Beta[-((e*x)/d), 3, 3/2])/(e^3*Sqrt[1 + (e*x)/d])) +
 (b*(-((c*(e + d/x)*x*((4*(5*c^2*d^2 + 9*e^2)*Sqrt[1 + 1/(c^2*x^2)])/(105*e
^2) - (16*c^3*d^3*ArcCsch[c*x])/(105*e^3) - (2*c^3*x^3*ArcCsch[c*x])/7 - (2
*c^2*x^2*(2*e*Sqrt[1 + 1/(c^2*x^2)] + c*d*ArcCsch[c*x]))/(35*e) - (8*c*x*(c
*d*e*Sqrt[1 + 1/(c^2*x^2)] - c^2*d^2*ArcCsch[c*x]))/(105*e^2)))/Sqrt[d + e*
x]) - (2*Sqrt[e + d/x]*Sqrt[c*x]*(-((Sqrt[2]*(9*c^3*d^3*e + c*d*e^3)*Sqrt[1
 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-
((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*S
qrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(
c*d - I*e)*(8*c^4*d^4 - 5*c^2*d^2*e^2 - 9*e^4)*Sqrt[1 + I*c*x]*Sqrt[(e*(I +
 c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-
((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]
*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*(-5*c^3*d^3*e - 9*c*d*e^3)*Cosh[2*ArcCsch[
c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*I)*c*x]*(I +
c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/
(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*
(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*EllipticE[ArcSin[Sqr
t[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*EllipticF[Arc
Sin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)]) + (I*c*d +
e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(e*(I + c*x)
*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(
I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*(I + c*x))/(c*d
 - I*e))])))/(c*d*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(2 + c^2*x^
2))))/(105*e^3*Sqrt[d + e*x])))/c^4
```

Maple [C] Result contains complex when optimal does not.
time = 1.09, size = 2517, normalized size = 2.74

method	result	size
derivativedivides	Expression too large to display	2517
default	Expression too large to display	2517

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^3*(a*(1/7*(e*x+d)^(7/2)-2/5*d*(e*x+d)^(5/2)+1/3*d^2*(e*x+d)^(3/2))+b*(1
/7*arccsch(c*x)*(e*x+d)^(7/2)-2/5*arccsch(c*x)*d*(e*x+d)^(5/2)+1/3*arccsch(
c*x)*d^2*(e*x+d)^(3/2)-2/105/c^4*(I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c*d*e
^3*(e*x+d)^(1/2)+3*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^4*d*(e*x+d)^(7/2)-9*
I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e
*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(
1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e
^2))^(1/2))*c^3*d^3*e-7*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^2*(e*x+d)^(
5/2)+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^3*e*(e*x+d)^(1/2)-5*I*((I*e+
c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^2*e*(e*x+d)^(3/2)-3*I*((I*e+c*d)*c/(c^2*d
^2+e^2))^(1/2)*c*e^3*(e*x+d)^(3/2)+5*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^4*
d^3*(e*x+d)^(3/2)+4*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^
2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*E
llipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d
^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^4*d^4+5*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*
d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c
^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2
),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^4*d^4-8*(-(I*c*(e*x+d)*
e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*
x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*
c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2
+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^4*d^4-3*I*((I*e+c*d)*c/(c
^2*d^2+e^2))^(1/2)*c^3*e*(e*x+d)^(7/2)-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^
4*d^4*(e*x+d)^(1/2)+7*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d*e*(e*x+d)^(
5/2)-I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I
*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x
+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*
d^2+e^2))^(1/2))*c*d*e^3+3*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d*e^2*(e*x
+d)^(3/2)-13*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/
2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*Elliptic
F((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)
/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e^2+14*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^
2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2
*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),
(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e^2+8*I*(-(I*c*(e*x
+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d
*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c
*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2
*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^3*d^3*e-((I*e+c*d)*c/
(c^2*d^2+e^2))^(1/2)*c^2*d^2*e^2*(e*x+d)^(1/2)-9*(-(I*c*(e*x+d)*e+c^2*d*(e*
x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^
2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e
^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^4+9*(-(I*c*(e*
```

```
x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*
d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c
*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*
e^4)/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((I*
e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/105*(15*(x*e + d)^(7/2)*e^(-3) - 42*(x*e + d)^(5/2)*d*e^(-3) + 35*(x*e +
d)^(3/2)*d^2*e^(-3))*a - 1/11025*(1680*c^2*d^3*(e^(-3)*integrate(((x*e + d)
*c^2*d - c^2*d^2 - e^2)*e^(1/2*log(x*e + d) + 1)/(((x*e + d)^2*c^2 - 2*(x*e
 + d)*c^2*d + c^2*d^2 + e^2)*(x*e + d)), x)/c^2 + 2*e^(1/2*log(x*e + d) - 3
)/c^2) + 280*c^2*d^2*(3*e^(-1)*integrate(e^(3/2*log(x*e + d) + 1)/(((x*e +
d)^2*c^2 - 2*(x*e + d)*c^2*d + c^2*d^2 + e^2)*(x*e + d)), x)/c^2 - 2*e^(3/2
*log(x*e + d) - 3)/c^2) + 42*c^2*d*(2*(3*(x*e + d)^(5/2)*c^2 - 5*(x*e + d)^
(3/2)*c^2*d - 15*e^(1/2*log(x*e + d) + 2))*e^(-3)/c^4 - 15*e^(-1)*integrate
(((x*e + d)*c^2*d - c^2*d^2 - e^2)*e^(1/2*log(x*e + d) + 1)/(((x*e + d)^2*c
^2 - 2*(x*e + d)*c^2*d + c^2*d^2 + e^2)*(x*e + d)), x)/c^4) + 105*c^2*(2*(1
5*(x*e + d)^(7/2)*c^2 - 42*(x*e + d)^(5/2)*c^2*d + 35*(c^2*d^2 - e^2)*(x*e
+ d)^(3/2))*e^(-3)/c^4 + 105*e*integrate(e^(3/2*log(x*e + d) + 1)/(((x*e +
d)^2*c^2 - 2*(x*e + d)*c^2*d + c^2*d^2 + e^2)*(x*e + d)), x)/c^4)*log(c) -
210*(15*x^3*e^3 + 3*d*x^2*e^2 - 4*d^2*x*e + 8*d^3)*sqrt(x*e + d)*e^(-3)*log
(sqrt(c^2*x^2 + 1) + 1) + 30*c^2*(2*(15*(x*e + d)^(7/2)*c^2 - 42*(x*e + d)^
(5/2)*c^2*d + 35*(c^2*d^2 - e^2)*(x*e + d)^(3/2))*e^(-3)/c^4 + 105*e*integr
ate(e^(3/2*log(x*e + d) + 1)/(((x*e + d)^2*c^2 - 2*(x*e + d)*c^2*d + c^2*d^
2 + e^2)*(x*e + d)), x)/c^4) + 1157625*c^2*integrate(1/105*x^4*e^(1/2*log(x
*e + d) + 3)*log(x)/(c^2*x^2*e^3 + e^3), x) - 3675*(3*e*integrate(e^(3/2*lo
g(x*e + d) + 1)/(((x*e + d)^2*c^2 - 2*(x*e + d)*c^2*d + c^2*d^2 + e^2)*(x*e
 + d)), x)/c^2 - 2*e^(3/2*log(x*e + d) - 1)/c^2)*log(c) + 1157625*integrate
(1/105*x^2*e^(1/2*log(x*e + d) + 3)*log(x)/(c^2*x^2*e^3 + e^3), x) - 11025*
integrate(2/105*(15*c^2*x^4*e^3 + 3*c^2*d*x^3*e^2 - 4*c^2*d^2*x^2*e + 8*c^2
*d^3*x)*sqrt(x*e + d)/(c^2*x^2*e^3 + (c^2*x^2*e^3 + e^3)*sqrt(c^2*x^2 + 1)
+ e^3), x))*b
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))*(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a)*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d + e\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*asinh(1/(c*x)))*(d + e*x)^(1/2),x)
```

```
[Out] int(x^2*(a + b*asinh(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.52 $\int x\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=679

$$\frac{4b\sqrt{d+ex}\,(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}\,x}-\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2}+\frac{8bcd\sqrt{d+ex}\,\sqrt{\ }}{1}$$

```
[Out] -2/3*d*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^2+2/5*(e*x+d)^(5/2)*(a+b*arccsch(
c*x))/e^2+4/15*b*(c^2*x^2+1)*(e*x+d)^(1/2)/c^3/x/(1+1/c^2/x^2)^(1/2)+8/15*b
*d^3*EllipticPi(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)
^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e
))^(1/2)/c/e^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+8/15*b*c*d*EllipticE(1/2
*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2))
)^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/e/x/(1+1/c^2/x^2)^(1/
2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)-8/15*b*c*d^2*EllipticF(1/2*(1
-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(
1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(
3/2)/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/15*b*c*(c^2*d^2+e^2)*EllipticF
(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1
/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(
-c^2)^(5/2)/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]

time = 1.67, antiderivative size = 679, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 15, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.790$, Rules used = {45, 6445, 12, 6853, 6874, 757, 858, 733, 435, 430, 972, 946, 174, 552, 551}

$$-\frac{2d(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2}+\frac{8bd^3\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}d+e}\right)}{15ce^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}+\frac{4bc\sqrt{c^2x^2+1}\,(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15(-c^2)^{5/2}ex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}-\frac{8bcd^2\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15(-c^2)^{3/2}ex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}+\frac{8bcd\sqrt{c^2x^2+1}\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15(-c^2)^{3/2}ex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}+\frac{4b(c^2x^2+1)\sqrt{d+ex}}{15c^3x\sqrt{\frac{1}{c^2x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (4*b*Sqrt[d + e*x]*(1 + c^2*x^2))/(15*c^3*Sqrt[1 + 1/(c^2*x^2)]*x) - (2*d*(
d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^2) + (2*(d + e*x)^(5/2)*(a + b*Ar
cCsch[c*x]))/(5*e^2) + (8*b*c*d*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[A
rcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2
]*e)])/(15*(-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2
*d - Sqrt[-c^2]*e)]) - (8*b*c*d^2*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*
e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2
*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*
x^2)]*x*Sqrt[d + e*x]) + (4*b*c*(c^2*d^2 + e^2)*Sqrt[(c^2*(d + e*x))/(c^2*d
```

```
- Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]
/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(5/2)*e*Sq
rt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (8*b*d^3*Sqrt[(Sqrt[-c^2]*(d + e*x))
/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-
c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(15*c*e^2*Sqrt[1 + 1/(c^2*x^2)
]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m - 1)*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 1))), x] + Dist[1/(c
*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - a*e^2*(m
- 1) + 2*c*d*e*(m + p)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m,
 p}, x] && NeQ[c*d^2 + a*e^2, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ
[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}$$

$$=-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}$$

$$=-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}$$

$$=-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}$$

$$=-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}$$

$$=\frac{4b\sqrt{d+ex}\,(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}\,x}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex}{}$$

$$=\frac{4b\sqrt{d+ex}\,(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}\,x}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex}{}$$

$$=\frac{4b\sqrt{d+ex}\,(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}\,x}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex}{}$$

$$=\frac{4b\sqrt{d+ex}\,(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}\,x}-\frac{2d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{2(d+ex}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.97, size = 418, normalized size = 0.62

$$\frac{1}{15}\left(\frac{4b\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}{c}+\frac{2a\sqrt{d+ex}\,(-2d^2+dex+3e^2x^2)}{e^2}+\frac{2b\sqrt{d+ex}\,(-2d^2+dex+3e^2x^2)\operatorname{csch}^{-1}(cx)}{e^2}+\frac{4ib\sqrt{-\frac{e(-i+cx)}{cd+ie}}\sqrt{-\frac{e(i+cx)}{cd-ie}}\left(2cd(cd+ie)E\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)+(c^2d^2-2icde+e^2)F\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)-2c^2d^2\Pi\left(1-\frac{ie}{cd};i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)\right)}{c^3\sqrt{-\frac{c}{cd-ie}}\,e^2\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] ((4*b*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + (2*a*Sqrt[d + e*x]*(-2*d^2
 + d*e*x + 3*e^2*x^2))/e^2 + (2*b*Sqrt[d + e*x]*(-2*d^2 + d*e*x + 3*e^2*x^2
)*ArcCsch[c*x])/e^2 + ((4*I)*b*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*Sqrt[-((
e*(I + c*x))/(c*d - I*e))]*(2*c*d*(c*d + I*e)*EllipticE[I*ArcSinh[Sqrt[-(c/
(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] + (c^2*d^2 - (2*I)*c
*d*e + e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d
 - I*e)/(c*d + I*e)] - 2*c^2*d^2*EllipticPi[1 - (I*e)/(c*d), I*ArcSinh[Sqrt
[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)]))/(c^3*Sqrt[-(c
/(c*d - I*e))]*e^2*Sqrt[1 + 1/(c^2*x^2)]*x))/15
```

Maple [C] Result contains complex when optimal does not.
time = 1.05, size = 1966, normalized size = 2.90

method	result	size
derivativedivides	Expression too large to display	1966
default	Expression too large to display	1966

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^2*(-a*(-1/5*(e*x+d)^(5/2)+1/3*(e*x+d)^(3/2)*d)-b*(-1/5*arccsch(c*x)*(e*
x+d)^(5/2)+1/3*arccsch(c*x)*(e*x+d)^(3/2)*d-2/15/c^3*(-3*I*(-(I*c*(e*x+d)*e
+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x
+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/
(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^
2*e-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d*(e*x+d)^(5/2)-2*I*((I*e+c*d)*c/
(c^2*d^2+e^2))^(1/2)*c^2*d*e*(e*x+d)^(3/2)+2*I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+
d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+
e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^
2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/
((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e-I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+
d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+
e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2
))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^3+2*((I*e+c*d)*c
/(c^2*d^2+e^2))^(1/2)*c^3*d^2*(e*x+d)^(3/2)+(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-
c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2
```

```
)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^
(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3+2*(-(I*c*(e*x
+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d
*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*
d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c
^3*d^3-2*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*(
(I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((
e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/
d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^3
*d^3+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*e^3*(e*x+d)^(1/2)-((I*e+c*d)*c/(c^
2*d^2+e^2))^(1/2)*c^3*d^3*(e*x+d)^(1/2)+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)
*c^2*d^2*e*(e*x+d)^(1/2)-(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d
^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1
/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-
c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2+2*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)
-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^
2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))
^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2+I*((I*e+c*d)
*c/(c^2*d^2+e^2))^(1/2)*c^2*e*(e*x+d)^(5/2)-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/
2)*c*d*e^2*(e*x+d)^(1/2))/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/
e^2/x^2)^(1/2)/x/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*(x*e + d)^(5/2)*e^(-2) - 5*(x*e + d)^(3/2)*d*e^(-2))*a + 1/225*(60*
c^2*d^2*(e^(-2)*integrate(((x*e + d)*c^2*d - c^2*d^2 - e^2)*e^(1/2*log(x*e
+ d) + 1)/(((x*e + d)^2*c^2 - 2*(x*e + d)*c^2*d + c^2*d^2 + e^2)*(x*e + d))
, x)/c^2 + 2*e^(1/2*log(x*e + d) - 2)/c^2) - 10*c^2*d*(2*e^(3/2*log(x*e + d
) - 2)/c^2 - 3*integrate(e^(3/2*log(x*e + d) + 1)/(((x*e + d)^2*c^2 - 2*(x*
e + d)*c^2*d + c^2*d^2 + e^2)*(x*e + d)), x)/c^2) - 15*c^2*(2*(3*(x*e + d)^
(5/2)*c^2 - 5*(x*e + d)^(3/2)*c^2*d - 15*e^(1/2*log(x*e + d) + 2))*e^(-2)/c
^4 - 15*integrate(((x*e + d)*c^2*d - c^2*d^2 - e^2)*e^(1/2*log(x*e + d) + 1
)/(((x*e + d)^2*c^2 - 2*(x*e + d)*c^2*d + c^2*d^2 + e^2)*(x*e + d)), x)/c^4
)*log(c) + 30*(3*x^2*e^2 + d*x*e - 2*d^2)*sqrt(x*e + d)*e^(-2)*log(sqrt(c^2
*x^2 + 1) + 1) - 6*c^2*(2*(3*(x*e + d)^(5/2)*c^2 - 5*(x*e + d)^(3/2)*c^2*d
- 15*e^(1/2*log(x*e + d) + 2))*e^(-2)/c^4 - 15*integrate(((x*e + d)*c^2*d -
 c^2*d^2 - e^2)*e^(1/2*log(x*e + d) + 1)/(((x*e + d)^2*c^2 - 2*(x*e + d)*c^
2*d + c^2*d^2 + e^2)*(x*e + d)), x)/c^4) - 3375*c^2*integrate(1/15*x^3*e^(1
/2*log(x*e + d) + 2)*log(x)/(c^2*x^2*e^2 + e^2), x) - 225*(integrate(((x*e
```

```
+ d)*c^2*d - c^2*d^2 - e^2)*e^(1/2*log(x*e + d) + 1)/(((x*e + d)^2*c^2 - 2*
(x*e + d)*c^2*d + c^2*d^2 + e^2)*(x*e + d)), x)/c^2 + 2*sqrt(x*e + d)/c^2)*
log(c) - 3375*integrate(1/15*x*e^(1/2*log(x*e + d) + 2)*log(x)/(c^2*x^2*e^2
 + e^2), x) + 225*integrate(2/15*(3*c^2*x^3*e^2 + c^2*d*x^2*e - 2*c^2*d^2*x
)*sqrt(x*e + d)/(c^2*x^2*e^2 + (c^2*x^2*e^2 + e^2)*sqrt(c^2*x^2 + 1) + e^2)
, x))*b
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b\,\mathrm{acsch}\,(cx))\,\sqrt{d + ex}\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))*(e*x+d)**(1/2),x)
[Out] Integral(x*(a + b*acsch(c*x))*sqrt(d + e*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a)*x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d + e\,x}\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*asinh(1/(c*x)))*(d + e*x)^(1/2),x)
[Out] int(x*(a + b*asinh(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.53 $\int \sqrt{d+ex} \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

Optimal. Leaf size=429

$$\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}+\frac{4bc\sqrt{d+ex}\,\sqrt{1+c^2x^2}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right)}{3\left(-c^2\right)^{3/2}\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}}+$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e+4/3*b*c*EllipticE(1/2*(1-(-c^2)^(1/2
)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d
)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/x/(1+1/c^2/x^2)^(1/2)/((e*x+d)/(d+e/
(-c^2)^(1/2)))^(1/2)+4/3*b*c*d*EllipticF(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/
2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*((e*
x+d)/(d+e/(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(
1/2)-4/3*b*d^2*EllipticPi(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),2,2^(1/2)*(e
/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^
2)^(1/2)+e))^(1/2)/c/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]

time = 0.45, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {6425, 1588, 972, 733, 430, 947, 174, 552, 551, 858, 435}

$$\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{4bd^2\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}\,(d+ex)}{\sqrt{-c^2}\,d+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}\,d+e}\right)}{3cex\sqrt{\frac{1}{c^2x^2}+1}\,\sqrt{d+ex}}+\frac{4bcd\sqrt{c^2x^2+1}\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right)}{3\left(-c^2\right)^{3/2}x\sqrt{\frac{1}{c^2x^2}+1}\,\sqrt{d+ex}}+\frac{4bc\sqrt{c^2x^2+1}\,\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right)}{3\left(-c^2\right)^{3/2}x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e) + (4*b*c*Sqrt[d + e*x]*Sqrt[
1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^
2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt
[(d + e*x)/(d + e/Sqrt[-c^2])]) + (4*b*c*d*Sqrt[(d + e*x)/(d + e/Sqrt[-c^2]
)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*
Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)*Sqrt[1 + 1/(c^2*x^2)
]*x*Sqrt[d + e*x]) - (4*b*d^2*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e
)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]],
(2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
```

```
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{(2b)\int \frac{(d+ex)^{3/2}}{\sqrt{1+\frac{1}{c^2x^2}}\,x^2}\,dx}{3ce}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{(d+ex)^{3/2}}{x\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3ce\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \left(\frac{2de}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}\cdots}}\cdots\right.}{\cdots}$$

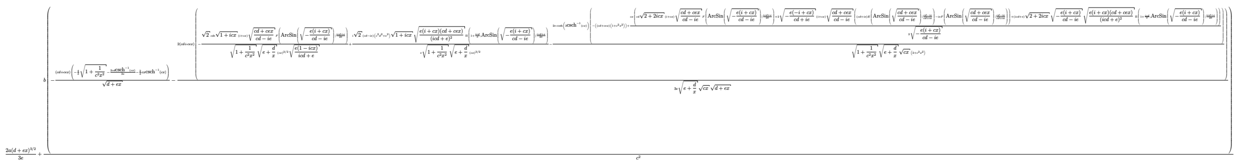
$$= \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{\left(4bd\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}+\cdots}}\cdots}{3c\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3c\sqrt{1+\frac{1}{c^2x^2}}\,x} - \cdots$$

$$= \frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e} + \frac{8b\sqrt{-c^2}\,d\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\cdots}{3c^3\sqrt{1+\cdots}}$$

$$4b\sqrt{-c^2}\sqrt{d+ex}\sqrt{1+c^2x^2}\,E\left(\sin^{\cdots}\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 29.45, size = 926, normalized size = 2.16



Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (2*a*(d + e*x)^(3/2))/(3*e) + (b*(-(((c*d + c*e*x)*((-4*Sqrt[1 + 1/(c^2*x^2
)])/3 - (2*c*d*ArcCsch[c*x])/(3*e) - (2*c*x*ArcCsch[c*x])/3))/Sqrt[d + e*x]
) - (2*(c*d + c*e*x)*(-((Sqrt[2]*c*d*e*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d
+ c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]],
 (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[
(e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(c^2*d^2 + e^2)*Sqr
t[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 +
 (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])
/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*e*Cosh[2*ArcCsch[
c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*I)*c*x]*(I +
c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/
(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*
(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*EllipticE[ArcSin[Sqr
t[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*EllipticF[Arc
Sin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)]) + (I*c*d +
e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(e*(I + c*x)
*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(
I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*(I + c*x))/(c*d
 - I*e))])))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(2 + c^2*x^2)))
)/(3*e*Sqrt[e + d/x]*Sqrt[c*x]*Sqrt[d + e*x])))/c^2
```

Maple [C] Result contains complex when optimal does not.
time = 0.91, size = 840, normalized size = 1.96

method	result
derivativedivides	$\frac{\frac{2(ex+d)^{\frac{3}{2}}a}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}+\frac{2\sqrt{-\frac{ic(ex+d)e+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ic(ex+d)e-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}}{}\right.}{}$

default	$\frac{2(ex+d)^{\frac{3}{2}}a}{3}+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}+\frac{2\sqrt{-\frac{ic(ex+d)e+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ic(ex+d)e-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}\left(\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))*(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(1/3*(e*x+d)^(3/2)*a+b*(1/3*(e*x+d)^(3/2)*arccsch(c*x)+2/3/c^2*(-(I*c*(
e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^
2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*(I*EllipticF((e*x+d)^(1/2)*((
I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1
/2))*c*d*e-I*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(
I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/
(c^2*d^2+e^2))^(1/2))*c*d*e-2*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2
+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2+Ellipt
icE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^
2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2+EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2
*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))
^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2-EllipticF((e*x+d)^(1/2)*(
(I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(
1/2))*e^2+EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*
c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^2)/((c^2*(e*x+d)^2-2*c^2*d*(e*x+
d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(I*e
-c*d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*(x*e + d)^(3/2)*a*e^(-1) - 1/9*(6*c^2*d*(e^(-1)*integrate(((x*e + d)*c^
2*d - c^2*d^2 - e^2)*e^(1/2*log(x*e + d) + 1)/(((x*e + d)^2*c^2 - 2*(x*e +
d)*c^2*d + c^2*d^2 + e^2)*(x*e + d)), x)/c^2 + 2*e^(1/2*log(x*e + d) - 1)/c
^2) - 3*c^2*(3*e*integrate(e^(3/2*log(x*e + d) + 1)/(((x*e + d)^2*c^2 - 2*(
x*e + d)*c^2*d + c^2*d^2 + e^2)*(x*e + d)), x)/c^2 - 2*e^(3/2*log(x*e + d)
- 1)/c^2)*log(c) - 6*(x*e + d)^(3/2)*e^(-1)*log(sqrt(c^2*x^2 + 1) + 1) - 2*
c^2*(3*e*integrate(e^(3/2*log(x*e + d) + 1)/(((x*e + d)^2*c^2 - 2*(x*e + d)
*c^2*d + c^2*d^2 + e^2)*(x*e + d)), x)/c^2 - 2*e^(3/2*log(x*e + d) - 1)/c^2
```

```
) + 27*c^2*integrate(1/3*x^2*e^(1/2*log(x*e + d) + 1)*log(x)/(c^2*x^2*e + e
), x) + 9*e*integrate(e^(3/2*log(x*e + d) + 1)/(((x*e + d)^2*c^2 - 2*(x*e +
 d)*c^2*d + c^2*d^2 + e^2)*(x*e + d)), x)*log(c) + 27*integrate(1/3*e^(1/2*
log(x*e + d) + 1)*log(x)/(c^2*x^2*e + e), x) - 9*integrate(2/3*(c^2*x^2*e +
 c^2*d*x)*sqrt(x*e + d)/(c^2*x^2*e + (c^2*x^2*e + e)*sqrt(c^2*x^2 + 1) + e)
, x))*b
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acsch}(cx)) \sqrt{d + ex}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))*(e*x+d)**(1/2),x)
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a + b \operatorname{asinh}\left(\frac{1}{c\,x}\right)\right) \sqrt{d + e\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2),x)
[Out] int((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2), x)
```

3.54 $\int \frac{\sqrt{d+ex}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{\sqrt{d+ex}\,(a+b\text{csch}^{-1}(cx))}{x},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex}\,(a+b\text{csch}^{-1}(cx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] Defer[Int][(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,(a+b\text{csch}^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{d+ex}\,(a+b\text{csch}^{-1}(cx))}{x}\,dx$$

Mathematica [F]
time = 180.00, size = 0, normalized size = 0.00

$$\$\text{Aborted}$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] $Aborted
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{arccsch}(cx))\sqrt{ex+d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x)
```

```
[Out] int((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] (sqrt(d)*log((sqrt(x*e + d) - sqrt(d))/(sqrt(x*e + d) + sqrt(d))) + 2*sqrt(
x*e + d))*a - ((sqrt(d)*log((sqrt(x*e + d) - sqrt(d))/(sqrt(x*e + d) + sqrt
(d))) + 2*sqrt(x*e + d))*log(c) + integrate(sqrt(x*e + d)*log(x)/x, x) - in
tegrate(sqrt(x*e + d)*log(sqrt(c^2*x^2 + 1) + 1)/x, x))*b
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)*sqrt(x*e + d)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsch}\,(cx))\,\sqrt{d + ex}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))*(e*x+d)**(1/2)/x,x)
```

```
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2))/x,x)
[Out] int(((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2))/x, x)
```

3.55 $\int \frac{\sqrt{d+ex}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=24

$$\mathrm{Int}\left(\frac{\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x^2,x]
[Out] Defer[Int][(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x^2, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx = \int \frac{\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Mathematica [A]
time = 6.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x^2,x]
[Out] Integrate[(Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/x^2, x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{arccsch}(cx)\right)\sqrt{ex+d}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x)
```

```
[Out] int((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(e*log((sqrt(x*e + d) - sqrt(d))/(sqrt(x*e + d) + sqrt(d)))/sqrt(d) - 2
*sqrt(x*e + d)/x)*a - 1/2*((e*log((sqrt(x*e + d) - sqrt(d))/(sqrt(x*e + d)
+ sqrt(d)))/sqrt(d) - 2*sqrt(x*e + d)/x)*log(c) + 2*integrate(sqrt(x*e + d)
*log(x)/x^2, x) - 2*integrate(sqrt(x*e + d)*log(sqrt(c^2*x^2 + 1) + 1)/x^2,
 x))*b
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)*sqrt(x*e + d)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsch}\,(cx))\,\sqrt{d + ex}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))*(e*x+d)**(1/2)/x**2,x)
```

```
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x)/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x+d)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(e*x + d)*(b*arccsch(c*x) + a)/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,\sqrt{d+e\,x}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2))/x^2,x)
[Out] int(((a + b*asinh(1/(c*x)))*(d + e*x)^(1/2))/x^2, x)
```

3.56 $\int (d+ex)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

Optimal. Leaf size=486

$$\frac{4be\sqrt{d+ex}\,(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}+\frac{28bcd\sqrt{d+ex}\,\sqrt{1+c^2x^2}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-}}}{\sqrt{2}}\right.\right.}{15\left(-c^2\right)^{3/2}\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{\frac{d+}{d+}}}$$

```
[Out] 2/5*(e*x+d)^(5/2)*(a+b*arccsch(c*x))/e+4/15*b*e*(c^2*x^2+1)*(e*x+d)^(1/2)/c
^3/x/(1+1/c^2/x^2)^(1/2)+28/15*b*c*d*EllipticE(1/2*(1-(-c^2)^(1/2)*x)^(1/2)
*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c
^2*x^2+1)^(1/2)/(-c^2)^(3/2)/x/(1+1/c^2/x^2)^(1/2)/((e*x+d)/(d+e/(-c^2)^(1/
2)))^(1/2)-4/15*b*c*(2*c^2*d^2-e^2)*EllipticF(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*
2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)
*((e*x+d)/(d+e/(-c^2)^(1/2)))^(1/2)/(-c^2)^(5/2)/x/(1+1/c^2/x^2)^(1/2)/(e*x
+d)^(1/2)-4/5*b*d^3*EllipticPi(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),2,2^(1/
2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d
*(-c^2)^(1/2)+e))^(1/2)/c/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]

time = 0.68, antiderivative size = 486, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {6425, 1588, 972, 733, 430, 947, 174, 552, 551, 858, 435, 945, 1598}

$$\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}-\frac{4bd^3\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}d+e}\right)}{5cex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}-\frac{4bc\sqrt{c^2x^2+1}\left(2c^2d^2-e^2\right)\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{5/2}x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}+\frac{28bcd\sqrt{c^2x^2+1}\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15\left(-c^2\right)^{3/2}x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}}+\frac{4be\left(c^2x^2+1\right)\sqrt{d+ex}}{15c^3x\sqrt{\frac{1}{c^2x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (4*b*e*Sqrt[d + e*x]*(1 + c^2*x^2))/(15*c^3*Sqrt[1 + 1/(c^2*x^2)]*x) + (2*(
d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e) + (28*b*c*d*Sqrt[d + e*x]*Sqrt[1
 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2
]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(3/2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt
[(d + e*x)/(d + e/Sqrt[-c^2])]) - (4*b*c*(2*c^2*d^2 - e^2)*Sqrt[(d + e*x)/(
d + e/Sqrt[-c^2])]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x
]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(5/2)*Sqr
t[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*d^3*Sqrt[(Sqrt[-c^2]*(d + e*x))/
(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c
^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(5*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*
Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 945

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*(Sqrt[a +
c*x^2]/(c*g*(2*m - 1))), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3
*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x +
2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int (d+ex)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx = \frac{2(d+ex)^{5/2} \left(a+b\text{csch}^{-1}(cx)\right)}{5e} + \frac{(2b)\int \frac{(d+ex)^{5/2}}{\sqrt{1+\frac{1}{c^2x^2}}\,x^2}dx}{5ce}$$

$$= \frac{2(d+ex)^{5/2} \left(a+b\text{csch}^{-1}(cx)\right)}{5e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{(d+ex)^{5/2}}{x\sqrt{\frac{1}{c^2}+x^2}}dx}{5ce\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2(d+ex)^{5/2} \left(a+b\text{csch}^{-1}(cx)\right)}{5e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \left(\frac{3d^2e}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}}}\right.}{}$$

$$= \frac{2(d+ex)^{5/2} \left(a+b\text{csch}^{-1}(cx)\right)}{5e} + \frac{\left(6bd^2\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\sqrt{\frac{1}{c^2}}}}{5c\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= \frac{4be\sqrt{d+ex}\,(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}\,x} + \frac{2(d+ex)^{5/2} \left(a+b\text{csch}^{-1}(cx)\right)}{5e} + \frac{\left(6bd\sqrt{\frac{1}{c^2}}\right.}{}$$

$$= \frac{4be\sqrt{d+ex}\,(1+c^2x^2)}{15c^3\sqrt{1+\frac{1}{c^2x^2}}\,x} + \frac{2(d+ex)^{5/2} \left(a+b\text{csch}^{-1}(cx)\right)}{5e} + \frac{12b\sqrt{-c^2}}{}$$

$$12b\sqrt{-c^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.93, size = 380, normalized size = 0.78

$$\frac{2\left(\frac{2be^2\sqrt{1+\frac{1}{c^2x^2}}x\sqrt{d+ex}}{c}+3a(d+ex)^{5/2}+3b(d+ex)^{5/2}\operatorname{csch}^{-1}(cx)+\frac{2ib\sqrt{-\frac{e(-i+cx)}{cd+ie}}\sqrt{-\frac{e(i+cx)}{cd-ie}}\left(7cd(cd+ie)E\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)+(-9c^2d^2-7icde+e^2)F\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)+3c^2d^2\Pi\left(1-\frac{ie}{cd};i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)\right)}{c^3\sqrt{-\frac{c}{cd-ie}}\sqrt{1+\frac{1}{c^2x^2}}x}\right)}{15e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (2*((2*b*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])/c + 3*a*(d + e*x)^(5/2)
 + 3*b*(d + e*x)^(5/2)*ArcCsch[c*x] + ((2*I)*b*Sqrt[-((e*(-I + c*x))/(c*d +
 I*e))]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*(7*c*d*(c*d + I*e)*EllipticE[I*A
rcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] + (-
9*c^2*d^2 - (7*I)*c*d*e + e^2)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*S
qrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] + 3*c^2*d^2*EllipticPi[1 - (I*e)/(c
*d), I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*
e)]))/(c^3*Sqrt[-(c/(c*d - I*e))]*Sqrt[1 + 1/(c^2*x^2)]*x)))/(15*e)
```

Maple [C] Result contains complex when optimal does not.
time = 0.86, size = 1939, normalized size = 3.99

method	result	size
derivativedivides	Expression too large to display	1939
default	Expression too large to display	1939

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(1/5*(e*x+d)^(5/2)*a+b*(1/5*arccsch(c*x)*(e*x+d)^(5/2)+2/15/c^3*(-3*I*(
-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+
d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/
2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c
*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e-((I
*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d*(e*x+d)^(5/2)+I*((I*e+c*d)*c/(c^2*d^2+
e^2))^(1/2)*c^2*d^2*e*(e*x+d)^(1/2)-2*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c
^2*d*e*(e*x+d)^(3/2)+2*I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d
^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1
/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-
c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e-9*(-(I*c*(e*x+d)*e+c^2*d*(e*x+
d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+
e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2
))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3+7*(-(I*c*(
e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^
2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e
```

```
+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)
)*c^3*d^3+3*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2
)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticP
i((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^
2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*
c^3*d^3+2*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^2*(e*x+d)^(3/2)+I*((I*e+c
*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*e*(e*x+d)^(5/2)-((I*e+c*d)*c/(c^2*d^2+e^2))^
(1/2)*c^3*d^3*(e*x+d)^(1/2)+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*e^3*(e*x+d)
^(1/2)-6*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*(
(I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e
*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^
2*d^2+e^2))^(1/2))*c*d*e^2+7*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c
^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2)
)^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*
d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2-I*(-(I*c*(e*x+d)*e+c^2*d*(e*
x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^
2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e
^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^3-((I*e+c*d)*c
/(c^2*d^2+e^2))^(1/2)*c*d*e^2*(e*x+d)^(1/2))/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d
)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(I*e-
c*d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 2/5*(x*e + d)^(5/2)*a*e^(-1) - 1/75*(30*c^2*d^2*(e^(-1)*integrate(((x*e + d
)*c^2*d - c^2*d^2 - e^2)*e^(1/2*log(x*e + d) + 1)/(((x*e + d)^2*c^2 - 2*(x*
e + d)*c^2*d + c^2*d^2 + e^2)*(x*e + d)), x)/c^2 + 2*e^(1/2*log(x*e + d) -
1)/c^2) - 25*c^2*d*(3*e*integrate(e^(3/2*log(x*e + d) + 1)/(((x*e + d)^2*c^
2 - 2*(x*e + d)*c^2*d + c^2*d^2 + e^2)*(x*e + d)), x)/c^2 - 2*e^(3/2*log(x*
e + d) - 1)/c^2)*log(c) - 20*c^2*d*(3*e*integrate(e^(3/2*log(x*e + d) + 1)/
(((x*e + d)^2*c^2 - 2*(x*e + d)*c^2*d + c^2*d^2 + e^2)*(x*e + d)), x)/c^2 -
 2*e^(3/2*log(x*e + d) - 1)/c^2) + 375*c^2*d*integrate(1/5*x^2*e^(1/2*log(x
*e + d) + 1)*log(x)/(c^2*x^2*e + e), x) - 5*c^2*(2*(5*c^2*d*e^(3/2*log(x*e
+ d) + 1) - 3*c^2*e^(5/2*log(x*e + d) + 1) + 15*e^(1/2*log(x*e + d) + 3))*e
^(-2)/c^4 + 15*e*integrate(((x*e + d)*c^2*d - c^2*d^2 - e^2)*e^(1/2*log(x*e
 + d) + 1)/(((x*e + d)^2*c^2 - 2*(x*e + d)*c^2*d + c^2*d^2 + e^2)*(x*e + d)
), x)/c^4)*log(c) + 75*d*e*integrate(e^(3/2*log(x*e + d) + 1)/(((x*e + d)^2
*c^2 - 2*(x*e + d)*c^2*d + c^2*d^2 + e^2)*(x*e + d)), x)*log(c) - 30*(x^2*e
^2 + 2*d*x*e + d^2)*sqrt(x*e + d)*e^(-1)*log(sqrt(c^2*x^2 + 1) + 1) - 2*c^2
```

```
*(2*(5*c^2*d*e^(3/2*log(x*e + d) + 1) - 3*c^2*e^(5/2*log(x*e + d) + 1) + 15
*e^(1/2*log(x*e + d) + 3))*e^(-2)/c^4 + 15*e*integrate(((x*e + d)*c^2*d - c
^2*d^2 - e^2)*e^(1/2*log(x*e + d) + 1)/(((x*e + d)^2*c^2 - 2*(x*e + d)*c^2*
d + c^2*d^2 + e^2)*(x*e + d)), x)/c^4) + 375*c^2*integrate(1/5*x^3*e^(1/2*l
og(x*e + d) + 2)*log(x)/(c^2*x^2*e + e), x) + 375*d*integrate(1/5*e^(1/2*lo
g(x*e + d) + 1)*log(x)/(c^2*x^2*e + e), x) + 75*(e*integrate(((x*e + d)*c^2
*d - c^2*d^2 - e^2)*e^(1/2*log(x*e + d) + 1)/(((x*e + d)^2*c^2 - 2*(x*e + d
)*c^2*d + c^2*d^2 + e^2)*(x*e + d)), x)/c^2 + 2*e^(1/2*log(x*e + d) + 1)/c^
2)*log(c) + 375*integrate(1/5*x*e^(1/2*log(x*e + d) + 2)*log(x)/(c^2*x^2*e
+ e), x) - 75*integrate(2/5*(c^2*x^3*e^2 + 2*c^2*d*x^2*e + c^2*d^2*x)*sqrt(
x*e + d)/(c^2*x^2*e + (c^2*x^2*e + e)*sqrt(c^2*x^2 + 1) + e), x))*b
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral((a*x*e + a*d + (b*x*e + b*d)*arccsch(c*x))*sqrt(x*e + d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(a+b*acsch(c*x)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x + d)^(3/2)*(b*arccsch(c*x) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,{\left(d+e\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))*(d + e*x)^(3/2),x)
[Out] int((a + b*asinh(1/(c*x)))*(d + e*x)^(3/2), x)
```

3.57 $\int \frac{x^3\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=939

$$\frac{4b\sqrt{d+ex}\,(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}}-\frac{4bd\sqrt{d+ex}\,(1+c^2x^2)}{21c^3e^2\sqrt{1+\frac{1}{c^2x^2}}\,x}-\frac{2d^3\sqrt{d+ex}\,\left(a+b\mathrm{csch}^{-1}(cx)\right)}{e^4}+\frac{2d^2(d+ex)^{3/2}\,\left(a+b\mathrm{csch}\right.}{e^4}$$

```
[Out] 2*d^2*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^4-6/5*d*(e*x+d)^(5/2)*(a+b*arccsch
(c*x))/e^4+2/7*(e*x+d)^(7/2)*(a+b*arccsch(c*x))/e^4-2*d^3*(a+b*arccsch(c*x)
)*(e*x+d)^(1/2)/e^4+4/35*b*(c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e/(1+1/c^2/x^2)^(1
/2)-4/21*b*d*(c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e^2/x/(1+1/c^2/x^2)^(1/2)+64/35*
b*d^4*EllipticPi(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2
)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+
e))^(1/2)/c/e^4/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+24/35*b*c*d^2*EllipticE
(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1
/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/e^3/x/(1+1/c^2/x^
2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)+4/105*b*c*(2*c^2*d^2+9*
e^2)*EllipticE(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2
*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(5/2)/e^3
/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)-64/35*b*c
*d^3*EllipticF(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2
*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(
1/2)))^(1/2)/(-c^2)^(3/2)/e^3/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-32/105*b*
c*d*(c^2*d^2+e^2)*EllipticF(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^
2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2
*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(5/2)/e^3/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1
/2)
```

Rubi [A]
time = 1.97, antiderivative size = 939, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 17, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {45, 6445, 12, 6853, 6874, 733, 435, 958, 430, 946, 174, 552, 551, 847, 858, 956, 1668}

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (4*b*Sqrt[d + e*x]*(1 + c^2*x^2))/(35*c^3*e*Sqrt[1 + 1/(c^2*x^2)]) - (4*b*d
*Sqrt[d + e*x]*(1 + c^2*x^2))/(21*c^3*e^2*Sqrt[1 + 1/(c^2*x^2)]*x) - (2*d^3
*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e^4 + (2*d^2*(d + e*x)^(3/2)*(a + b*Ar
cCsch[c*x]))/e^4 - (6*d*(d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e^4) + (2*
(d + e*x)^(7/2)*(a + b*ArcCsch[c*x]))/(7*e^4) + (24*b*c*d^2*Sqrt[d + e*x]*S
qrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt
[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(35*(-c^2)^(3/2)*e^3*Sqrt[1 + 1/(c^2*x^2
)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) + (4*b*c*(2*c^2*d^2 + 9*
e^2)*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x
]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(-c^2)^(5/2)*e^
3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) - (
64*b*c*d^3*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*E
llipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d -
 Sqrt[-c^2]*e)])/(35*(-c^2)^(3/2)*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]
) - (32*b*c*d*(c^2*d^2 + e^2)*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*
Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqr
t[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(105*(-c^2)^(5/2)*e^3*Sqrt[1 + 1/(c^2*x
^2)]*x*Sqrt[d + e*x]) + (64*b*d^4*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d
 + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2
]], (2*e)/(Sqrt[-c^2]*d + e)])/(35*c*e^4*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e
*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
```

```
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] && !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] && !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] && !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) && !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 956

```
Int[(((d_.) + (e_.)*(x_))^(m_)*Sqrt[(f_.) + (g_.)*(x_)])/Sqrt[(a_) + (c_.)*
(x_)^2], x_Symbol] :> Simp[2*e*(d + e*x)^(m - 1)*Sqrt[f + g*x]*(Sqrt[a + c*
x^2]/(c*(2*m + 1))), x] - Dist[1/(c*(2*m + 1)), Int[((d + e*x)^(m - 2)/(Sqr
t[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e*(d*g + 2*e*f*(m - 1)) - c*d^2*f*(2*m
+ 1) + (a*e^2*g*(2*m - 1) - c*d*(4*e*f*m + d*g*(2*m + 1)))*x - c*e*(e*f + d
*g*(4*m - 1))*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f -
d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GtQ[m, 1]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 1668

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(d + e*x)
^(m + q - 1)*((a + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q + 2*p + 1))), x] + Di
st[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + c*x^2)^p*ExpandToSum[c
*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1)*(d + e*x)^q - f*(d + e*x)
^(q - 2)*(a*e^2*(m + q - 1) - c*d^2*(m + q + 2*p + 1) - 2*c*d*e*(m + q + p)
*x), x], x], x] /; GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, c, d,
 e, m, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] &&  !(EqQ[d, 0] && T
rue) &&  !(IGtQ[m, 0] && RationalQ[a, c, d, e] && (IntegerQ[p] || ILtQ[p +
1/2, 0]))
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
```

```
FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = -\frac{2d^3\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} - \frac{6d(d}{}$$

$$= -\frac{2d^3\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} - \frac{6d(d}{}$$

$$= -\frac{2d^3\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} - \frac{6d(d}{}$$

$$= -\frac{2d^3\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} - \frac{6d(d}{}$$

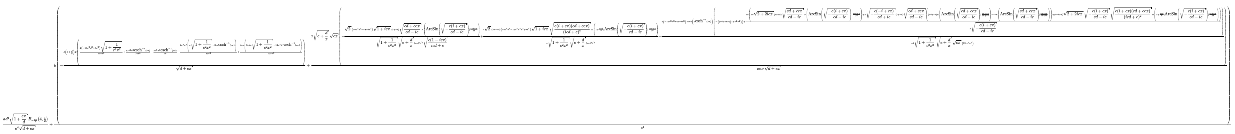
$$= -\frac{2d^3\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} + \frac{2d^2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} - \frac{6d(d}{}$$

$$= \frac{4b\sqrt{d+ex}\,(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{8bd\sqrt{d+ex}\,(1+c^2x^2)}{35c^3e^2\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{2d^3\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}\right.}{e^4}$$

$$= \frac{4b\sqrt{d+ex}\,(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{4bd\sqrt{d+ex}\,(1+c^2x^2)}{21c^3e^2\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{2d^3\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}\right.}{e^4}$$

$$= \frac{4b\sqrt{d+ex}\,(1+c^2x^2)}{35c^3e\sqrt{1+\frac{1}{c^2x^2}}} - \frac{4bd\sqrt{d+ex}\,(1+c^2x^2)}{21c^3e^2\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{2d^3\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}\right.}{e^4}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 29.46, size = 1098, normalized size = 1.17



Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (a*d^4*Sqrt[1 + (e*x)/d]*Beta[-((e*x)/d), 4, 1/2])/(e^4*Sqrt[d + e*x]) + (b
*(-((c*(e + d/x)*x*((4*(-16*c^2*d^2 + 9*e^2)*Sqrt[1 + 1/(c^2*x^2)])/(105*e^
3) + (32*c^3*d^3*ArcCsch[c*x])/(35*e^4) - (2*c^3*x^3*ArcCsch[c*x])/(7*e) -
(4*c^2*x^2*(e*Sqrt[1 + 1/(c^2*x^2)] - 3*c*d*ArcCsch[c*x]))/(35*e^2) + (4*c*
x*(5*c*d*e*Sqrt[1 + 1/(c^2*x^2)] - 12*c^2*d^2*ArcCsch[c*x]))/(105*e^3)))/Sq
rt[d + e*x]) + (2*Sqrt[e + d/x]*Sqrt[c*x]*(-((Sqrt[2]*(40*c^3*d^3*e - 8*c*d
*e^3)*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[A
rcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/
(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) +
(I*Sqrt[2]*(c*d - I*e)*(48*c^4*d^4 - 16*c^2*d^2*e^2 + 9*e^4)*Sqrt[1 + I*c*x
]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e,
 ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(e*Sqrt[1
+ 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*(-16*c^3*d^3*e + 9*c*d*e^3)*
Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (
2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-
((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))
/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*Ellip
ticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*
e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e
)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sq
rt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, Arc
Sin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*
(I + c*x))/(c*d - I*e))])))/(c*d*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c
*x]*(2 + c^2*x^2))))/(105*e^4*Sqrt[d + e*x])))/c^4
```

Maple [C] Result contains complex when optimal does not.
time = 0.93, size = 2545, normalized size = 2.71

method	result	size
derivativedivides	Expression too large to display	2545
default	Expression too large to display	2545

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^4*(-a*(-1/7*(e*x+d)^(7/2)+3/5*d*(e*x+d)^(5/2)-d^2*(e*x+d)^(3/2)+d^3*(e*
x+d)^(1/2))-b*(-1/7*arccsch(c*x)*(e*x+d)^(7/2)+3/5*arccsch(c*x)*d*(e*x+d)^(
5/2)-arccsch(c*x)*d^2*(e*x+d)^(3/2)+arccsch(c*x)*d^3*(e*x+d)^(1/2)+2/105/c^
4*(8*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^3*e*(e*x+d)^(1/2)+3*((I*e+c*
d)*c/(c^2*d^2+e^2))^(1/2)*c^4*d*(e*x+d)^(7/2)-8*I*(-(I*c*(e*x+d)*e+c^2*d*(e
*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d
^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+
e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^3-14*((I*
e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^2*(e*x+d)^(5/2)-19*I*((I*e+c*d)*c/(c^2*
d^2+e^2))^(1/2)*c^3*d^2*e*(e*x+d)^(3/2)-24*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c
^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)
/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(
1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^4*d^4-16*(-(I*c*(e*x
+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d
*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*
d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c
^4*d^4+40*I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2
)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF
((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/
(c^2*d^2+e^2))^(1/2))*c^3*d^3*e+48*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e
^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^
2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/
(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c
/(c^2*d^2+e^2))^(1/2))*c^4*d^4+14*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d
*e*(e*x+d)^(5/2)+19*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^3*(e*x+d)^(3/2)
+8*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c*d*e^3*(e*x+d)^(1/2)-8*((I*e+c*d)*c
/(c^2*d^2+e^2))^(1/2)*c^4*d^4*(e*x+d)^(1/2)-3*I*((I*e+c*d)*c/(c^2*d^2+e^2))
^(1/2)*c*e^3*(e*x+d)^(3/2)+15*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(
c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2
))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c
*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e^2-7*(-(I*c*(e*x+d)*e+c^2*
d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c
^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*
d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e^2
-48*I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*
c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x
+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(
-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^3*d^
3*e+3*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d*e^2*(e*x+d)^(3/2)-3*I*((I*e+c
*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*e*(e*x+d)^(7/2)-8*((I*e+c*d)*c/(c^2*d^2+e^2)
)^(1/2)*c^2*d^2*e^2*(e*x+d)^(1/2)-9*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-
e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d
^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-
```

```
(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^4+9*(-(I*c*(e*x+d)*e+c^2*d*
(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2
*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^
2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^4)/((c^2*(e
*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((I*e+c*d)*c/(c^2
*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/35*(5*(x*e + d)^(7/2)*e^(-4) - 21*(x*e + d)^(5/2)*d*e^(-4) + 35*(x*e + d)
^(3/2)*d^2*e^(-4) - 35*sqrt(x*e + d)*d^3*e^(-4))*a + 1/35*(2*(5*x^4*e^4 - d
*x^3*e^3 + 2*d^2*x^2*e^2 - 8*d^3*x*e - 16*d^4)*e^(-4)*log(sqrt(c^2*x^2 + 1)
 + 1)/sqrt(x*e + d) + 35*integrate(2/35*(5*c^2*x^5*e^4 - c^2*d*x^4*e^3 + 2*
c^2*d^2*x^3*e^2 - 8*c^2*d^3*x^2*e - 16*c^2*d^4*x)/((c^2*x^2*e^4 + e^4)*sqrt
(c^2*x^2 + 1)*sqrt(x*e + d) + (c^2*x^2*e^4 + e^4)*sqrt(x*e + d)), x) - 35*i
ntegrate(1/35*(5*c^2*x^5*(7*log(c) + 2)*e^4 - 2*c^2*d*x^4*e^3 - 16*c^2*d^3*
x^2*e - 32*c^2*d^4*x + (4*c^2*d^2*e^2 + 35*e^4*log(c))*x^3 + 35*(c^2*x^5*e^
4 + x^3*e^4)*log(x))/((c^2*x^2*e^4 + e^4)*sqrt(x*e + d)), x))*b
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccsch(c*x) + a*x^3)/sqrt(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\,\mathrm{acsch}\,(cx))}{\sqrt{d + ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))/sqrt(d + e*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^3/sqrt(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d + e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2),x)
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2), x)
```

3.58 $\int \frac{x^2\left(a+b\textbf{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=707

$$\frac{4b\sqrt{d+ex}\,(1+c^2x^2)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{2d^2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{5/2}\left(a+\right.}{5e^3}$$

```
[Out] -4/3*d*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^3+2/5*(e*x+d)^(5/2)*(a+b*arccsch(
c*x))/e^3+2*d^2*(a+b*arccsch(c*x))*(e*x+d)^(1/2)/e^3+4/15*b*(c^2*x^2+1)*(e*
x+d)^(1/2)/c^3/e/x/(1+1/c^2/x^2)^(1/2)-32/15*b*d^3*EllipticPi(1/2*(1-(-c^2)
^(1/2)*x)^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)
^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e^3/x/(1+1/c^2/x^2
)^(1/2)/(e*x+d)^(1/2)-4/5*b*c*d*EllipticE(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1
/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^
2+1)^(1/2)/(-c^2)^(3/2)/e^2/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c
^2)^(1/2)))^(1/2)+32/15*b*c*d^2*EllipticF(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1
/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^
2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/e^2/x/(1+1/c^2/x^2)^(1
/2)/(e*x+d)^(1/2)+4/15*b*c*(c^2*d^2+e^2)*EllipticF(1/2*(1-(-c^2)^(1/2)*x)^(
1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^
(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(5/2)/e^2/x/(1+1/c^
2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]
time = 1.48, antiderivative size = 707, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {45, 6445, 12, 6853, 6874, 733, 435, 958, 430, 946, 174, 552, 551, 847, 858}

$$\frac{2d^2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}-\frac{32bd^3\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}d+e}\right)}{15ce^3x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}+\frac{32bcd^2\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15(-c^2)^{3/2}e^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}+\frac{4bc\sqrt{c^2x^2+1}(c^2d^2+e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15(-c^2)^{5/2}e^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}-\frac{4bcd\sqrt{c^2x^2+1}\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{5(-c^2)^{3/2}e^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}+\frac{4b(c^2x^2+1)\sqrt{d+ex}}{15c^3ex\sqrt{\frac{1}{c^2x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (4*b*Sqrt[d + e*x]*(1 + c^2*x^2))/(15*c^3*e*Sqrt[1 + 1/(c^2*x^2)]*x) + (2*d
^2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e^3 - (4*d*(d + e*x)^(3/2)*(a + b*Ar
cCsch[c*x]))/(3*e^3) + (2*(d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e^3) - (
4*b*c*d*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2
]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(5*(-c^2)^(3/2)*e
^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) +
```

```
(32*b*c*d^2*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*
EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d
- Sqrt[-c^2]*e)])/(15*(-c^2)^(3/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x
]) + (4*b*c*(c^2*d^2 + e^2)*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sq
rt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[
-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(5/2)*e^2*Sqrt[1 + 1/(c^2*x^2)
]*x*Sqrt[d + e*x]) - (32*b*d^3*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d +
e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]],
 (2*e)/(Sqrt[-c^2]*d + e)])/(15*c*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
```

```
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
```

```
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx=\frac{2d^2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex}{}$$

$$=\frac{2d^2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex}{}$$

$$=\frac{2d^2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex}{}$$

$$=\frac{2d^2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex}{}$$

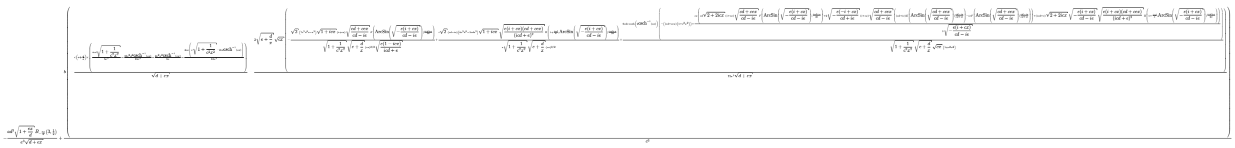
$$=\frac{2d^2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}-\frac{4d(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{2(d+ex}{}$$

$$=\frac{4b\sqrt{d+ex}\,(1+c^2x^2)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{2d^2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}-\frac{4d(d+ex)^{3/2}\left(a+\right.}{3e^3}$$

$$=\frac{4b\sqrt{d+ex}\,(1+c^2x^2)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{2d^2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}-\frac{4d(d+ex)^{3/2}\left(a+\right.}{3e^3}$$

$$=\frac{4b\sqrt{d+ex}\,(1+c^2x^2)}{15c^3e\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{2d^2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}-\frac{4d(d+ex)^{3/2}\left(a+\right.}{3e^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 29.69, size = 1012, normalized size = 1.43



Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] -((a*d^3*Sqrt[1 + (e*x)/d]*Beta[-((e*x)/d), 3, 1/2])/(e^3*Sqrt[d + e*x])) +
 (b*(-((c*(e + d/x)*x*((4*c*d*Sqrt[1 + 1/(c^2*x^2)])/(5*e^2) - (16*c^2*d^2*
ArcCsch[c*x])/(15*e^3) - (2*c^2*x^2*ArcCsch[c*x])/(5*e) - (4*c*x*(e*Sqrt[1
+ 1/(c^2*x^2)] - 2*c*d*ArcCsch[c*x]))/(15*e^2)))/Sqrt[d + e*x]) - (2*Sqrt[e
 + d/x]*Sqrt[c*x]*(-((Sqrt[2]*(7*c^2*d^2*e - e^3)*Sqrt[1 + I*c*x]*(I + c*x)
*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d
 - I*e))]], (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^
(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(8*c^3*d
^3 - 3*c*d*e^2)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e
)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]],
(I*c*d + e)/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (
6*c*d*e*Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sq
rt[2 + (2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSi
n[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I
 + c*x))/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*
e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*
e)] - I*e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c
*d + I*e)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I
*e))]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d
)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sq
rt[-((e*(I + c*x))/(c*d - I*e))])))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sq
rt[c*x]*(2 + c^2*x^2))))/(15*e^3*Sqrt[d + e*x])))/c^3
```

Maple [C] Result contains complex when optimal does not.
time = 1.00, size = 1991, normalized size = 2.82

method	result	size
derivativedivides	Expression too large to display	1991
default	Expression too large to display	1991

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^3*(a*(1/5*(e*x+d)^(5/2)-2/3*(e*x+d)^(3/2)*d+d^2*(e*x+d)^(1/2))+b*(1/5*a
rccsch(c*x)*(e*x+d)^(5/2)-2/3*arccsch(c*x)*(e*x+d)^(3/2)*d+arccsch(c*x)*d^2
*(e*x+d)^(1/2)+2/15/c^3*(I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^2*e*(e*x
+d)^(1/2)-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d*(e*x+d)^(5/2)+I*((I*e+c*d
)*c/(c^2*d^2+e^2))^(1/2)*e^3*(e*x+d)^(1/2)+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1
/2)*c^2*e*(e*x+d)^(5/2)+7*I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^
2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))
^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d
*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e+2*((I*e+c*d)*c/(c^2*d^2+e^2
))^(1/2)*c^3*d^2*(e*x+d)^(3/2)-4*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2
)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+
e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*
I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3-3*(-(I*c*(e*x+d)*e+c^2*d
*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^
2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d
^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3+8*(-
(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d
)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2
)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*
d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^3*d^3-2*I*((
I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d*e*(e*x+d)^(3/2)-((I*e+c*d)*c/(c^2*d^2
+e^2))^(1/2)*c^3*d^3*(e*x+d)^(1/2)-8*I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d
^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^
2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2
),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*
d)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e+4*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d
^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^
2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)
,(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2-3*(-(I*c*(e*x+d)*e
+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x
+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/
(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^
2-I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*
(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)
^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2
+e^2))^(1/2))*e^3-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c*d*e^2*(e*x+d)^(1/2))/
((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((I*e+c*d
)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*(3*(x*e + d)^(5/2)*e^(-3) - 10*(x*e + d)^(3/2)*d*e^(-3) + 15*sqrt(x*e
+ d)*d^2*e^(-3))*a + 1/15*(2*(3*x^3*e^3 - d*x^2*e^2 + 4*d^2*x*e + 8*d^3)*e^
(-3)*log(sqrt(c^2*x^2 + 1) + 1)/sqrt(x*e + d) + 15*integrate(2/15*(3*c^2*x^
4*e^3 - c^2*d*x^3*e^2 + 4*c^2*d^2*x^2*e + 8*c^2*d^3*x)/((c^2*x^2*e^3 + e^3)
*sqrt(c^2*x^2 + 1)*sqrt(x*e + d) + (c^2*x^2*e^3 + e^3)*sqrt(x*e + d)), x) -
 15*integrate(1/15*(3*c^2*x^4*(5*log(c) + 2)*e^3 - 2*c^2*d*x^3*e^2 + 16*c^2
*d^3*x + (8*c^2*d^2*e + 15*e^3*log(c))*x^2 + 15*(c^2*x^4*e^3 + x^2*e^3)*log
(x))/((c^2*x^2*e^3 + e^3)*sqrt(x*e + d)), x))*b
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)/sqrt(x*e + d), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\operatorname{acsch}(cx))}{\sqrt{d + ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))/sqrt(d + e*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/sqrt(e*x + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2),x)
```

```
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2), x)
```

3.59 $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=474

$$-\frac{2d\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+\frac{4bc\sqrt{d+ex}\,\sqrt{1+c^2x^2}\,E\left(\operatorname{ArcSin}\left(\frac{\sqrt{1}}{}\right.\right.}{3\left(-c^2\right)^{3/2}e\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{}}$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^2-2*d*(a+b*arccsch(c*x))*(e*x+d)^(1/
2)/e^2+8/3*b*d^2*EllipticPi(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),2,2^(1/2)*
(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-
c^2)^(1/2)+e))^(1/2)/c/e^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/3*b*c*Elli
pticE(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^
2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/e/x/(1+1/c^2
/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)-8/3*b*c*d*EllipticF(
1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/
2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-
c^2)^(3/2)/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]

time = 1.20, antiderivative size = 474, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {45, 6445, 12, 6853, 6874, 733, 435, 958, 430, 946, 174, 552, 551}

$$-\frac{2d\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}+\frac{2(d+ex)^{3/2}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+\frac{8bd^2\sqrt{c^2x^2+1}\,\sqrt{\frac{\sqrt{-c^2}\,(d+ex)}{\sqrt{-c^2}\,d+e}}\,\Pi\left(2;\operatorname{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}\,d+e}\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}\,\sqrt{d+ex}}-\frac{8bcd\sqrt{c^2x^2+1}\,\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}\,e}}\,F\left(\operatorname{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right)}{3\left(-c^2\right)^{3/2}ex\sqrt{\frac{1}{c^2x^2}+1}\,\sqrt{d+ex}}+\frac{4bc\sqrt{c^2x^2+1}\,\sqrt{d+ex}\,E\left(\operatorname{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right)}{3\left(-c^2\right)^{3/2}ex\sqrt{\frac{1}{c^2x^2}+1}\,\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}\,e}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (-2*d*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e^2 + (2*(d + e*x)^(3/2)*(a + b*A
rcCsch[c*x]))/(3*e^2) + (4*b*c*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[Ar
cSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]
*e)])/(3*(-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d
 - Sqrt[-c^2]*e)]) - (8*b*c*d*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*
Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqr
t[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*x^2)]
*x*Sqrt[d + e*x]) + (8*b*d^2*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)
]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (
2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex}}\,dx = -\frac{2d\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{b\int \frac{2(-2d\cdots}{\cdots}}{3e^2\sqrt{\cdots}}$$

$$= -\frac{2d\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{(2b)\int \frac{(-\cdots}{\cdots}}{\cdots}$$

$$= -\frac{2d\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{\left(2b\sqrt{1+\cdots}\right.}{\cdots}$$

$$= -\frac{2d\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{\left(2b\sqrt{1+\cdots}\right.}{\cdots}$$

$$= -\frac{2d\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} - \frac{\left(4bd\sqrt{1\cdots}\right.}{\cdots}$$

$$= -\frac{2d\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} - \frac{\left(4bd^2\sqrt{1\cdots}\right.}{\cdots}$$

$$= -\frac{2d\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{4b\sqrt{-c^2\cdots}}{\cdots}$$

$$= -\frac{2d\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2} + \frac{4b\sqrt{-c^2\cdots}}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.83, size = 343, normalized size = 0.72

$$\frac{2\left(a(-2d+ex)\sqrt{d+ex}+b(-2d+ex)\sqrt{d+ex}\,\mathrm{csch}^{-1}(cx)+\frac{2b\sqrt{-\frac{e(-i+cx)}{cd+ie}}\sqrt{-\frac{e(i+cx)}{cd-ie}}\left((icd-e)E\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)+(icd+e)F\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)-2icd\Pi\left(1-\frac{ie}{cd};i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)\right)}{c^2\sqrt{-\frac{c}{cd-ie}}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x],x]
```

```
[Out] (2*(a*(-2*d + e*x)*Sqrt[d + e*x] + b*(-2*d + e*x)*Sqrt[d + e*x]*ArcCsch[c*x
] + (2*b*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*Sqrt[-((e*(I + c*x))/(c*d - I*
e))]*((I*c*d - e)*EllipticE[I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]]
, (c*d - I*e)/(c*d + I*e)] + (I*c*d + e)*EllipticF[I*ArcSinh[Sqrt[-(c/(c*d
- I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] - (2*I)*c*d*EllipticPi[1
- (I*e)/(c*d), I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)
/(c*d + I*e)]))/(c^2*Sqrt[-(c/(c*d - I*e))]*Sqrt[1 + 1/(c^2*x^2)]*x)))/(3*e
^2)
```

Maple [C] Result contains complex when optimal does not.
time = 0.82, size = 868, normalized size = 1.83

method	result
derivativedivides	$\frac{-2a\left(-\frac{(ex+d)^{\frac{3}{2}}}{3}+d\sqrt{ex+d}\right)-2b\left(-\frac{(ex+d)^{\frac{3}{2}}\mathrm{arccsch}(cx)}{3}+\mathrm{arccsch}(cx)d\sqrt{ex+d}+\frac{2\sqrt{-\frac{ic(ex+d)e+c^2d(ex+d)}{c^2d^2+e^2}}}{}\right.}{}$
default	$\frac{-2a\left(-\frac{(ex+d)^{\frac{3}{2}}}{3}+d\sqrt{ex+d}\right)-2b\left(-\frac{(ex+d)^{\frac{3}{2}}\mathrm{arccsch}(cx)}{3}+\mathrm{arccsch}(cx)d\sqrt{ex+d}+\frac{2\sqrt{-\frac{ic(ex+d)e+c^2d(ex+d)}{c^2d^2+e^2}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^2*(-a*(-1/3*(e*x+d)^(3/2)+d*(e*x+d)^(1/2))-b*(-1/3*(e*x+d)^(3/2)*arccsc
h(c*x)+arccsch(c*x)*d*(e*x+d)^(1/2)+2/3/c^2*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-
c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2
)/(c^2*d^2+e^2))^(1/2)*(2*I*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e
```

```
^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e-2*I*Ellipt
icPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2
+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2
))*c*d*e-EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c
*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2-EllipticE((e*x+d)^(1/2)*((I
*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/
2))*c^2*d^2+2*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/
(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c
/(c^2*d^2+e^2))^(1/2))*c^2*d^2+EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^
2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^2-EllipticE
((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/
(c^2*d^2+e^2))^(1/2))*e^2)/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2
/e^2/x^2)^(1/2)/x/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*((x*e + d)^(3/2)*e^(-2) - 3*sqrt(x*e + d)*d*e^(-2))*a + 1/3*(2*(x^2*e^2
 - d*x*e - 2*d^2)*e^(-2)*log(sqrt(c^2*x^2 + 1) + 1)/sqrt(x*e + d) + 3*integ
rate(2/3*(c^2*x^3*e^2 - c^2*d*x^2*e - 2*c^2*d^2*x)/((c^2*x^2*e^2 + e^2)*sqr
t(c^2*x^2 + 1)*sqrt(x*e + d) + (c^2*x^2*e^2 + e^2)*sqrt(x*e + d)), x) - 3*i
ntegrate(1/3*(c^2*x^3*(3*log(c) + 2)*e^2 - 2*c^2*d*x^2*e - (4*c^2*d^2 - 3*e
^2*log(c))*x + 3*(c^2*x^3*e^2 + x*e^2)*log(x))/((c^2*x^2*e^2 + e^2)*sqrt(x*
e + d)), x))*b
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccsch(c*x) + a*x)/sqrt(x*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b \operatorname{acsch}(cx))}{\sqrt{d + ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))/sqrt(d + e*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/sqrt(e*x + d), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2),x)
```

```
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x)^(1/2), x)
```

3.60 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx$

Optimal. Leaf size=284

$$\frac{2\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e}+\frac{4bc\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\,\sqrt{1+c^2x^2}\,F\left(\operatorname{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right.}{\left(-c^2\right)^{3/2}\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

```
[Out] 2*(a+b*arccsch(c*x))*(e*x+d)^(1/2)/e+4*b*c*EllipticF(1/2*(1-(-c^2)^(1/2)*x)
^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1
)^(1/2)*((e*x+d)/(d+e/(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/x/(1+1/c^2/x^2)^(1/
2)/(e*x+d)^(1/2)-4*b*d*EllipticPi(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),2,2^
(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)
/(d*(-c^2)^(1/2)+e))^(1/2)/c/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6425, 1588, 958, 733, 430, 947, 174, 552, 551}

$$\frac{2\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e}+\frac{4bc\sqrt{c^2x^2+1}\,\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}\,F\left(\operatorname{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right)}{\left(-c^2\right)^{3/2}x\sqrt{\frac{1}{c^2x^2}+1}\,\sqrt{d+ex}}-\frac{4bd\sqrt{c^2x^2+1}\,\sqrt{\frac{\sqrt{-c^2}\,(d+ex)}{\sqrt{-c^2}\,d+e}}\,\Pi\left(2;\operatorname{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}\,d+e}\right)}{cex\sqrt{\frac{1}{c^2x^2}+1}\,\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/Sqrt[d + e*x],x]
```

```
[Out] (2*Sqrt[d + e*x]*(a + b*ArcCsch[c*x]))/e + (4*b*c*Sqrt[(d + e*x)/(d + e/Sqr
t[-c^2])]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]
], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/((-c^2)^(3/2)*Sqrt[1 + 1/(c^2
*x^2)]*x*Sqrt[d + e*x]) - (4*b*d*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d
+ e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]
], (2*e)/(Sqrt[-c^2]*d + e)])/(c*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 958

```
Int[Sqrt[(f_.) + (g_.)*(x_)]/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2
]), x_Symbol] :> Dist[g/e, Int[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x] +
Dist[(e*f - d*g)/e, Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x], x]
 /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{(2b)\int \frac{\sqrt{d+ex}}{\sqrt{1+\frac{1}{c^2x^2}}\,x^2}\,dx}{ce}$$

$$= \frac{2\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{\sqrt{d+ex}}{x\sqrt{\frac{1}{c^2}+x^2}}\,dx}{ce\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{\sqrt{d+ex}\,\sqrt{\frac{1}{c^2}+x^2}}\,dx}{c\sqrt{1+\frac{1}{c^2x^2}}\,x} + \frac{\left(2b\right.}{}$$

$$= \frac{2\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{\left(2bd\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}\,x}\,\sqrt{1+\sqrt{-c^2}\,x}}}{ce\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{4b\sqrt{-c^2}\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\,\sqrt{1+c^2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1}}{}\right.\right.}{c^3\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+e}}$$

$$= \frac{2\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{4b\sqrt{-c^2}\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\,\sqrt{1+c^2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1}}{}\right.\right.}{c^3\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+e}}$$

$$= \frac{2\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e} + \frac{4b\sqrt{-c^2}\sqrt{\frac{d+ex}{d+\frac{e}{\sqrt{-c^2}}}}\,\sqrt{1+c^2x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{1}}{}\right.\right.}{c^3\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+e}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.45, size = 307, normalized size = 1.08

$$\frac{2\left(ae(d+ex)-\frac{b\left(e+\frac{d}{x}\right)\left(-cex\,\mathrm{csch}^{-1}(cx)+\frac{\sqrt{2}\sqrt{1+icx}\left(-e^2(i+cx)\sqrt{\frac{c(d+ex)}{cd-ie}}F\left(\mathrm{ArcSin}\left(\sqrt{-\frac{e(i+cx)}{cd-ie}}\right)\middle|\frac{icd+e}{2e}\right)+cd(icd+e)\sqrt{-\frac{e(i+cx)}{cd-ie}}\sqrt{\frac{ce(i+cx)(d+ex)}{(icd+e)^2}}\Pi\left(1+\frac{icd}{e};\mathrm{ArcSin}\left(\sqrt{-\frac{e(i+cx)}{cd-ie}}\right)\middle|\frac{icd+e}{2e}\right)\right)}{\sqrt{1+\frac{1}{c^2x^2}}\sqrt{-\frac{e(i+cx)}{cd-ie}}(cd+cex)}\right)}{c}\right)}{e^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/Sqrt[d + e*x],x]
```

```
[Out] (2*(a*e*(d + e*x) - (b*(e + d/x)*(-(c*e*x*ArcCsch[c*x]) + (Sqrt[2]*Sqrt[1 +
 I*c*x]*(-(e^2*(I + c*x)*Sqrt[(c*(d + e*x))/(c*d - I*e)]*EllipticF[ArcSin[S
qrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]) + c*d*(I*c*d + e)*S
qrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(c*e*(I + c*x)*(d + e*x))/(I*c*d + e
)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]],
(I*c*d + e)/(2*e)]))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[-((e*(I + c*x))/(c*d - I*e
))]*(c*d + c*e*x))))/c))/(e^2*Sqrt[d + e*x])
```

Maple [C] Result contains complex when optimal does not.
time = 0.78, size = 395, normalized size = 1.39

method	result
derivativedivides	$2\sqrt{ex+d}\,a+2b\left(\sqrt{ex+d}\,\mathrm{arccsch}(cx)+\frac{2\sqrt{-\frac{ic(ex+d)e+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ic(ex+d)e-c^2d(ex+d)+}{c^2d^2+e^2}}}{}\right.$
default	$2\sqrt{ex+d}\,a+2b\left(\sqrt{ex+d}\,\mathrm{arccsch}(cx)+\frac{2\sqrt{-\frac{ic(ex+d)e+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ic(ex+d)e-c^2d(ex+d)+}{c^2d^2+e^2}}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*((e*x+d)^(1/2)*a+b*((e*x+d)^(1/2)*arccsch(c*x)+2/c*(-(I*c*(e*x+d)*e+c^2
*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+
```

```
c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*(EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^
2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))-EllipticP
i((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^
2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)))
/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((I*e+c*
d)*c/(c^2*d^2+e^2))^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(x*e + d)*a*e^(-1) + (2*sqrt(x*e + d)*e^(-1)*log(sqrt(c^2*x^2 + 1) +
1) + integrate(2*(c^2*x^2*e + c^2*d*x)/((c^2*x^2*e + e)*sqrt(c^2*x^2 + 1)*s
qrt(x*e + d) + (c^2*x^2*e + e)*sqrt(x*e + d)), x) - integrate((c^2*x^2*(log
(c) + 2)*e + 2*c^2*d*x + e*log(c) + (c^2*x^2*e + e)*log(x))/((c^2*x^2*e + e
)*sqrt(x*e + d)), x))*b
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}{\left(cx \right)}}{\sqrt{d + ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/sqrt(d + e*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/sqrt(e*x + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x)^(1/2),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x)^(1/2), x)
```

3.61 $\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x]),x]
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x]), x]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

Mathematica [A]
time = 4.52, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x]),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x]), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsch}(cx)}{x\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -(log(c)*log((sqrt(x*e + d) - sqrt(d))/(sqrt(x*e + d) + sqrt(d)))/sqrt(d) +
 integrate(log(x)/(sqrt(x*e + d)*x), x) - integrate(log(sqrt(c^2*x^2 + 1) +
 1)/(sqrt(x*e + d)*x), x))*b + a*log((sqrt(x*e + d) - sqrt(d))/(sqrt(x*e +
d) + sqrt(d)))/sqrt(d)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)*sqrt(x*e + d)/(x^2*e + d*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x\sqrt{d + ex}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x*sqrt(d + e*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(e*x + d)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{d + e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x)^(1/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x)^(1/2)), x)
```

3.62 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x]),x]
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x]), x]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

Mathematica [A]
time = 7.03, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x]),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x]), x]
```

Maple [A]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsch}(cx)}{x^2\sqrt{ex+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*((2*sqrt(x*e + d)*e/((x*e + d)*d - d^2) + e*log((sqrt(x*e + d) - sqrt(d
))/(sqrt(x*e + d) + sqrt(d)))/d^(3/2))*log(c) - 2*integrate(log(x)/(sqrt(x*
e + d)*x^2), x) + 2*integrate(log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(x*e + d)*x^2
), x))*b - 1/2*a*(2*sqrt(x*e + d)*e/((x*e + d)*d - d^2) + e*log((sqrt(x*e +
 d) - sqrt(d))/(sqrt(x*e + d) + sqrt(d)))/d^(3/2))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)*sqrt(x*e + d)/(x^3*e + d*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}{\left(cx \right)}}{x^{2} \sqrt{d + ex}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x**2*sqrt(d + e*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(e*x + d)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x^2\,\sqrt{d+e\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x)^(1/2)),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x)^(1/2)), x)
```

3.63 $\int \frac{x^3\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=731

$$\frac{4b\sqrt{d+ex}\,(1+c^2x^2)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{2d^3\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\,\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(\right.}{e^4}$$

```
[Out] -2*d*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^4+2/5*(e*x+d)^(5/2)*(a+b*arccsch(c*
x))/e^4+2*d^3*(a+b*arccsch(c*x))/e^4/(e*x+d)^(1/2)+6*d^2*(a+b*arccsch(c*x))
*(e*x+d)^(1/2)/e^4+4/15*b*(c^2*x^2+1)*(e*x+d)^(1/2)/c^3/e^2/x/(1+1/c^2/x^2)
^(1/2)-64/5*b*d^3*EllipticPi(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),2,2^(1/2)
*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(
-c^2)^(1/2)+e))^(1/2)/c/e^4/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-32/15*b*c*d
*EllipticE(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e
*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/e^3/x/(
1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)+8*b*c*d^2*Ell
ipticF(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c
^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(
1/2)/(-c^2)^(3/2)/e^3/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/15*b*c*(2*c^2*d
^2-e^2)*EllipticF(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(
c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2
)^(1/2)))^(1/2)/(-c^2)^(5/2)/e^3/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]
time = 1.79, antiderivative size = 731, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$,
Rules used = {45, 6445, 12, 6853, 6874, 733, 430, 946, 174, 552, 551, 858, 435, 945, 1598}

$$\frac{2d^3\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}+\frac{6d^2\sqrt{d+ex}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^4}-\frac{2d(d+ex)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{5/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{5e^4}-\frac{64bd^3\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\Pi\left(2;\mathrm{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}d+e}\right)}{5ce^4x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}+\frac{8bcd^2\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{(-c^2)^{3/2}e^3x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}-\frac{4bc\sqrt{c^2x^2+1}(2c^2d^2-e^2)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15(-c^2)^{5/2}e^3x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}-\frac{32bcd\sqrt{c^2x^2+1}\sqrt{d+ex}\,E\left(\mathrm{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{15(-c^2)^{3/2}e^3x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}+\frac{4b(c^2x^2+1)\sqrt{d+ex}}{15c^3e^2x\sqrt{\frac{1}{c^2x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (4*b*Sqrt[d + e*x]*(1 + c^2*x^2))/(15*c^3*e^2*Sqrt[1 + 1/(c^2*x^2)]*x) + (2
*d^3*(a + b*ArcCsch[c*x]))/(e^4*Sqrt[d + e*x]) + (6*d^2*Sqrt[d + e*x]*(a +
b*ArcCsch[c*x]))/e^4 - (2*d*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/e^4 + (2*
(d + e*x)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e^4) - (32*b*c*d*Sqrt[d + e*x]*Sqr
t[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-
c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*(-c^2)^(3/2)*e^3*Sqrt[1 + 1/(c^2*x^2)]
*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) + (8*b*c*d^2*Sqrt[(c^2*(d
```

```
+ e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 -
 Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/((-c^2)
^(3/2)*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*c*(2*c^2*d^2 - e^2
)*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[
ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^
2]*e)])/(15*(-c^2)^(5/2)*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (64*b
*d^3*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*Elli
pticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)]
)/(5*c*e^4*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
```

```
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 945

```
Int[((d_.) + (e_.)*(x_))^(m_)/(Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(
x_)^2]), x_Symbol] :> Simp[2*e^2*(d + e*x)^(m - 2)*Sqrt[f + g*x]*(Sqrt[a +
c*x^2]/(c*g*(2*m - 1))), x] - Dist[1/(c*g*(2*m - 1)), Int[((d + e*x)^(m - 3
)/(Sqrt[f + g*x]*Sqrt[a + c*x^2]))*Simp[a*e^2*(d*g + 2*e*f*(m - 2)) - c*d^3
*g*(2*m - 1) + e*(e*(a*e*g*(2*m - 3)) + c*d*(2*e*f - 3*d*g*(2*m - 1)))*x +
2*e^2*(c*e*f - 3*c*d*g)*(m - 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, g}
, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[2*m] && GeQ[
m, 2]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 1598

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} - \frac{2d(d+ex)^{3/2}\,(a+b}{e^4}$$

$$= \frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} - \frac{2d(d+ex)^{3/2}\,(a+b}{e^4}$$

$$= \frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} - \frac{2d(d+ex)^{3/2}\,(a+b}{e^4}$$

$$= \frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} - \frac{2d(d+ex)^{3/2}\,(a+b}{e^4}$$

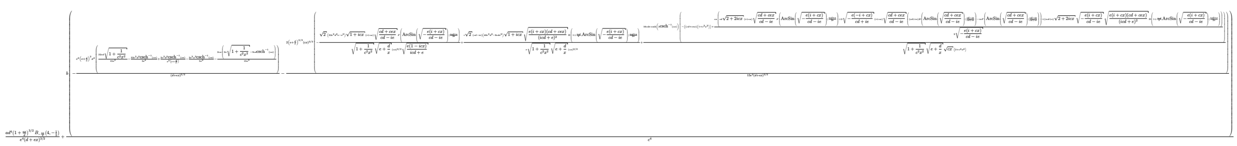
$$= \frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^4} - \frac{2d(d+ex)^{3/2}\,(a+b}{e^4}$$

$$= \frac{4b\sqrt{d+ex}\,(1+c^2x^2)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}\,x} + \frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx\right.}{e^4}$$

$$= \frac{4b\sqrt{d+ex}\,(1+c^2x^2)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}\,x} + \frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx\right.}{e^4}$$

$$= \frac{4b\sqrt{d+ex}\,(1+c^2x^2)}{15c^3e^2\sqrt{1+\frac{1}{c^2x^2}}\,x} + \frac{2d^3\left(a+b\text{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} + \frac{6d^2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx\right.}{e^4}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 29.87, size = 1042, normalized size = 1.43



Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (a*d^4*(1 + (e*x)/d)^(3/2)*Beta[-((e*x)/d), 4, -1/2])/(e^4*(d + e*x)^(3/2))
 + (b*(-((c^2*(e + d/x)^2*x^2*((32*c*d*Sqrt[1 + 1/(c^2*x^2)])/(15*e^3) - (3
2*c^2*d^2*ArcCsch[c*x])/(5*e^4) + (2*c^2*d^2*ArcCsch[c*x])/(e^3*(e + d/x))
- (2*c^2*x^2*ArcCsch[c*x])/(5*e^2) - (2*c*x*(2*e*Sqrt[1 + 1/(c^2*x^2)] - 9*
c*d*ArcCsch[c*x]))/(15*e^3)))/(d + e*x)^(3/2)) - (2*(e + d/x)^(3/2)*(c*x)^(
3/2)*(-((Sqrt[2]*(32*c^2*d^2*e - e^3)*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d +
 c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]],
(I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(
e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(48*c^3*d^3 - 8*c*d*
e^2)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*Ellipt
icPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)
/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (16*c*d*e*Co
sh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*
I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((
e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(
c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*Ellipti
cE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*
EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)]
) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt
[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSi
n[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*(I
 + c*x))/(c*d - I*e))])))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(2
 + c^2*x^2))))/(15*e^4*(d + e*x)^(3/2))))/c^4
```

Maple [C] Result contains complex when optimal does not.
time = 0.87, size = 2021, normalized size = 2.76

method	result	size
derivativedivides	Expression too large to display	2021
default	Expression too large to display	2021

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^4*(-a*(-1/5*(e*x+d)^(5/2)+(e*x+d)^(3/2)*d-3*d^2*(e*x+d)^(1/2)-d^3/(e*x+
d)^(1/2))-b*(-1/5*arccsch(c*x)*(e*x+d)^(5/2)+arccsch(c*x)*(e*x+d)^(3/2)*d-3
*arccsch(c*x)*d^2*(e*x+d)^(1/2)-arccsch(c*x)*d^3/(e*x+d)^(1/2)-2/15/c^3*(-2
*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d*e*(e*x+d)^(3/2)-((I*e+c*d)*c/(c^
2*d^2+e^2))^(1/2)*c^3*d*(e*x+d)^(5/2)-48*I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c
^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)
/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^
(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*
e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2
)*e^3*(e*x+d)^(1/2)+32*I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d
^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1
/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-
c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e+2*((I*e+c*d)*c/(c^2*d^2+e^2))^
(1/2)*c^3*d^2*(e*x+d)^(3/2)-24*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/
(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^
2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*
c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3-8*(-(I*c*(e*x+d)*e+c^2*d*(
e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*
d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2
+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3+48*(-(
I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)
*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)
*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d
)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^3*d^3+I*((I*e
+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^2*e*(e*x+d)^(1/2)-((I*e+c*d)*c/(c^2*d^2+
e^2))^(1/2)*c^3*d^3*(e*x+d)^(1/2)+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*e
*(e*x+d)^(5/2)+9*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))
^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*Elli
pticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+
e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2-8*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2
-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*
d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(
-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2-I*(-(I*c*(e*x+d)*e+c
^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d
)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c
^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^3-((I*
e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c*d*e^2*(e*x+d)^(1/2))/((c^2*(e*x+d)^2-2*c^2*
d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/
2)/(I*e-c*d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/5*((x*e + d)^(5/2)*e^(-4) - 5*(x*e + d)^(3/2)*d*e^(-4) + 15*sqrt(x*e + d)
*d^2*e^(-4) + 5*d^3*e^(-4)/sqrt(x*e + d))*a + 1/5*(2*(x^3*e^3 - 2*d*x^2*e^2
 + 8*d^2*x*e + 16*d^3)*e^(-4)*log(sqrt(c^2*x^2 + 1) + 1)/sqrt(x*e + d) + 5*
integrate(2/5*(c^2*x^4*e^3 - 2*c^2*d*x^3*e^2 + 8*c^2*d^2*x^2*e + 16*c^2*d^3
*x)/((c^2*x^2*e^4 + e^4)*sqrt(c^2*x^2 + 1)*sqrt(x*e + d) + (c^2*x^2*e^4 + e
^4)*sqrt(x*e + d)), x) - 5*integrate(1/5*(c^2*x^5*(5*log(c) + 2)*e^4 - 2*c^
2*d*x^4*e^3 + 48*c^2*d^3*x^2*e + 32*c^2*d^4*x + (12*c^2*d^2*e^2 + 5*e^4*log
(c))*x^3 + 5*(c^2*x^5*e^4 + x^3*e^4)*log(x))/((c^2*x^3*e^5 + c^2*d*x^2*e^4
+ x*e^5 + d*e^4)*sqrt(x*e + d)), x))*b
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\operatorname{acsch}(cx))}{(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))/(d + e*x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x + d)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(d+e\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2),x)
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2), x)
```

3.64 $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=499

$$-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\,\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{4bc\sqrt{d+ex}\,\sqrt{1}}{3}$$

```
[Out] 2/3*(e*x+d)^(3/2)*(a+b*arccsch(c*x))/e^3-2*d^2*(a+b*arccsch(c*x))/e^3/(e*x+
d)^(1/2)-4*d*(a+b*arccsch(c*x))*(e*x+d)^(1/2)/e^3+32/3*b*d^2*EllipticPi(1/2
*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(
c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e^3/x/(1
+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/3*b*c*EllipticE(1/2*(1-(-c^2)^(1/2)*x)^(1
/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)
*(c^2*x^2+1)^(1/2)/(-c^2)^(3/2)/e^2/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2
*d-e*(-c^2)^(1/2)))^(1/2)-20/3*b*c*d*EllipticF(1/2*(1-(-c^2)^(1/2)*x)^(1/2)
*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^2+1)^(1/2
)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/e^2/x/(1+1/c^2/x^
2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]
time = 1.36, antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {45, 6445, 12, 6853, 6874, 733, 430, 946, 174, 552, 551, 858, 435}

$$-\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}-\frac{4d\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{2(d+ex)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}+\frac{32bd^2\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}-\frac{20bcd\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3(-c^2)^{3/2}e^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}+\frac{4bc\sqrt{c^2x^2+1}\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3(-c^2)^{3/2}e^2x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (-2*d^2*(a + b*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x]) - (4*d*Sqrt[d + e*x]*(a +
 b*ArcCsch[c*x]))/e^3 + (2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) +
(4*b*c*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]
*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)*e^
2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) - (
20*b*c*d*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*Ell
ipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - S
qrt[-c^2]*e)])/(3*(-c^2)^(3/2)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) +
 (32*b*d^2*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2
]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d
 + e)])/(3*c*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
```

```
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = -\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\,\left(a+b\right.}{3e^3}$$

$$= -\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\,\left(a+b\right.}{3e^3}$$

$$= -\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\,\left(a+b\right.}{3e^3}$$

$$= -\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\,\left(a+b\right.}{3e^3}$$

$$= -\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\,\left(a+b\right.}{3e^3}$$

$$= -\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\,\left(a+b\right.}{3e^3}$$

$$= -\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\,\left(a+b\right.}{3e^3}$$

$$= -\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} - \frac{4d\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3} + \frac{2(d+ex)^{3/2}\,\left(a+b\right.}{3e^3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 27.56, size = 820, normalized size = 1.64

$$-\frac{ad^3\left(1+\frac{ex}{d}\right)^{3/2}B_{-\frac{ex}{d}}\left(3,-\frac{1}{2}\right)}{e^3(d+ex)^{3/2}}-\frac{b\left(e+\frac{d}{x}\right)\left(-\frac{2c^3x(-8d^2-4dex+e^2x^2)\operatorname{csch}^{-1}(cx)}{e^3}+\frac{10c^3d\sqrt{2+\frac{2}{c^2x^2}}x^2\sqrt{1+icx}\sqrt{\frac{c(d+ex)}{cd-ie}}F\left(\operatorname{ArcSin}\left(\sqrt{-\frac{e(i+cx)}{cd-ie}}\right)|\frac{icd+e}{2e}\right)}{e^2\sqrt{\frac{e(1-icx)}{icd+e}}(-i+cx)}-\frac{2cd\sqrt{2+2icx}(i+cx)\sqrt{\frac{c(d+ex)}{cd-ie}}F\left(\operatorname{ArcSin}\left(\sqrt{-\frac{e(i+cx)}{cd-ie}}\right)|\frac{icd+e}{2e}\right)}{e^2\sqrt{1+\frac{1}{c^2x^2}}\sqrt{\frac{e(1-icx)}{icd+e}}}-\frac{4\sqrt{-\frac{e(-i+cx)}{cd+ie}}(i+cx)\sqrt{\frac{c(d+ex)}{cd-ie}}\left((cd+ie)E\left(\operatorname{ArcSin}\left(\sqrt{\frac{c(d+ex)}{cd-ie}}\right)|\frac{cd-ie}{cd+ie}\right)-ieF\left(\operatorname{ArcSin}\left(\sqrt{\frac{c(d+ex)}{cd-ie}}\right)|\frac{cd-ie}{cd+ie}\right)\right)}{e^2\sqrt{1+\frac{1}{c^2x^2}}\sqrt{\frac{e(1-icx)}{icd+e}}}-\frac{2(icd+e)\sqrt{2+2icx}\sqrt{\frac{ce(i+cx)(d+ex)}{(icd+e)^2}}\Pi\left(1+\frac{icd}{e};\operatorname{ArcSin}\left(\sqrt{-\frac{e(i+cx)}{cd-ie}}\right)|\frac{icd+e}{2e}\right)}{e^2\sqrt{1+\frac{1}{c^2x^2}}}-\frac{2i(cd-ie)(8c^2d^2-e^2)\sqrt{2+2icx}\sqrt{\frac{ce(i+cx)(d+ex)}{(icd+e)^2}}\Pi\left(1+\frac{icd}{e};\operatorname{ArcSin}\left(\sqrt{-\frac{e(i+cx)}{cd-ie}}\right)|\frac{icd+e}{2e}\right)}{e^4\sqrt{1+\frac{1}{c^2x^2}}}\right)}{3c^3(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] -((a*d^3*(1 + (e*x)/d)^(3/2)*Beta[-((e*x)/d), 3, -1/2])/(e^3*(d + e*x)^(3/2
))) - (b*(e + d/x)*((-2*c^3*x*(-8*d^2 - 4*d*e*x + e^2*x^2)*ArcCsch[c*x])/e^
3 + (10*c^3*d*Sqrt[2 + 2/(c^2*x^2)]*x^2*Sqrt[1 + I*c*x]*Sqrt[(c*(d + e*x))/
(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d +
 e)/(2*e)])/(e^2*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)]*(-I + c*x)) - (2*c*d*Sqr
t[2 + (2*I)*c*x]*(I + c*x)*Sqrt[(c*(d + e*x))/(c*d - I*e)]*EllipticF[ArcSin
[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(e^2*Sqrt[1 + 1/(
c^2*x^2)]*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)]) - (4*Sqrt[-((e*(-I + c*x))/(c*
d + I*e))]*(I + c*x)*Sqrt[(c*(d + e*x))/(c*d - I*e)]*((c*d + I*e)*EllipticE
[ArcSin[Sqrt[(c*(d + e*x))/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*El
lipticF[ArcSin[Sqrt[(c*(d + e*x))/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)]))
/(e^2*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)]) - (2*(I*c*d
+ e)*Sqrt[2 + (2*I)*c*x]*Sqrt[(c*e*(I + c*x)*(d + e*x))/(I*c*d + e)^2]*Elli
pticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d +
e)/(2*e)])/(e^2*Sqrt[1 + 1/(c^2*x^2)]) - ((2*I)*(c*d - I*e)*(8*c^2*d^2 - e^
2)*Sqrt[2 + (2*I)*c*x]*Sqrt[(c*e*(I + c*x)*(d + e*x))/(I*c*d + e)^2]*Ellipt
icPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)
/(2*e)])/(e^4*Sqrt[1 + 1/(c^2*x^2)])))/(3*c^3*(d + e*x)^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.84, size = 896, normalized size = 1.80

method	result
derivativedivides	$2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-2d\sqrt{ex+d}-\frac{d^2}{\sqrt{ex+d}}\right)+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}-2\operatorname{arccsch}(cx)d\sqrt{ex+d}-\frac{\operatorname{arccsch}(cx)d^2}{\sqrt{ex+d}}-\right.$

default	$$\frac{2a\left(\frac{(ex+d)^{\frac{3}{2}}}{3}-2d\sqrt{ex+d}-\frac{d^2}{\sqrt{ex+d}}\right)+2b\left(\frac{(ex+d)^{\frac{3}{2}}\operatorname{arccsch}(cx)}{3}-2\operatorname{arccsch}(cx)d\sqrt{ex+d}-\frac{\operatorname{arccsch}(cx)d^2}{\sqrt{ex+d}}-\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^3*(a*(1/3*(e*x+d)^(3/2)-2*d*(e*x+d)^(1/2)-d^2/(e*x+d)^(1/2))+b*(1/3*(e*
x+d)^(3/2)*arccsch(c*x)-2*arccsch(c*x)*d*(e*x+d)^(1/2)-arccsch(c*x)*d^2/(e*
x+d)^(1/2)-2/3/c^2*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2
))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*(5
*I*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c
^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e-8*I*EllipticPi((e*x+d)^(1/2)*((I*e+
c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^
2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c*d*e-4*EllipticF((e*x
+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*
d^2+e^2))^(1/2))*c^2*d^2-EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2)
)^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2+8*EllipticP
i((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^
2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*
c^2*d^2+EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*
d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^2-EllipticE((e*x+d)^(1/2)*((I*e+c*
d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e
^2)/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((I*e
+c*d)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2/3*((x*e + d)^(3/2)*e^(-3) - 6*sqrt(x*e + d)*d*e^(-3) - 3*d^2*e^(-3)/sqrt(
x*e + d))*a + 1/3*(2*(x^2*e^2 - 4*d*x*e - 8*d^2)*e^(-3)*log(sqrt(c^2*x^2 +
1) + 1)/sqrt(x*e + d) + 3*integrate(2/3*(c^2*x^3*e^2 - 4*c^2*d*x^2*e - 8*c^
2*d^2*x)/((c^2*x^2*e^3 + e^3)*sqrt(c^2*x^2 + 1)*sqrt(x*e + d) + (c^2*x^2*e^
3 + e^3)*sqrt(x*e + d)), x) - 3*integrate(1/3*(c^2*x^4*(3*log(c) + 2)*e^3 -
 6*c^2*d*x^3*e^2 - 16*c^2*d^3*x - 3*(8*c^2*d^2*e - e^3*log(c))*x^2 + 3*(c^2
```

```
*x^4*e^3 + x^2*e^3)*log(x))/((c^2*x^3*e^4 + c^2*d*x^2*e^3 + x*e^4 + d*e^3)*
sqrt(x*e + d)), x))*b
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\operatorname{acsch}(cx))}{(d + ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x+d)**(3/2),x)
[Out] Integral(x**2*(a + b*acsch(c*x))/(d + e*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2),x)
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2), x)
```

3.65 $\int \frac{x\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=318

$$\frac{2d\left(a+b\mathrm{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\,\left(a+b\mathrm{csch}^{-1}(cx)\right)}{e^2}+\frac{4bc\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2x^2}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt{1-\sqrt{}}}{\sqrt{2}}\right.\right.}{(-c^2)^{3/2}\,e\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+e}}$$

```
[Out] 2*d*(a+b*arccsch(c*x))/e^2/(e*x+d)^(1/2)+2*(a+b*arccsch(c*x))*(e*x+d)^(1/2)
/e^2-8*b*d*EllipticPi(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*
(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(
1/2)+e))^(1/2)/c/e^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4*b*c*EllipticF(1/
2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)
))^(1/2))*(c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^
2)^(3/2)/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]
time = 1.14, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {45, 6445, 12, 6853, 6874, 733, 430, 946, 174, 552, 551}

$$\frac{2\sqrt{d+ex}\,\left(a+b\mathrm{csch}^{-1}(cx)\right)}{e^2}+\frac{2d\left(a+b\mathrm{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}-\frac{8bd\sqrt{c^2x^2+1}\,\sqrt{\frac{\sqrt{-c^2}\,(d+ex)}{\sqrt{-c^2}\,d+e}}\,\Pi\left(2;\mathrm{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}\,d+e}\right)}{ce^2x\sqrt{\frac{1}{c^2x^2}+1}\,\sqrt{d+ex}}+\frac{4bc\sqrt{c^2x^2+1}\,\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}\,e}}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right)}{(-c^2)^{3/2}\,ex\sqrt{\frac{1}{c^2x^2}+1}\,\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (2*d*(a + b*ArcCsch[c*x]))/(e^2*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*Ar
cCsch[c*x]))/e^2 + (4*b*c*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]*Sqrt
[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c
^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/((-c^2)^(3/2)*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqr
t[d + e*x]) - (8*b*d*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1
 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sq
rt[-c^2]*d + e)])/(c*e^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
```

```
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{3/2}}\,dx = \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{b\int \frac{2(2d+ex)}{e^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2\sqrt{d \cdots}}}{c}$$

$$= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{(2b)\int \frac{2d+ex}{\sqrt{1+\frac{1}{c^2x^2}}\,x^2\sqrt{\cdots}}}{ce^2}$$

$$= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{\cdots}{x\sqrt{\cdots}}}{ce^2\sqrt{1+\cdots}}$$

$$= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \left(\frac{\cdots}{\sqrt{\cdots}}\cdots\right.}{\cdots}$$

$$= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int \frac{\cdots}{x\sqrt{\cdots}}}{ce^2\sqrt{1+\cdots}}$$

$$= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int \frac{\cdots}{x\sqrt{\cdots}}}{\cdots}$$

$$= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{4b\sqrt{-c^2}\,\sqrt{\frac{c^2(d+e\cdots}{c^2d-\sqrt{-\cdots}}}\cdots}{\cdots}$$

$$= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2} + \frac{4b\sqrt{-c^2}\,\sqrt{\frac{c^2(d+e\cdots}{c^2d-\sqrt{-\cdots}}}\cdots}{\cdots}$$

$$= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\cdots} + \frac{2\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\cdots} + \frac{4b\sqrt{-c^2}\,\sqrt{\frac{c^2(d+e\cdots}{c^2d-\sqrt{-\cdots}}}\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.03, size = 264, normalized size = 0.83

$$\frac{2\left(\frac{a(2d+ex)}{\sqrt{d+ex}}+\frac{b(2d+ex)\operatorname{csch}^{-1}(cx)}{\sqrt{d+ex}}-\frac{2ib\sqrt{-\frac{e(-i+cx)}{cd+ie}}\sqrt{-\frac{e(i+cx)}{cd-ie}}\left(F\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)-2\Pi\left(1-\frac{ie}{cd};i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)\right)}{c\sqrt{-\frac{c}{cd-ie}}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x)^(3/2),x]
```

```
[Out] (2*((a*(2*d + e*x))/Sqrt[d + e*x] + (b*(2*d + e*x)*ArcCsch[c*x])/Sqrt[d + e
*x] - ((2*I)*b*Sqrt[-((e*(-I + c*x))/(c*d + I*e))]*Sqrt[-((e*(I + c*x))/(c*
d - I*e))]*(EllipticF[I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d
 - I*e)/(c*d + I*e)] - 2*EllipticPi[1 - (I*e)/(c*d), I*ArcSinh[Sqrt[-(c/(c*
d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)]))/(c*Sqrt[-(c/(c*d - I*
e))]*Sqrt[1 + 1/(c^2*x^2)]*x)))/e^2
```

Maple [C] Result contains complex when optimal does not.
time = 0.78, size = 425, normalized size = 1.34

method	result
derivativedivides	$-2a\left(-\sqrt{ex+d}-\frac{d}{\sqrt{ex+d}}\right)-2b\left(-\sqrt{ex+d}\,\operatorname{arccsch}(cx)-\frac{\operatorname{arccsch}(cx)d}{\sqrt{ex+d}}-\frac{2\sqrt{-\frac{ic(ex+d)e+c^2d(ex+d)-}{c^2d^2+e^2}}}{}\right.$
default	$-2a\left(-\sqrt{ex+d}-\frac{d}{\sqrt{ex+d}}\right)-2b\left(-\sqrt{ex+d}\,\operatorname{arccsch}(cx)-\frac{\operatorname{arccsch}(cx)d}{\sqrt{ex+d}}-\frac{2\sqrt{-\frac{ic(ex+d)e+c^2d(ex+d)-}{c^2d^2+e^2}}}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^2*(-a*(-(e*x+d)^(1/2)-d/(e*x+d)^(1/2))-b*(-(e*x+d)^(1/2)*arccsch(c*x)-a
rccsch(c*x)*d/(e*x+d)^(1/2)-2/c*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)
/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e
^2))^(1/2)*(EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*
I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))-2*EllipticPi((e*x+d)^(1/2)*((I*e
```

```
+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c
^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)))/((c^2*(e*x+d)^2-2*c^
2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/((I*e+c*d)*c/(c^2*d^2+e^2))^(
1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*(sqrt(x*e + d)*e^(-2) + d*e^(-2)/sqrt(x*e + d))*a + (2*(x*e + 2*d)*e^(-2)
*log(sqrt(c^2*x^2 + 1) + 1)/sqrt(x*e + d) + integrate(2*(c^2*x^2*e + 2*c^2*
d*x)/((c^2*x^2*e^2 + e^2)*sqrt(c^2*x^2 + 1)*sqrt(x*e + d) + (c^2*x^2*e^2 +
e^2)*sqrt(x*e + d)), x) - integrate((c^2*x^3*(log(c) + 2)*e^2 + 6*c^2*d*x^2
*e + (4*c^2*d^2 + e^2*log(c))*x + (c^2*x^3*e^2 + x*e^2)*log(x))/((c^2*x^3*e
^3 + c^2*d*x^2*e^2 + x*e^3 + d*e^2)*sqrt(x*e + d)), x))*b
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccsch(c*x) + a*x)*sqrt(x*e + d)/(x^2*e^2 + 2*d*x*e + d^2),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\operatorname{acsch}(cx))}{(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))/(d + e*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(d+e\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2),x)
```

```
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x)^(3/2), x)
```

3.66 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}}+\frac{4b\sqrt{\frac{\sqrt{-c^2}\,(d+ex)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2x^2}\,\Pi\left(2;\operatorname{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}\,d+e}\right)}{ce\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

```
[Out] -2*(a+b*arccsch(c*x))/e/(e*x+d)^(1/2)+4*b*EllipticPi(1/2*(1-(-c^2)^(1/2)*x)
^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((
e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e/x/(1+1/c^2/x^2)^(1/2)/(e*
x+d)^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6425, 1588, 947, 174, 552, 551}

$$\frac{4b\sqrt{c^2x^2+1}\,\sqrt{\frac{\sqrt{-c^2}\,(d+ex)}{\sqrt{-c^2}\,d+e}}\,\Pi\left(2;\operatorname{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}\,d+e}\right)}{cex\sqrt{\frac{1}{c^2x^2}+1}\,\sqrt{d+ex}}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCsch[c*x]))/(e*Sqrt[d + e*x]) + (4*b*Sqrt[(Sqrt[-c^2]*(d + e*
x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqr
t[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(c*e*Sqrt[1 + 1/(c^2*x^2)]*
x*Sqrt[d + e*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
```

```
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{3/2}}\,dx = -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} - \frac{(2b)\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x^2\sqrt{d+ex}}\,dx}{ce}$$

$$= -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{x\sqrt{d+ex}\,\sqrt{\frac{1}{c^2}+x^2}}\,dx}{ce\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} - \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{1}{x\sqrt{1-\sqrt{-c^2}\,x}\,\sqrt{1+\sqrt{-c^2}\,x}\,\sqrt{d+}}}{ce\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} + \frac{\left(4b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{2-x^2}\,\sqrt{d+\frac{e}{\sqrt{-c^2}}-}}\right.}{ce\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} + \frac{\left(4b\sqrt{1+c^2x^2}\,\sqrt{1+\frac{e\left(-1+\sqrt{-c^2}\,x\right)}{\sqrt{-c^2}\,d+e}}\right)\text{Subst}\left(\int \frac{}{(1-}\right.}{ce\sqrt{1+}}$$

$$= -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{e\sqrt{d+ex}} + \frac{4b\sqrt{1+c^2x^2}\,\sqrt{1-\frac{e\left(1-\sqrt{-c^2}\,x\right)}{\sqrt{-c^2}\,d+e}}\;\Pi\left(2;\sin^{-1}\left(\frac{\sqrt{1-}}{}\right.\right.}{ce\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 0.47, size = 166, normalized size = 1.11

$$\frac{-2e(1+c^2x^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)+2bc(icd+e)\sqrt{2+\frac{2}{c^2x^2}}\,x\sqrt{1+icx}\,\sqrt{\frac{ce(i+cx)(d+ex)}{(icd+e)^2}}\,\Pi\left(1+\frac{icd}{e};\operatorname{ArcSin}\left(\sqrt{-\frac{e(i+cx)}{cd-ie}}\right)|\frac{icd+e}{2e}\right)}{e^2\sqrt{d+ex}\,(1+c^2x^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^(3/2),x]
```

```
[Out] (-2*e*(1 + c^2*x^2)*(a + b*ArcCsch[c*x]) + 2*b*c*(I*c*d + e)*Sqrt[2 + 2/(c^
2*x^2)]*x*Sqrt[1 + I*c*x]*Sqrt[(c*e*(I + c*x)*(d + e*x))/(I*c*d + e)^2]*Ell
ipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d +
 e)/(2*e)])/(e^2*Sqrt[d + e*x]*(1 + c^2*x^2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.75, size = 328, normalized size = 2.20

method	result
derivativedivides	$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arccsch}(cx)}{\sqrt{ex+d}}+\frac{2\sqrt{-\frac{ic(ex+d)e+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ic(ex+d)e-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2}{c^2e^2x^2}}}\right.}{e}$
default	$\frac{-\frac{2a}{\sqrt{ex+d}}+2b\left(-\frac{\operatorname{arccsch}(cx)}{\sqrt{ex+d}}+\frac{2\sqrt{-\frac{ic(ex+d)e+c^2d(ex+d)-c^2d^2-e^2}{c^2d^2+e^2}}\sqrt{\frac{ic(ex+d)e-c^2d(ex+d)+c^2d^2+e^2}{c^2d^2+e^2}}}{c\sqrt{\frac{c^2(ex+d)^2-2c^2d(ex+d)+c^2d^2}{c^2e^2x^2}}}\right.}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(-a/(e*x+d)^(1/2)+b*(-1/(e*x+d)^(1/2)*arccsch(c*x)+2/c/((c^2*(e*x+d)^2-
2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/d/((I*e+c*d)*c/(c^2*d^2+e
^2))^(1/2)*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)
*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi
((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2
)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -(2*c^2*integrate(x/((c^2*x^2*e + e)*sqrt(c^2*x^2 + 1)*sqrt(x*e + d) + (c^2
*x^2*e + e)*sqrt(x*e + d)), x) + 2*e^(-1)*log(sqrt(c^2*x^2 + 1) + 1)/sqrt(x
*e + d) + integrate((c^2*x^2*(log(c) - 2)*e - 2*c^2*d*x + e*log(c) + (c^2*x
^2*e + e)*log(x))/((c^2*x^3*e^2 + c^2*d*x^2*e + x*e^2 + d*e)*sqrt(x*e + d))
, x))*b - 2*a*e^(-1)/sqrt(x*e + d)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)*sqrt(x*e + d)/(x^2*e^2 + 2*d*x*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acsch}\,(cx)}{(d + ex)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(d + e*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x)^(3/2),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x)^(3/2), x)
```

3.67 $\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=24

$$\mathrm{Int}\left(\frac{a+b\,\mathrm{csch}^{-1}(cx)}{x(d+ex)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(3/2)),x]
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*(d + e*x)^(3/2)), x]
```

Rubi steps

$$\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

Mathematica [A]
time = 14.47, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{x(d+ex)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(3/2)),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(3/2)), x]
```

Maple [A]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsch}(cx)}{x\,(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -((e*log((sqrt(x*e + d) - sqrt(d))/(sqrt(x*e + d) + sqrt(d)))/d^(3/2) + 2*e
/(sqrt(x*e + d)*d))*e^(-1)*log(c) + integrate(log(x)/(sqrt(x*e + d)*x^2*e +
 sqrt(x*e + d)*d*x), x) - integrate(log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(x*e +
d)*x^2*e + sqrt(x*e + d)*d*x), x))*b + a*(log((sqrt(x*e + d) - sqrt(d))/(sq
rt(x*e + d) + sqrt(d)))/d^(3/2) + 2/(sqrt(x*e + d)*d))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)*sqrt(x*e + d)/(x^3*e^2 + 2*d*x^2*e + d^2*x),
x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}{(cx)}}{x (d + ex)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x*(d + e*x)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x + d)^(3/2)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x)^(3/2)),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x)^(3/2)), x)
```

3.68 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(3/2)),x]
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(3/2)), x]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

Mathematica [A]
time = 16.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(3/2)),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(3/2)), x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsch}(cx)}{x^2\,(ex+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*((2*(3*(x*e + d)*e^2 - 2*d*e^2)/((x*e + d)^(3/2)*d^2 - sqrt(x*e + d)*d^
3) + 3*e^2*log((sqrt(x*e + d) - sqrt(d))/(sqrt(x*e + d) + sqrt(d)))/d^(5/2)
)*e^(-1)*log(c) - 2*integrate(log(x)/(sqrt(x*e + d)*x^3*e + sqrt(x*e + d)*d
*x^2), x) + 2*integrate(log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(x*e + d)*x^3*e + s
qrt(x*e + d)*d*x^2), x))*b - 1/2*a*(2*(3*(x*e + d)*e - 2*d*e)/((x*e + d)^(3
/2)*d^2 - sqrt(x*e + d)*d^3) + 3*e*log((sqrt(x*e + d) - sqrt(d))/(sqrt(x*e
+ d) + sqrt(d)))/d^(5/2))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)*sqrt(x*e + d)/(x^4*e^2 + 2*d*x^3*e + d^2*x^2)
, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}{\left(cx \right)}}{x^{2} \left(d + ex\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x**2*(d + e*x)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x + d)^(3/2)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x^2\,{\left(d+e\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x)^(3/2)),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x)^(3/2)), x)
```

3.69 $\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=777

$$\frac{4bd^2(1+c^2x^2)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}+\frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\,\left(a+bc\right.}{e^4}$$

```
[Out] 2/3*d^3*(a+b*arccsch(c*x))/e^4/(e*x+d)^(3/2)+2/3*(e*x+d)^(3/2)*(a+b*arccsch
(c*x))/e^4-6*d^2*(a+b*arccsch(c*x))/e^4/(e*x+d)^(1/2)+4/3*b*d^2*(c^2*x^2+1)
/c/e^2/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-6*d*(a+b*arccsch(c
*x))*(e*x+d)^(1/2)/e^4+64/3*b*d^2*EllipticPi(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2
^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(
-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e^4/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(
1/2)+4/3*b*c*(2*c^2*d^2+e^2)*EllipticE(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2)
,(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1
)^(1/2)/(-c^2)^(3/2)/e^3/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(
c^2*d-e*(-c^2)^(1/2)))^(1/2)-8/3*b*d^2*EllipticE(1/2*(1-(-c^2)^(1/2)*x)^(1/
2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(-c^2)^(1/2)*(
e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/c/e^3/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(c
^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)-32/3*b*c*d*EllipticF(1/2*(1-(-c^2)
^(1/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(
c^2*x^2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/e^
3/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]
time = 2.15, antiderivative size = 777, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 18, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {45, 6445, 12, 6853, 6874, 759, 21, 733, 435, 972, 946, 174, 552, 551, 849, 858, 430, 1665}

$$\frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}}-\frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}}-\frac{6d\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4}+\frac{2(d+ex)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4}+\frac{64bd^2\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}d+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}d+e}\right)}{3ce^4x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}-\frac{8b\sqrt{-c^2}d^2\sqrt{c^2x^2+1}\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}+\frac{4bc\sqrt{c^2x^2+1}\left(2c^2d^2+e^2\right)\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e^3x\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}}-\frac{32bcd\sqrt{c^2x^2+1}\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}e}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}e}{c^2d-\sqrt{-c^2}e}\right)}{3\left(-c^2\right)^{3/2}e^3x\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}+\frac{4bd^2\left(c^2x^2+1\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (4*b*d^2*(1 + c^2*x^2))/(3*c*e^2*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sq
rt[d + e*x]) + (2*d^3*(a + b*ArcCsch[c*x]))/(3*e^4*(d + e*x)^(3/2)) - (6*d^
2*(a + b*ArcCsch[c*x]))/(e^4*Sqrt[d + e*x]) - (6*d*Sqrt[d + e*x]*(a + b*Arc
Csch[c*x]))/e^4 + (2*(d + e*x)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^4) - (8*b*S
qrt[-c^2]*d^2*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqr
```

```
t[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*c*e^3*(c
^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c
^2]*e)]) + (4*b*c*(2*c^2*d^2 + e^2)*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*Ellipti
cE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[
-c^2]*e)])/(3*(-c^2)^(3/2)*e^3*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt
[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) - (32*b*c*d*Sqrt[(c^2*(d + e*x))/
(c^2*d - Sqrt[-c^2]*e)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c
^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*(-c^2)^(3/2)
*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) + (64*b*d^2*Sqrt[(Sqrt[-c^2]*(d
 + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1
- Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e^4*Sqrt[1 + 1/(c
^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
```

```
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1665

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :>
 With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq,
d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*
d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)
*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*
R*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x]
 && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
```

```
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx\right.}{e^4}$$

$$= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx\right.}{e^4}$$

$$= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx\right.}{e^4}$$

$$= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx\right.}{e^4}$$

$$= \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^4\sqrt{d+ex}} - \frac{6d\sqrt{d+ex}\,\left(a+b\operatorname{csch}^{-1}(cx\right.}{e^4}$$

$$= \frac{68bd^2(1+c^2x^2)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\right.}{e^4\sqrt{d}}$$

$$= \frac{68bd^2(1+c^2x^2)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\right.}{e^4\sqrt{d}}$$

$$= \frac{4bd^2(1+c^2x^2)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\right.}{e^4\sqrt{d}}$$

$$= \frac{4bd^2(1+c^2x^2)}{3ce^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^4(d+ex)^{3/2}} - \frac{6d^2\left(a+b\right.}{e^4\sqrt{d}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 29.61, size = 1108, normalized size = 1.43

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (a*d^4*(1 + (e*x)/d)^(5/2)*Beta[-((e*x)/d), 4, -3/2])/(e^4*(d + e*x)^(5/2))
 + (b*(-((c^3*(e + d/x)^3*x^3*((-4*Sqrt[1 + 1/(c^2*x^2)])/(3*e*(c^2*d^2 + e
^2)) + (32*c*d*ArcCsch[c*x])/(3*e^4) - (2*c*d*ArcCsch[c*x])/(3*e^2*(e + d/x
)^2) - (2*c*x*ArcCsch[c*x])/(3*e^3) - (2*(2*c^2*d^2*e*Sqrt[1 + 1/(c^2*x^2)]
 + 7*c^3*d^3*ArcCsch[c*x] + 7*c*d*e^2*ArcCsch[c*x]))/(3*e^3*(c^2*d^2 + e^2)
*(e + d/x))))/(d + e*x)^(5/2)) + (2*(e + d/x)^(5/2)*(c*x)^(5/2)*(-((Sqrt[2]
*(8*c^3*d^3*e + 8*c*d*e^3)*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*
d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)
/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*
x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(16*c^4*d^4 + 16*c^2*d^2*e^2 -
e^4)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*Ellipt
icPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)
/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) + (2*e^3*Cosh[
2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*I)*
c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(
I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(c*d
 + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*EllipticE[
ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*Ell
ipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)]) +
 (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[(e
*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[S
qrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*(I +
c*x))/(c*d - I*e))])))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]*(2 +
c^2*x^2))))/(3*e^4*(c^2*d^2 + e^2)*(d + e*x)^(5/2))))/c^4
```

Maple [C] Result contains complex when optimal does not.
time = 0.87, size = 2728, normalized size = 3.51

method	result	size
derivativedivides	Expression too large to display	2728
default	Expression too large to display	2728

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/e^4*(-a*(-1/3*(e*x+d)^(3/2)+3*d*(e*x+d)^(1/2)+3*d^2/(e*x+d)^(1/2)-1/3*d^3
/(e*x+d)^(3/2))-b*(-1/3*(e*x+d)^(3/2)*arccsch(c*x)+3*arccsch(c*x)*d*(e*x+d)
^(1/2)+3*arccsch(c*x)*d^2/(e*x+d)^(1/2)-1/3*arccsch(c*x)*d^3/(e*x+d)^(3/2)-
2/3/c^2*(16*I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1
/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*Ellipti
cPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+
e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)
)*c*d*e^3*(e*x+d)^(1/2)+16*I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c
^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2)
)^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c
*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*
d^2+e^2))^(1/2))*c^3*d^3*e*(e*x+d)^(1/2)+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2
)*c^3*d^4*e-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^3*(e*x+d)^2+8*(-(I*c*(e
*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2
*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+
c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))
*c^4*d^4*(e*x+d)^(1/2)-16*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*
d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(
1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)
/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2
+e^2))^(1/2))*c^4*d^4*(e*x+d)^(1/2)-2*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c
^3*d^3*e*(e*x+d)+2*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^4*(e*x+d)-8*I*(-
(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d
)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)
*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))
^(1/2))*c*d*e^3*(e*x+d)^(1/2)+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c*d^2*e^3
+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^2*e*(e*x+d)^2-((I*e+c*d)*c/(c^2*
d^2+e^2))^(1/2)*c^4*d^5+7*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*
d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(
1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e
-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e^2*(e*x+d)^(1/2)+(-(I*c*(e*x+d
)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(
e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)
*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2
*d^2*e^2*(e*x+d)^(1/2)-16*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*
d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(
1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)
/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2
+e^2))^(1/2))*c^2*d^2*e^2*(e*x+d)^(1/2)-8*I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-
c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2
)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^
(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3*e*(e*x+d)^(1/
2)-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^3*e^2-(-(I*c*(e*x+d)*e+c^2*d*(e*
```

```
x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^
2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e
^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^4*(e*x+d)^(1/2
)+(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e
*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(
1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e
^2))^(1/2))*e^4*(e*x+d)^(1/2))/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)
/c^2/e^2/x^2)^(1/2)/x/(c^2*d^2+e^2)/(e*x+d)^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2
))^(1/2)/(I*e-c*d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*((x*e + d)^(3/2)*e^(-4) - 9*sqrt(x*e + d)*d*e^(-4) - 9*d^2*e^(-4)/sqrt(
x*e + d) + d^3*e^(-4)/(x*e + d)^(3/2))*a + 1/3*b*(2*(x^3*e^3 - 6*d*x^2*e^2
- 24*d^2*x*e - 16*d^3)*log(sqrt(c^2*x^2 + 1) + 1)/((x*e^5 + d*e^4)*sqrt(x*e
 + d)) + 3*integrate(2/3*(c^2*x^4*e^3 - 6*c^2*d*x^3*e^2 - 24*c^2*d^2*x^2*e
- 16*c^2*d^3*x)/((c^2*x^3*e^5 + c^2*d*x^2*e^4 + x*e^5 + d*e^4)*sqrt(c^2*x^2
 + 1)*sqrt(x*e + d) + (c^2*x^3*e^5 + c^2*d*x^2*e^4 + x*e^5 + d*e^4)*sqrt(x*
e + d)), x) - 3*integrate(1/3*(c^2*x^5*(3*log(c) + 2)*e^4 - 10*c^2*d*x^4*e^
3 - 80*c^2*d^3*x^2*e - 32*c^2*d^4*x - 3*(20*c^2*d^2*e^2 - e^4*log(c))*x^3 +
 3*(c^2*x^5*e^4 + x^3*e^4)*log(x))/((c^2*x^4*e^6 + 2*c^2*d*x^3*e^5 + (c^2*d
^2*e^4 + e^6)*x^2 + 2*d*x*e^5 + d^2*e^4)*sqrt(x*e + d)), x))
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\operatorname{acsch}(cx))}{(d + ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))/(d + e*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3 \left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x)^(5/2),x)
```

```
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x)^(5/2), x)
```

3.70 $\int \frac{x^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=569

$$-\frac{4bd(1+c^2x^2)}{3ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}-\frac{2d^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\,\left(a+b\mathrm{csc}\right.}{e^3}$$

```
[Out] -2/3*d^2*(a+b*arccsch(c*x))/e^3/(e*x+d)^(3/2)+4*d*(a+b*arccsch(c*x))/e^3/(e
*x+d)^(1/2)-4/3*b*d*(c^2*x^2+1)/c/e/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(e*
x+d)^(1/2)+2*(a+b*arccsch(c*x))*(e*x+d)^(1/2)/e^3-32/3*b*d*EllipticPi(1/2*(
1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^
2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/e^3/x/(1+1
/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/3*b*d*EllipticE(1/2*(1-(-c^2)^(1/2)*x)^(1/2
)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(-c^2)^(1/2)*(e
*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/c/e^2/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(c^
2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)+4*b*c*EllipticF(1/2*(1-(-c^2)^(1/2)
*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(c^2*x^
2+1)^(1/2)*(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)/(-c^2)^(3/2)/e^2/x/(1
+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]
time = 1.68, antiderivative size = 569, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 17, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.810$, Rules used = {45, 6445, 12, 6853, 6874, 759, 21, 733, 435, 972, 946, 174, 552, 551, 849, 858, 430}

$$-\frac{2d^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}}+\frac{4d\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}}+\frac{2\sqrt{d+ex}\,\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^3}+\frac{4b\sqrt{-c^2}\,d\sqrt{c^2x^2+1}\,\sqrt{d+ex}\,E\left(\mathrm{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}\,\left(c^2d^2+e^2\right)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}\,e}}}-\frac{32bd\sqrt{c^2x^2+1}\,\sqrt{\frac{\sqrt{-c^2}\,(d+ex)}{\sqrt{-c^2}\,d+e}}\,\Pi\left(2;\mathrm{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}\,d+e}\right)}{3ce^3x\sqrt{\frac{1}{c^2x^2}+1}\,\sqrt{d+ex}}+\frac{4bc\sqrt{c^2x^2+1}\,\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}\,e}}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right)}{\left(-c^2\right)^{3/2}e^2x\sqrt{\frac{1}{c^2x^2}+1}\,\sqrt{d+ex}}-\frac{4bd(c^2x^2+1)}{3cex\sqrt{\frac{1}{c^2x^2}+1}\,\left(c^2d^2+e^2\right)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (-4*b*d*(1 + c^2*x^2))/(3*c*e*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[
d + e*x]) - (2*d^2*(a + b*ArcCsch[c*x]))/(3*e^3*(d + e*x)^(3/2)) + (4*d*(a
+ b*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x]) + (2*Sqrt[d + e*x]*(a + b*ArcCsch[c*
x]))/e^3 + (4*b*Sqrt[-c^2]*d*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcS
in[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e
)])/(3*c*e^2*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(c^2*(d + e*x))/(
c^2*d - Sqrt[-c^2]*e)]) + (4*b*c*Sqrt[(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e
)]*Sqrt[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*
```

```
Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/((-c^2)^(3/2)*e^2*Sqrt[1 + 1/(c^2*x^
2)]*x*Sqrt[d + e*x]) - (32*b*d*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d +
e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]],
 (2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e^3*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = -\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$= -\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$= -\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$= -\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$= -\frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex}} + \frac{2\sqrt{d+ex}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

$$= -\frac{12bd(1+c^2x^2)}{ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\text{csc}\right.}{e^3\sqrt{d+}}$$

$$= -\frac{12bd(1+c^2x^2)}{ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\text{csc}\right.}{e^3\sqrt{d+}}$$

$$= -\frac{4bd(1+c^2x^2)}{3ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\text{cs}\right.}{e^3\sqrt{d}}$$

$$= -\frac{4bd(1+c^2x^2)}{3ce\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3(d+ex)^{3/2}} + \frac{4d\left(a+b\text{cs}\right.}{e^3\sqrt{d}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.
time = 29.62, size = 1076, normalized size = 1.89

$$-\frac{ad^3\left(1+\frac{ex}{d}\right)^{5/2}B_{-\frac{ex}{d}}\left(3,-\frac{3}{2}\right)}{e^3(d+ex)^{5/2}}+$$ [illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] -((a*d^3*(1 + (e*x)/d)^(5/2)*Beta[-((e*x)/d), 3, -3/2])/(e^3*(d + e*x)^(5/2
))) + (b*(-((c^3*(e + d/x)^3*x^3*((-4*c*d*Sqrt[1 + 1/(c^2*x^2)])/(3*e^2*(c^
2*d^2 + e^2)) - (16*ArcCsch[c*x])/(3*e^3) + (2*ArcCsch[c*x])/(3*e*(e + d/x)
^2) + (4*(c*d*e*Sqrt[1 + 1/(c^2*x^2)] + 2*c^2*d^2*ArcCsch[c*x] + 2*e^2*ArcC
sch[c*x]))/(3*e^2*(c^2*d^2 + e^2)*(e + d/x))))/(d + e*x)^(5/2)) - (2*(e + d
/x)^(5/2)*(c*x)^(5/2)*(-((Sqrt[2]*(3*c^2*d^2*e + 3*e^3)*Sqrt[1 + I*c*x]*(I
+ c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x)
)/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*
(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(8
*c^3*d^3 + 9*c*d*e^2)*Sqrt[1 + I*c*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c
*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e
))]], (I*c*d + e)/(2*e)])/(e*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2
)) - (2*c*d*e*Cosh[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(
c*d*Sqrt[2 + (2*I)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF
[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-(
(e*(-I + c*x))/(c*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*
d + I*e)*EllipticE[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*
d + I*e)] - I*e*EllipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I
*e)/(c*d + I*e)]) + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c
*d - I*e))]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 +
(I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))
/(2*Sqrt[-((e*(I + c*x))/(c*d - I*e))])))/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d
/x]*Sqrt[c*x]*(2 + c^2*x^2))))/(3*e^3*(c^2*d^2 + e^2)*(d + e*x)^(5/2))))/c^
3
```

Maple [C] Result contains complex when optimal does not.
time = 0.82, size = 2497, normalized size = 4.39

method	result	size
derivativedivides	Expression too large to display	2497
default	Expression too large to display	2497

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)

[Out] 2/e^3*(a*((e*x+d)^(1/2)+2*d/(e*x+d)^(1/2)-1/3*d^2/(e*x+d)^(3/2))+b*((e*x+d)
^(1/2)*arccsch(c*x)+2*arccsch(c*x)*d/(e*x+d)^(1/2)-1/3*arccsch(c*x)*d^2/(e*
x+d)^(3/2)-2/3/c*(8*I*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^
(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*
e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/
(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^
2))^(1/2)*e^3*(e*x+d)^(1/2)-2*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^2*e
*(e*x+d)+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^3*e-((I*e+c*d)*c/(c^2*d^
2+e^2))^(1/2)*c^3*d^2*(e*x+d)^2-8*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^
2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2)
)^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c
^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)
/(c^2*d^2+e^2))^(1/2)*c^3*d^3*(e*x+d)^(1/2)+4*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d
)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e
^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2)
)^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3*(e*x+d)^(1/
2)-(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(
e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^
(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+
e^2))^(1/2))*c^3*d^3*(e*x+d)^(1/2)-3*I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d
^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^
2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)
,(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*e^3*(e*x+d)^(1/2)+2*((I*e+
c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^3*(e*x+d)-3*I*(-(I*c*(e*x+d)*e+c^2*d*(e*x
+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2
+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^
2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e*(e*x+d)
^(1/2)+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d*e*(e*x+d)^2+I*((I*e+c*d)*c
/(c^2*d^2+e^2))^(1/2)*d*e^3-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^4-8*Ell
ipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*
d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(
1/2))*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*
c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*c*d*e^2*(e*x+d)
^(1/2)+4*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*(
(I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e
x+d)^(1/2)((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^
2*d^2+e^2))^(1/2))*c*d*e^2*(e*x+d)^(1/2)-(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2
*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(
c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/
2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2*(e*x+d)^(1/2)+8*
I*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*
(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^
2))^(1/2))*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)

```
*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*c^2*d^2*e*
(e*x+d)^(1/2)-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c*d^2*e^2)/((c^2*(e*x+d)^2-
2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/2)/x/(c^2*d^2+e^2)/(e*x+d)^(1/
2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(I*e-c*d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sqrt(x*e + d)*e^(-3) + 6*d*e^(-3)/sqrt(x*e + d) - d^2*e^(-3)/(x*e +
d)^(3/2))*a + 1/3*b*(2*(3*x^2*e^2 + 12*d*x*e + 8*d^2)*log(sqrt(c^2*x^2 + 1)
 + 1)/((x*e^4 + d*e^3)*sqrt(x*e + d)) + 3*integrate(2/3*(3*c^2*x^3*e^2 + 12
*c^2*d*x^2*e + 8*c^2*d^2*x)/((c^2*x^3*e^4 + c^2*d*x^2*e^3 + x*e^4 + d*e^3)*
sqrt(c^2*x^2 + 1)*sqrt(x*e + d) + (c^2*x^3*e^4 + c^2*d*x^2*e^3 + x*e^4 + d*
e^3)*sqrt(x*e + d)), x) - 3*integrate(1/3*(3*c^2*x^4*(log(c) + 2)*e^3 + 30*
c^2*d*x^3*e^2 + 16*c^2*d^3*x + (40*c^2*d^2*e + 3*e^3*log(c))*x^2 + 3*(c^2*x
^4*e^3 + x^2*e^3)*log(x))/((c^2*x^4*e^5 + 2*c^2*d*x^3*e^4 + (c^2*d^2*e^3 +
e^5)*x^2 + 2*d*x*e^4 + d^2*e^3)*sqrt(x*e + d)), x))
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\operatorname{acsch}(cx))}{(d + ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))/(d + e*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(5/2),x)
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x)^(5/2), x)
```

3.71 $\int \frac{x\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=393

$$\frac{4b(1+c^2x^2)}{3c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}+\frac{2d\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}-\frac{2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}-\frac{4b\sqrt{-c^2}\,\sqrt{d+ex}\,\sqrt{1-}}{3ce\left(c^2d\right.}$$

```
[Out] 2/3*d*(a+b*arccsch(c*x))/e^2/(e*x+d)^(3/2)-2*(a+b*arccsch(c*x))/e^2/(e*x+d)
^(1/2)+4/3*b*(c^2*x^2+1)/c/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2
)+8/3*b*EllipticPi(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c
^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2
)+e))^(1/2)/c/e^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/3*b*EllipticE(1/2*(
1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^
(1/2))*(-c^2)^(1/2)*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/c/e/(c^2*d^2+e^2)/x/(1+
1/c^2/x^2)^(1/2)/(c^2*(e*x+d)/(c^2*d-e*(-c^2)^(1/2)))^(1/2)
```

Rubi [A]

time = 1.48, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 14, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.737$, Rules used = {45, 6445, 12, 6853, 6874, 759, 21, 733, 435, 972, 946, 174, 552, 551}

$$-\frac{2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}}+\frac{2d\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}}-\frac{4b\sqrt{-c^2}\,\sqrt{c^2x^2+1}\,\sqrt{d+ex}\,E\left(\mathrm{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right)}{3cex\sqrt{\frac{1}{c^2x^2}+1}\,\left(c^2d^2+e^2\right)\sqrt{\frac{c^2(d+ex)}{c^2d-\sqrt{-c^2}\,e}}}+\frac{8b\sqrt{c^2x^2+1}\,\sqrt{\frac{\sqrt{-c^2}\,(d+ex)}{\sqrt{-c^2}\,d+e}}\,\Pi\left(2;\mathrm{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}\,d+e}\right)}{3ce^2x\sqrt{\frac{1}{c^2x^2}+1}\,\sqrt{d+ex}}+\frac{4b(c^2x^2+1)}{3cx\sqrt{\frac{1}{c^2x^2}+1}\,\left(c^2d^2+e^2\right)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (4*b*(1 + c^2*x^2))/(3*c*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e
*x]) + (2*d*(a + b*ArcCsch[c*x]))/(3*e^2*(d + e*x)^(3/2)) - (2*(a + b*ArcCs
ch[c*x]))/(e^2*Sqrt[d + e*x]) - (4*b*Sqrt[-c^2]*Sqrt[d + e*x]*Sqrt[1 + c^2*
x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c
^2*d - Sqrt[-c^2]*e)])/(3*c*e*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[
(c^2*(d + e*x))/(c^2*d - Sqrt[-c^2]*e)]) + (8*b*Sqrt[(Sqrt[-c^2]*(d + e*x))
/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-
c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(3*c*e^2*Sqrt[1 + 1/(c^2*x^2)]
*x*Sqrt[d + e*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
```

```
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 946

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[1/Sqrt[a], Int[1/((d + e*x)
*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x], x]] /; FreeQ[{a, c, d, e,
f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex)^{5/2}}\,dx = \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{b\int \frac{2(-2d-3ex)}{3e^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2(d+ex)^{3/2}}\,dx}{c}$$

$$= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{(2b)\int \frac{-2d-3ex}{\sqrt{1+\frac{1}{c^2x^2}}\,x^2(d+ex)^{3/2}}\,dx}{3ce^2}$$

$$= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int \frac{-2d-3e}{x(d+ex)^{3/2}\sqrt{1+}}}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} + \frac{\left(2b\sqrt{1+c^2x^2}\right)\int\left(-\frac{3}{(d+ex)^{3/2}\sqrt{}}\right.}{3ce^2\sqrt{}}$$

$$= \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex}} - \frac{\left(4bd\sqrt{1+c^2x^2}\right)\int \frac{1}{x(d+ex)^{3/2}\sqrt{1}}}{3ce^2\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= \frac{4b(1+c^2x^2)}{c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\operatorname{csch}^{-1}(\right.}{e^2\sqrt{d+ex}}$$

$$= \frac{4b(1+c^2x^2)}{c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\operatorname{csch}^{-1}(\right.}{e^2\sqrt{d+ex}}$$

$$= \frac{4b(1+c^2x^2)}{3c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\operatorname{csch}^{-1}\right.}{e^2\sqrt{d+ex}}$$

$$= \frac{4b(1+c^2x^2)}{3c\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2(d+ex)^{3/2}} - \frac{2\left(a+b\operatorname{csch}^{-1}\right.}{e^2\sqrt{d+ex}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.62, size = 390, normalized size = 0.99

$$\frac{2}{3}\left(\frac{2bc\sqrt{1+\frac{1}{c^2x^2}}\,x}{(c^2d^2+e^2)\sqrt{d+ex}}-\frac{a(2d+3ex)}{e^2(d+ex)^{3/2}}-\frac{b(2d+3ex)\operatorname{csch}^{-1}(cx)}{e^2(d+ex)^{3/2}}+\frac{2ib\sqrt{-\frac{c}{cd-ie}}\sqrt{-\frac{e(-i+cx)}{cd+ie}}\sqrt{-\frac{e(i+cx)}{cd-ie}}\left(cdE\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)-cdF\left(i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)+2(cd-ie)\Pi\left(1-\frac{ie}{cd};i\sinh^{-1}\left(\sqrt{-\frac{c}{cd-ie}}\sqrt{d+ex}\right)|\frac{cd-ie}{cd+ie}\right)\right)}{c^2de^2\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x)^(5/2),x]
```

```
[Out] (2*((2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x)/((c^2*d^2 + e^2)*Sqrt[d + e*x]) - (a*(2
*d + 3*e*x))/(e^2*(d + e*x)^(3/2)) - (b*(2*d + 3*e*x)*ArcCsch[c*x])/(e^2*(d
 + e*x)^(3/2)) + ((2*I)*b*Sqrt[-(c/(c*d - I*e))]*Sqrt[-((e*(-I + c*x))/(c*d
 + I*e))]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*(c*d*EllipticE[I*ArcSinh[Sqrt[
-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] - c*d*EllipticF[
I*ArcSinh[Sqrt[-(c/(c*d - I*e))]*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)] +
 2*(c*d - I*e)*EllipticPi[1 - (I*e)/(c*d), I*ArcSinh[Sqrt[-(c/(c*d - I*e))]
*Sqrt[d + e*x]], (c*d - I*e)/(c*d + I*e)]))/(c^2*d*e^2*Sqrt[1 + 1/(c^2*x^2)
]*x)))/3
```

Maple [C] Result contains complex when optimal does not.
time = 0.80, size = 2106, normalized size = 5.36

method	result	size
derivativedivides	Expression too large to display	2106
default	Expression too large to display	2106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e^2*(-a*(-1/3*d/(e*x+d)^(3/2)+1/(e*x+d)^(1/2))-b*(-1/3*arccsch(c*x)*d/(e*
x+d)^(3/2)+1/(e*x+d)^(1/2)*arccsch(c*x)-2/3/c*(I*((I*e+c*d)*c/(c^2*d^2+e^2)
)^(1/2)*d*e^3-2*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^2*e*(e*x+d)-((I*e
+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^2*(e*x+d)^2-2*EllipticPi((e*x+d)^(1/2)*(
(I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*
c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(-(I*c*(e*x+d)*e+
c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+
d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*c^3*d^3*(e*x+d)^(1/2)+(-(I*c*(e*x+d)*e
+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x
+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/
(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^
3*(e*x+d)^(1/2)-(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^
(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*Ellip
ticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e
^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3*(e*x+d)^(1/2)+2*I*(-(I*c*(e*x+d)*e+c^2*d*
(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2
```

```
*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d
^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(
1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e*(e*x+d)^(1/2)+2*((I*e+c*d
)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^3*(e*x+d)+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2
)*c^2*d^3*e+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d*e*(e*x+d)^2-((I*e+c*d
)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^4-2*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c
^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2
))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-
c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2
)/(c^2*d^2+e^2))^(1/2)*c*d*e^2*(e*x+d)^(1/2)+(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)
-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^
2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))
^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2*(e*x+d)^(1/2
)-(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e
*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(
1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e
^2))^(1/2))*c*d*e^2*(e*x+d)^(1/2)+2*I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^
2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2
*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)
,1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d
)*c/(c^2*d^2+e^2))^(1/2))*e^3*(e*x+d)^(1/2)-((I*e+c*d)*c/(c^2*d^2+e^2))^(1/
2)*c*d^2*e^2)/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^2/e^2/x^2)^(1/
2)/x/d/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(c^2*d^2+e^2)/(e*x+d)^(1/2)/(I*e-c
*d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -2/3*a*(3*e^(-2)/sqrt(x*e + d) - d*e^(-2)/(x*e + d)^(3/2)) - 1/3*b*(2*(3*x*
e + 2*d)*log(sqrt(c^2*x^2 + 1) + 1)/((x*e^3 + d*e^2)*sqrt(x*e + d)) + 3*int
egrate(2/3*(3*c^2*x^2*e + 2*c^2*d*x)/((c^2*x^3*e^3 + c^2*d*x^2*e^2 + x*e^3
+ d*e^2)*sqrt(c^2*x^2 + 1)*sqrt(x*e + d) + (c^2*x^3*e^3 + c^2*d*x^2*e^2 + x
*e^3 + d*e^2)*sqrt(x*e + d)), x) + 3*integrate(1/3*(3*c^2*x^3*(log(c) - 2)*
e^2 - 10*c^2*d*x^2*e - (4*c^2*d^2 - 3*e^2*log(c))*x + 3*(c^2*x^3*e^2 + x*e^
2)*log(x))/((c^2*x^4*e^4 + 2*c^2*d*x^3*e^3 + (c^2*d^2*e^2 + e^4)*x^2 + 2*d*
x*e^3 + d^2*e^2)*sqrt(x*e + d)), x))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccsch(c*x) + a*x)*sqrt(x*e + d)/(x^3*e^3 + 3*d*x^2*e^2 + 3*
d^2*x*e + d^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\,\text{acsch}\,(cx))}{(d + ex)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x+d)**(5/2),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))/(d + e*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(d + e\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x)^(5/2),x)
```

```
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x)^(5/2), x)
```

3.72 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{5/2}} \, dx$

Optimal. Leaf size=369

$$-\frac{4be(1+c^2x^2)}{3cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}+\frac{4b\sqrt{-c^2}\,\sqrt{d+ex}\,\sqrt{1+c^2x^2}\,E\left(\operatorname{ArcSin}\left(\right.\right.}{3cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}}$$

```
[Out] -2/3*(a+b*arccsch(c*x))/e/(e*x+d)^(3/2)-4/3*b*e*(c^2*x^2+1)/c/d/(c^2*d^2+e^
2)/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/3*b*EllipticE(1/2*(1-(-c^2)^(1/2)*
x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e*(-c^2)^(1/2)))^(1/2))*(-c^2)^(
1/2)*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/c/d/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2
)/((e*x+d)/(d+e/(-c^2)^(1/2)))^(1/2)+4/3*b*EllipticPi(1/2*(1-(-c^2)^(1/2)*x
)^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)+e))^(1/2))*(c^2*x^2+1)^(1/2)*(
(e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/2)/c/d/e/x/(1+1/c^2/x^2)^(1/2)/
(e*x+d)^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {6425, 1588, 972, 759, 21, 733, 435, 947, 174, 552, 551}

$$-\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}}+\frac{4b\sqrt{-c^2}\,\sqrt{c^2x^2+1}\,\sqrt{d+ex}\,E\left(\operatorname{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right)}{3cdx\sqrt{\frac{1}{c^2x^2}+1}\,\left(c^2d^2+e^2\right)\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}}+\frac{4b\sqrt{c^2x^2+1}\,\sqrt{\frac{\sqrt{-c^2}\,(d+ex)}{\sqrt{-c^2}\,d+e}}\,\Pi\left(2;\operatorname{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}\,d+e}\right)}{3cdex\sqrt{\frac{1}{c^2x^2}+1}\,\sqrt{d+ex}}-\frac{4be(c^2x^2+1)}{3cdx\sqrt{\frac{1}{c^2x^2}+1}\,\left(c^2d^2+e^2\right)\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x)^(5/2),x]
```

```
[Out] (-4*b*e*(1 + c^2*x^2))/(3*c*d*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[
d + e*x]) - (2*(a + b*ArcCsch[c*x]))/(3*e*(d + e*x)^(3/2)) + (4*b*Sqrt[-c^2
]*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/S
qrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(3*c*d*(c^2*d^2 + e^2)*
Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(d + e*x)/(d + e/Sqrt[-c^2])]) + (4*b*Sqrt[(Sq
rt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]*Sqrt[1 + c^2*x^2]*EllipticPi[2, Arc
Sin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2*e)/(Sqrt[-c^2]*d + e)])/(3*c*d*e*Sq
rt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
```

```
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex)^{5/2}}\,dx = -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{(2b)\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x^2(d+ex)^{3/2}}\,dx}{3ce}$$

$$= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{x(d+ex)^{3/2}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3ce\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \left(-\frac{e}{d(d+ex)^{3/2}\sqrt{\frac{1}{c^2}+x^2}} + \frac{1}{dx\sqrt{d+ex}\,\sqrt{}}\right.}{3ce\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{3cd\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right.}{3}$$

$$= -\frac{4be(1+c^2x^2)}{3cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} - \frac{\left(4bc\sqrt{\frac{1}{c^2}+x^2}\right)}{3d\left(c^2d^2+\right.}$$

$$= -\frac{4be(1+c^2x^2)}{3cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} + \frac{\left(2bc\sqrt{\frac{1}{c^2}+x^2}\right)}{3d\left(c^2d^2+e^2\right)}$$

$$= -\frac{4be(1+c^2x^2)}{3cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}} - \frac{2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e(d+ex)^{3/2}} + \frac{\left(4b\sqrt{-c^2}\,\sqrt{d+\right.}{}$$

$$4b\sqrt{-c^2}\,\sqrt{d+e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 18.87, size = 784, normalized size = 2.12

$$\frac{-\frac{2a(d+ex)}{e}+\frac{4bc\sqrt{1+\frac{1}{c^2x^2}}(d+ex)^3}{d(c^2d^2+e^2)}-\frac{2bex^2(d+ex)\operatorname{csch}^{-1}(cx)}{d^2}-\frac{2b(d+ex)^3\operatorname{csch}^{-1}(cx)}{d^2e}+\frac{4bx(d+ex)^2\left(-cde\sqrt{1+\frac{1}{c^2x^2}}+(c^2d^2+e^2)\operatorname{csch}^{-1}(cx)\right)}{d^2(c^2d^2+e^2)}+\frac{2ibcd\sqrt{2+2icx}(d+ex)^2\sqrt{\frac{ce(i+cx)(d+ex)}{(icd+e)^2}}\,\Pi\left(1+\frac{icd}{e};\operatorname{ArcSin}\left(\sqrt{-\frac{e(i+cx)}{cd-ie}}\right)\middle|\frac{icd+e}{2e}\right)}{(cd+ie)e^2\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{4b(d+ex)^2\cosh\left(2\operatorname{csch}^{-1}(cx)\right)\left(-c(d+ex)(1+c^2x^2)+\frac{cx\left(cd\sqrt{2+2icx}(i+cx)\sqrt{\frac{c(d+ex)}{cd-ie}}F\left(\operatorname{ArcSin}\left(\sqrt{-\frac{e(i+cx)}{cd-ie}}\right)\middle|\frac{icd+e}{2e}\right)+2\sqrt{\frac{e(1+icx)}{-icd+e}}(i+cx)\sqrt{\frac{c(d+ex)}{cd-ie}}\left((cd+ie)E\left(\operatorname{ArcSin}\left(\sqrt{\frac{c(d+ex)}{cd-ie}}\right)\middle|\frac{cd-ie}{cd+ie}\right)-ieF\left(\operatorname{ArcSin}\left(\sqrt{\frac{c(d+ex)}{cd-ie}}\right)\middle|\frac{cd-ie}{cd+ie}\right)\right)+(icd+e)\sqrt{\frac{e(1-icx)}{icd+e}}\sqrt{2+2icx}\sqrt{\frac{ce(i+cx)(d+ex)}{(icd+e)^2}}\,\Pi\left(1+\frac{icd}{e};\operatorname{ArcSin}\left(\sqrt{-\frac{e(i+cx)}{cd-ie}}\right)\middle|\frac{icd+e}{2e}\right)\right)}{2\sqrt{\frac{e(1-icx)}{icd+e}}}\right)}{d(c^2d^2+e^2)\sqrt{1+\frac{1}{c^2x^2}}(2+c^2x^2)}}{3(d+ex)^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^(5/2),x]
```

```
[Out] ((-2*a*(d + e*x))/e + (4*b*c*Sqrt[1 + 1/(c^2*x^2)]*(d + e*x)^3)/(d*(c^2*d^2
 + e^2)) - (2*b*e*x^2*(d + e*x)*ArcCsch[c*x])/d^2 - (2*b*(d + e*x)^3*ArcCsc
h[c*x])/(d^2*e) + (4*b*x*(d + e*x)^2*(-(c*d*e*Sqrt[1 + 1/(c^2*x^2)]) + (c^2
*d^2 + e^2)*ArcCsch[c*x]))/(d^2*(c^2*d^2 + e^2)) + ((2*I)*b*c*d*Sqrt[2 + (2
*I)*c*x]*(d + e*x)^2*Sqrt[(c*e*(I + c*x)*(d + e*x))/(I*c*d + e)^2]*Elliptic
Pi[1 + (I*c*d)/e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(
2*e)])/((c*d + I*e)*e^2*Sqrt[1 + 1/(c^2*x^2)]*x) + (4*b*(d + e*x)^2*Cosh[2*
ArcCsch[c*x]]*(-(c*(d + e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*I)*c*x]
*(I + c*x)*Sqrt[(c*(d + e*x))/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I +
c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[(e*(1 + I*c*x))/((-I)*c*d
 + e)]*(I + c*x)*Sqrt[(c*(d + e*x))/(c*d - I*e)]*((c*d + I*e)*EllipticE[Arc
Sin[Sqrt[(c*(d + e*x))/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*Ellipt
icF[ArcSin[Sqrt[(c*(d + e*x))/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)]) + (I
*c*d + e)*Sqrt[(e*(1 - I*c*x))/(I*c*d + e)]*Sqrt[2 + (2*I)*c*x]*Sqrt[(c*e*(
I + c*x)*(d + e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin[Sqrt[-(
(e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[(e*(1 - I*c*x))/
(I*c*d + e)])))/(d*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*(2 + c^2*x^2)))/(3
*(d + e*x)^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.80, size = 2079, normalized size = 5.63

method	result	size
derivativedivides	Expression too large to display	2079
default	Expression too large to display	2079

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(-1/3*a/(e*x+d)^(3/2)+b*(-1/3/(e*x+d)^(3/2)*arccsch(c*x)-2/3/c*(-2*I*((
I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^2*e*(e*x+d)-I*(-(I*c*(e*x+d)*e+c^2*d*
(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2
*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d
^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(
1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e*(e*x+d)^(1/2)-((I*e+c*d)*
c/(c^2*d^2+e^2))^(1/2)*c^3*d^2*(e*x+d)^2+EllipticPi((e*x+d)^(1/2)*((I*e+c*d
```

```
)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d
^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*(-(I*c*(e*x+d)*e+c^2*d*(e
*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d
^2+e^2)/(c^2*d^2+e^2))^(1/2)*c^3*d^3*(e*x+d)^(1/2)+(-(I*c*(e*x+d)*e+c^2*d*(
e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*
d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2
+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3*(e*x+d
)^(1/2)-(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((
I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*
x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2
*d^2+e^2))^(1/2))*c^3*d^3*(e*x+d)^(1/2)+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)
*c^2*d*e*(e*x+d)^2+2*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d^3*(e*x+d)-I*(-
(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d
)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2
)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*
d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*e^3*(e*x+d)^(1
/2)+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^3*e-((I*e+c*d)*c/(c^2*d^2+e^2
))^(1/2)*c^3*d^4+EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)
,1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d
)*c/(c^2*d^2+e^2))^(1/2))*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*
d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(
1/2)*c*d*e^2*(e*x+d)^(1/2)+(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2
*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^
(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*
e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^2*(e*x+d)^(1/2)-(-(I*c*(e*x+d)*e
+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x
+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/
(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^
2*(e*x+d)^(1/2)+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*d*e^3-((I*e+c*d)*c/(c^2
*d^2+e^2))^(1/2)*c*d^2*e^2)/((c^2*(e*x+d)^2-2*c^2*d*(e*x+d)+c^2*d^2+e^2)/c^
2/e^2/x^2)^(1/2)/x/d^2/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)/(c^2*d^2+e^2)/(e*x
+d)^(1/2)/(I*e-c*d)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(6*c^2*integrate(1/3*x/((c^2*x^3*e^2 + c^2*d*x^2*e + x*e^2 + d*e)*sqrt
(c^2*x^2 + 1)*sqrt(x*e + d) + (c^2*x^3*e^2 + c^2*d*x^2*e + x*e^2 + d*e)*sqr
t(x*e + d)), x) + 2*log(sqrt(c^2*x^2 + 1) + 1)/((x*e^2 + d*e)*sqrt(x*e + d)
) + 3*integrate(1/3*(c^2*x^2*(3*log(c) - 2)*e - 2*c^2*d*x + 3*e*log(c) + 3*
```

```
(c^2*x^2*e + e)*log(x))/((c^2*x^4*e^3 + 2*c^2*d*x^3*e^2 + (c^2*d^2*e + e^3)
*x^2 + 2*d*x*e^2 + d^2*e)*sqrt(x*e + d)), x))*b - 2/3*a*e^(-1)/(x*e + d)^(3
/2)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acsch}(cx)}{(d + ex)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**(5/2),x)
[Out] Integral((a + b*acsch(c*x))/(d + e*x)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{(d + e\,x)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x)^(5/2),x)
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x)^(5/2), x)
```

3.73 $\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(5/2)),x]
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*(d + e*x)^(5/2)), x]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

Mathematica [A]
time = 45.93, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(5/2)),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x)^(5/2)), x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsch}(cx)}{x\,(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*((3*e*log((sqrt(x*e + d) - sqrt(d))/(sqrt(x*e + d) + sqrt(d)))/d^(5/2)
 + 2*(3*(x*e + d)*e + d*e)/((x*e + d)^(3/2)*d^2))*e^(-1)*log(c) + 3*integra
te(log(x)/(sqrt(x*e + d)*x^3*e^2 + 2*sqrt(x*e + d)*d*x^2*e + sqrt(x*e + d)*
d^2*x), x) - 3*integrate(log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(x*e + d)*x^3*e^2
+ 2*sqrt(x*e + d)*d*x^2*e + sqrt(x*e + d)*d^2*x), x))*b + 1/3*a*(3*log((sqr
t(x*e + d) - sqrt(d))/(sqrt(x*e + d) + sqrt(d)))/d^(5/2) + 2*(3*x*e + 4*d)/
((x*e + d)^(3/2)*d^2))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)*sqrt(x*e + d)/(x^4*e^3 + 3*d*x^3*e^2 + 3*d^2*
x^2*e + d^3*x), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x + d)^(5/2)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x)^(5/2)),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x)^(5/2)), x)
```

3.74 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(5/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(5/2)), x]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

Mathematica [A]
time = 43.96, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(5/2)),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x)^(5/2)), x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsch}(cx)}{x^2\,(ex+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*((2*(15*(x*e + d)^2*e^2 - 10*(x*e + d)*d*e^2 - 2*d^2*e^2)/((x*e + d)^(5
/2)*d^3 - (x*e + d)^(3/2)*d^4) + 15*e^2*log((sqrt(x*e + d) - sqrt(d))/(sqrt
(x*e + d) + sqrt(d)))/d^(7/2))*e^(-1)*log(c) - 6*integrate(log(x)/(sqrt(x*e
 + d)*x^4*e^2 + 2*sqrt(x*e + d)*d*x^3*e + sqrt(x*e + d)*d^2*x^2), x) + 6*in
tegrate(log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(x*e + d)*x^4*e^2 + 2*sqrt(x*e + d)
*d*x^3*e + sqrt(x*e + d)*d^2*x^2), x))*b - 1/6*a*(2*(15*(x*e + d)^2*e - 10*
(x*e + d)*d*e - 2*d^2*e)/((x*e + d)^(5/2)*d^3 - (x*e + d)^(3/2)*d^4) + 15*e
*log((sqrt(x*e + d) - sqrt(d))/(sqrt(x*e + d) + sqrt(d)))/d^(7/2))
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)*sqrt(x*e + d)/(x^5*e^3 + 3*d*x^4*e^2 + 3*d^2*
x^3*e + d^3*x^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x + d)^(5/2)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x^2\,{\left(d+e\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x)^(5/2)),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x)^(5/2)), x)
```

3.75 $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{7/2}} \, dx$

Optimal. Leaf size=648

$$-\frac{4be(1+c^2x^2)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x(d+ex)^{3/2}}-\frac{16bce(1+c^2x^2)}{15\left(c^2d^2+e^2\right)^2\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}-\frac{4be(1+c^2}{5cd^2\left(c^2d^2+e^2\right)\sqrt{1+}}$$

```
[Out] -2/5*(a+b*arccsch(c*x))/e/(e*x+d)^(5/2)-4/15*b*e*(c^2*x^2+1)/c/d/(c^2*d^2+e
^2)/x/(e*x+d)^(3/2)/(1+1/c^2/x^2)^(1/2)-16/15*b*c*e*(c^2*x^2+1)/(c^2*d^2+e^
2)^2/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/5*b*e*(c^2*x^2+1)/c/d^2/(c^2*d^2
+e^2)/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)-4/15*b*c*(7*c^2*d^2+3*e^2)*Ellipt
icE(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),2^(1/2)*(e*(-c^2)^(1/2)/(-c^2*d+e*
(-c^2)^(1/2)))^(1/2))*(e*x+d)^(1/2)*(c^2*x^2+1)^(1/2)/d^2/(c^2*d^2+e^2)^2/x
/(-c^2)^(1/2)/(1+1/c^2/x^2)^(1/2)/((e*x+d)/(d+e/(-c^2)^(1/2)))^(1/2)-4/15*b
*EllipticF(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),(-2*e*(-c^2)^(1/2)/(c^2*d-e
*(-c^2)^(1/2)))^(1/2))*(-c^2)^(1/2)*(c^2*x^2+1)^(1/2)*((e*x+d)/(d+e/(-c^2)^
(1/2)))^(1/2)/c/d/(c^2*d^2+e^2)/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)+4/5*b*E
llipticPi(1/2*(1-(-c^2)^(1/2)*x)^(1/2)*2^(1/2),2,2^(1/2)*(e/(d*(-c^2)^(1/2)
+e))^(1/2))*(c^2*x^2+1)^(1/2)*((e*x+d)*(-c^2)^(1/2)/(d*(-c^2)^(1/2)+e))^(1/
2)/c/d^2/e/x/(1+1/c^2/x^2)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A]
time = 0.73, antiderivative size = 785, normalized size of antiderivative = 1.21, number of steps used = 19, number of rules used = 14, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {6425, 1588, 972, 759, 849, 858, 733, 435, 430, 21, 947, 174, 552, 551}

$$-\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}}-\frac{4b\sqrt{-c^2}\sqrt{c^2x^2+1}\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right)}{15cdx\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{d+ex}}+\frac{4b\sqrt{-c^2}\sqrt{c^2x^2+1}\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right)}{5cd^2x\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}}+\frac{16bc\sqrt{-c^2}\sqrt{c^2x^2+1}\sqrt{d+ex}\,E\left(\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|-\frac{2\sqrt{-c^2}\,e}{c^2d-\sqrt{-c^2}\,e}\right)}{15x\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)^2\sqrt{\frac{d+ex}{\frac{e}{\sqrt{-c^2}}+d}}}+\frac{4b\sqrt{c^2x^2+1}\sqrt{\frac{\sqrt{-c^2}(d+ex)}{\sqrt{-c^2}\,d+e}}\,\Pi\left(2;\text{ArcSin}\left(\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right)|\frac{2e}{\sqrt{-c^2}\,d+e}\right)}{5cd^2ex\sqrt{\frac{1}{c^2x^2}+1}\sqrt{d+ex}}-\frac{4be(c^2x^2+1)}{5cd^2x\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)\sqrt{d+ex}}-\frac{16bce(c^2x^2+1)}{15x\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)^2\sqrt{d+ex}}-\frac{4be(c^2x^2+1)}{15cdx\sqrt{\frac{1}{c^2x^2}+1}\left(c^2d^2+e^2\right)(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x)^(7/2),x]
```

```
[Out] (-4*b*e*(1 + c^2*x^2))/(15*c*d*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*(d +
 e*x)^(3/2)) - (16*b*c*e*(1 + c^2*x^2))/(15*(c^2*d^2 + e^2)^2*Sqrt[1 + 1/(c
^2*x^2)]*x*Sqrt[d + e*x]) - (4*b*e*(1 + c^2*x^2))/(5*c*d^2*(c^2*d^2 + e^2)*
Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x]) - (2*(a + b*ArcCsch[c*x]))/(5*e*(d +
 e*x)^(5/2)) + (16*b*c*Sqrt[-c^2]*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE
[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c
^2]*e)])/(15*(c^2*d^2 + e^2)^2*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(d + e*x)/(d +
e/Sqrt[-c^2])]) + (4*b*Sqrt[-c^2]*Sqrt[d + e*x]*Sqrt[1 + c^2*x^2]*EllipticE
```

```
[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-c^2]*e)/(c^2*d - Sqrt[-c
^2]*e)])/(5*c*d^2*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[(d + e*x)/(d
 + e/Sqrt[-c^2])]) - (4*b*Sqrt[-c^2]*Sqrt[(d + e*x)/(d + e/Sqrt[-c^2])]*Sqr
t[1 + c^2*x^2]*EllipticF[ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (-2*Sqrt[-
c^2]*e)/(c^2*d - Sqrt[-c^2]*e)])/(15*c*d*(c^2*d^2 + e^2)*Sqrt[1 + 1/(c^2*x^
2)]*x*Sqrt[d + e*x]) + (4*b*Sqrt[(Sqrt[-c^2]*(d + e*x))/(Sqrt[-c^2]*d + e)]
*Sqrt[1 + c^2*x^2]*EllipticPi[2, ArcSin[Sqrt[1 - Sqrt[-c^2]*x]/Sqrt[2]], (2
*e)/(Sqrt[-c^2]*d + e)])/(5*c*d^2*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[d + e*x])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 174

```
Int[1/(((a_.) + (b_.)*(x_))*Sqrt[(c_.) + (d_.)*(x_)]*Sqrt[(e_.) + (f_.)*(x_
)]*Sqrt[(g_.) + (h_.)*(x_)]), x_Symbol] :> Dist[-2, Subst[Int[1/(Simp[b*c -
 a*d - b*x^2, x]*Sqrt[Simp[(d*e - c*f)/d + f*(x^2/d), x]]*Sqrt[Simp[(d*g -
c*h)/d + h*(x^2/d), x]]), x], x, Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e
, f, g, h}, x] && GtQ[(d*e - c*f)/d, 0]
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 733

```
Int[((d_) + (e_.)*(x_))^(m_)/Sqrt[(a_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2
*a*Rt[-c/a, 2]*(d + e*x)^m*(Sqrt[1 + c*(x^2/a)]/(c*Sqrt[a + c*x^2]*(c*((d +
 e*x)/(c*d - a*e*Rt[-c/a, 2])))^m)), Subst[Int[(1 + 2*a*e*Rt[-c/a, 2]*(x^2/
(c*d - a*e*Rt[-c/a, 2])))^m/Sqrt[1 - x^2], x], x, Sqrt[(1 - Rt[-c/a, 2]*x)/
2]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m^2, 1/4]
```

Rule 759

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*Simp[d*(m + 1) - e*(
m + 2*p + 3)*x, x]*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] &&
 NeQ[c*d^2 + a*e^2, 0] && NeQ[m, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, 0,
 c, d, e, m, p, x]) || (SumSimplerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[
m + 2*p + 3], 0])
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 947

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(f_.) + (g_.)*(x_)]*Sqrt[(a_) + (c_.)*(x_)
^2]), x_Symbol] :> With[{q = Rt[-c/a, 2]}, Dist[Sqrt[1 + c*(x^2/a)]/Sqrt[a
+ c*x^2], Int[1/((d + e*x)*Sqrt[f + g*x]*Sqrt[1 - q*x]*Sqrt[1 + q*x]), x],
x]] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e
```

```
^2, 0] &&  !GtQ[a, 0]
```

Rule 972

```
Int[((f_.) + (g_.)*(x_))^(n_)/(((d_.) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^
2]), x_Symbol] :> Int[ExpandIntegrand[1/(Sqrt[f + g*x]*Sqrt[a + c*x^2]), (f
 + g*x)^(n + 1/2)/(d + e*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[n + 1/2]
```

Rule 1588

```
Int[(x_)^(m_.)*((a_.) + (c_.)*(x_)^(mn2_.))^(p_)*((d_) + (e_.)*(x_)^(n_.))^
(q_.), x_Symbol] :> Dist[x^(2*n*FracPart[p])*((a + c/x^(2*n))^FracPart[p]/(
c + a*x^(2*n))^FracPart[p]), Int[x^(m - 2*n*p)*(d + e*x^n)^q*(c + a*x^(2*n)
)^p, x], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[mn2, -2*n] &&  !In
tegerQ[p] &&  !IntegerQ[q] && PosQ[n]
```

Rule 6425

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcCsch[c*x])/(e*(m + 1))), x] + Dist[
b/(c*e*(m + 1)), Int[(d + e*x)^(m + 1)/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x]
/; FreeQ[{a, b, c, d, e, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex)^{7/2}}\,dx = -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} - \frac{(2b)\int \frac{1}{\sqrt{1+\frac{1}{c^2x^2}}\,x^2(d+ex)^{5/2}}\,dx}{5ce}$$

$$= -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{x(d+ex)^{5/2}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5ce\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} - \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \left(-\frac{e}{d(d+ex)^{5/2}\sqrt{\frac{1}{c^2}+x^2}} - \frac{e}{d^2(d+ex)^{3/2}\sqrt{\vphantom{\frac{1}{c^2}}}}\right.}{5ce\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

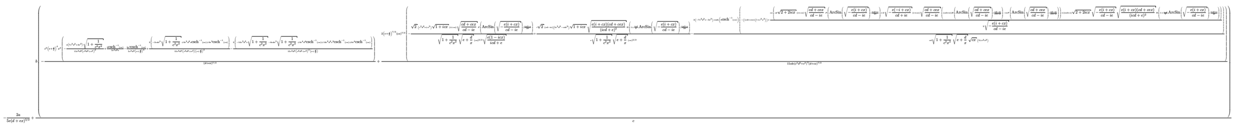
$$= -\frac{2\left(a+b\text{csch}^{-1}(cx)\right)}{5e(d+ex)^{5/2}} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right)\int \frac{1}{(d+ex)^{3/2}\sqrt{\frac{1}{c^2}+x^2}}\,dx}{5cd^2\sqrt{1+\frac{1}{c^2x^2}}\,x} + \frac{\left(2b\sqrt{\frac{1}{c^2}+x^2}\right.}{5c}$$

$$= -\frac{4be(1+c^2x^2)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x(d+ex)^{3/2}} - \frac{4be(1+c^2x^2)}{5cd^2\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+e}}$$

$$= -\frac{4be(1+c^2x^2)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x(d+ex)^{3/2}} - \frac{16bce(1+c^2x^2)}{15\left(c^2d^2+e^2\right)^2\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

$$= -\frac{4be(1+c^2x^2)}{15cd\left(c^2d^2+e^2\right)\sqrt{1+\frac{1}{c^2x^2}}\,x(d+ex)^{3/2}} - \frac{16bce(1+c^2x^2)}{15\left(c^2d^2+e^2\right)^2\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+ex}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 30.05, size = 1217, normalized size = 1.88



Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x)^(7/2),x]
[Out] (-2*a)/(5*e*(d + e*x)^(5/2)) + (b*(-((c^4*(e + d/x)^4*x^4*((-4*(7*c^2*d^2 +
 3*e^2)*Sqrt[1 + 1/(c^2*x^2)])/(15*c^2*d^2*(c^2*d^2 + e^2)^2) + (2*ArcCsch[
c*x])/(5*c^3*d^3*e) - (2*e^2*ArcCsch[c*x])/(5*c^3*d^3*(e + d/x)^3) + (2*(-2
*c*d*e^2*Sqrt[1 + 1/(c^2*x^2)] + 9*c^2*d^2*e*ArcCsch[c*x] + 9*e^3*ArcCsch[c
*x]))/(15*c^3*d^3*(c^2*d^2 + e^2)*(e + d/x)^2) - (2*(-16*c^3*d^3*e*Sqrt[1 +
 1/(c^2*x^2)] - 8*c*d*e^3*Sqrt[1 + 1/(c^2*x^2)] + 9*c^4*d^4*ArcCsch[c*x] +
18*c^2*d^2*e^2*ArcCsch[c*x] + 9*e^4*ArcCsch[c*x]))/(15*c^3*d^3*(c^2*d^2 + e
^2)^2*(e + d/x))))/(d + e*x)^(7/2)) + (2*(e + d/x)^(7/2)*(c*x)^(7/2)*(-((Sq
rt[2]*(c^2*d^2*e + e^3)*Sqrt[1 + I*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d -
 I*e)]*EllipticF[ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2
*e)])/(Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)*Sqrt[(e*(1 - I*c*x))
/(I*c*d + e)])) + (I*Sqrt[2]*(c*d - I*e)*(3*c^3*d^3 - c*d*e^2)*Sqrt[1 + I*c
*x]*Sqrt[(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/
e, ArcSin[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)])/(e*Sqrt[
1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*(c*x)^(3/2)) - (2*(-7*c^2*d^2*e - 3*e^3)*Cos
h[2*ArcCsch[c*x]]*(-((c*d + c*e*x)*(1 + c^2*x^2)) + (c*x*(c*d*Sqrt[2 + (2*I
)*c*x]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*EllipticF[ArcSin[Sqrt[-((e
*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)] + 2*Sqrt[-((e*(-I + c*x))/(c
*d + I*e))]*(I + c*x)*Sqrt[(c*d + c*e*x)/(c*d - I*e)]*((c*d + I*e)*Elliptic
E[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)] - I*e*E
llipticF[ArcSin[Sqrt[(c*d + c*e*x)/(c*d - I*e)]], (c*d - I*e)/(c*d + I*e)])
 + (I*c*d + e)*Sqrt[2 + (2*I)*c*x]*Sqrt[-((e*(I + c*x))/(c*d - I*e))]*Sqrt[
(e*(I + c*x)*(c*d + c*e*x))/(I*c*d + e)^2]*EllipticPi[1 + (I*c*d)/e, ArcSin
[Sqrt[-((e*(I + c*x))/(c*d - I*e))]], (I*c*d + e)/(2*e)]))/(2*Sqrt[-((e*(I
+ c*x))/(c*d - I*e))])))/(c*d*Sqrt[1 + 1/(c^2*x^2)]*Sqrt[e + d/x]*Sqrt[c*x]
*(2 + c^2*x^2))))/(15*c*d*e*(c^2*d^2 + e^2)^2*(d + e*x)^(7/2))))/c
```

Maple [C] Result contains complex when optimal does not.
time = 0.98, size = 3782, normalized size = 5.84

method	result	size
derivativedivides	Expression too large to display	3782
default	Expression too large to display	3782

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/e*(-1/5*a/(e*x+d)^(5/2)+b*(-1/5/(e*x+d)^(5/2)*arccsch(c*x)-2/15/c*(-((I*e
+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^5*d^7+3*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*
c^2*d*e^3*(e*x+d)^3+9*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+
e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)
*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2
*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*d^3*e^2*(e*x+d)^(3/2)-10*(-(I*c*(e*x+d)
*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e
*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*
c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^3*
d^3*e^2*(e*x+d)^(3/2)+3*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^
2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/
2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c
*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e
^2))^(1/2))*c^5*d^5*(e*x+d)^(3/2)+6*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-
e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d
^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-
(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^5*d^5*(e*x+d)^(3/2)+5*I*((I
*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^5*e*(e*x+d)-3*I*(-(I*c*(e*x+d)*e+c^2*d
*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^
2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*
d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^
(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*e^5*(e*x+d)^(3/2)-7*(-(I*c*(e*x+d)
*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e
*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*
c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c^5*
d^5*(e*x+d)^(3/2)+7*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^3*e*(e*x+d)^3
-13*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^4*e*(e*x+d)^2+3*(-(I*c*(e*x+d
)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(
e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d
)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d
^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c*d*e^4*(e*x+d)^(3/2)+3*(
-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+
d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d)^(1/2
)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2)
)^(1/2))*c*d*e^4*(e*x+d)^(3/2)-3*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2
)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+
e^2))^(1/2)*EllipticE((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*
I*c*d*e-c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2))*c*d*e^4*(e*x+d)^(3/2)+6*(-(I*c*(
e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^
2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*
e+c*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(
c^2*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^3*d^3*e^2*(e*x+d)^
(3/2)+I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((
```

```
I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*
x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2
*d^2+e^2))^(1/2))*c^2*d^2*e^3*(e*x+d)^(3/2)-3*I*(-(I*c*(e*x+d)*e+c^2*d*(e*x
+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2
+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e
^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2*d^2+e^2))^(1/2)
/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^4*d^4*e*(e*x+d)^(3/2)-6*I*(-(I*c*(e*x
+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c*(e*x+d)*e-c^2*d
*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticPi((e*x+d)^(1/2)*((I*e+c
*d)*c/(c^2*d^2+e^2))^(1/2),1/(I*e+c*d)/c*(c^2*d^2+e^2)/d,(-(I*e-c*d)*c/(c^2
*d^2+e^2))^(1/2)/((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2))*c^2*d^2*e^3*(e*x+d)^(3/
2)+I*(-(I*c*(e*x+d)*e+c^2*d*(e*x+d)-c^2*d^2-e^2)/(c^2*d^2+e^2))^(1/2)*((I*c
*(e*x+d)*e-c^2*d*(e*x+d)+c^2*d^2+e^2)/(c^2*d^2+e^2))^(1/2)*EllipticF((e*x+d
)^(1/2)*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2),(-(2*I*c*d*e-c^2*d^2+e^2)/(c^2*d^
2+e^2))^(1/2))*c^4*d^4*e*(e*x+d)^(3/2)-5*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2
)*c^2*d^2*e^3*(e*x+d)^2+8*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^3*e^3*(
e*x+d)+2*I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^2*d^4*e^3+3*I*((I*e+c*d)*c/(
c^2*d^2+e^2))^(1/2)*d*e^5*(e*x+d)-3*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^3*d
^2*e^2*(e*x+d)^3+I*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^4*d^6*e+5*((I*e+c*d)
*c/(c^2*d^2+e^2))^(1/2)*c^3*d^3*e^2*(e*x+d)^2-8*((I*e+c*d)*c/(c^2*d^2+e^2))
^(1/2)*c^3*d^4*e^2*(e*x+d)-3*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c*d^2*e^4*(e
*x+d)-7*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)*c^5*d^4*(e*x+d)^3+13*((I*e+c*d)*c
/(c^2*d^2+e^2))^(1/2)*c^5*d^5*(e*x+d)^2-5*((I*e+c*d)*c/(c^2*d^2+e^2))^(1/2)
*c^5*d^6*(e*x+d)-2*((I*e+c*d)*c/(c^2*d^2+e^2))^...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/5*(10*c^2*integrate(1/5*x/((c^2*x^4*e^3 + 2*c^2*d*x^3*e^2 + (c^2*d^2*e +
 e^3)*x^2 + 2*d*x*e^2 + d^2*e)*sqrt(c^2*x^2 + 1)*sqrt(x*e + d) + (c^2*x^4*e
^3 + 2*c^2*d*x^3*e^2 + (c^2*d^2*e + e^3)*x^2 + 2*d*x*e^2 + d^2*e)*sqrt(x*e
+ d)), x) + 2*log(sqrt(c^2*x^2 + 1) + 1)/((x^2*e^3 + 2*d*x*e^2 + d^2*e)*sqr
t(x*e + d)) + 5*integrate(1/5*(c^2*x^2*(5*log(c) - 2)*e - 2*c^2*d*x + 5*e*l
og(c) + 5*(c^2*x^2*e + e)*log(x))/((c^2*x^5*e^4 + 3*c^2*d*x^4*e^3 + (3*c^2*
d^2*e^2 + e^4)*x^3 + 3*d^2*x*e^2 + d^3*e + (c^2*d^3*e + 3*d*e^3)*x^2)*sqrt(
x*e + d)), x))*b - 2/5*a*e^(-1)/(x*e + d)^(5/2)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(7/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)*sqrt(x*e + d)/(x^4*e^4 + 4*d*x^3*e^3 + 6*d^2*
x^2*e^2 + 4*d^3*x*e + d^4), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x+d)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x + d)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{{\left(d+e\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x)^(7/2),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x)^(7/2), x)
```

3.76 $\int x^4(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=214

$$-\frac{b(42c^2d-25e)\,x^2\sqrt{-1-c^2x^2}}{560c^5\sqrt{-c^2x^2}}+\frac{b(42c^2d-25e)\,x^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}+\frac{bex^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{5}dx^5\left(a+b\text{csch}^{-1}(c\right.$$

```
[Out] 1/5*d*x^5*(a+b*arccsch(c*x))+1/7*e*x^7*(a+b*arccsch(c*x))-1/560*b*(42*c^2*d
-25*e)*x*arctan(c*x/(-c^2*x^2-1)^(1/2))/c^6/(-c^2*x^2)^(1/2)-1/560*b*(42*c^
2*d-25*e)*x^2*(-c^2*x^2-1)^(1/2)/c^5/(-c^2*x^2)^(1/2)+1/840*b*(42*c^2*d-25*
e)*x^4*(-c^2*x^2-1)^(1/2)/c^3/(-c^2*x^2)^(1/2)+1/42*b*e*x^6*(-c^2*x^2-1)^(1
/2)/c/(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {14, 6437, 12, 470, 327, 223, 209}

$$\frac{1}{5}dx^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bx\text{ArcTan}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)(42c^2d-25e)}{560c^6\sqrt{-c^2x^2}}+\frac{bex^6\sqrt{-c^2x^2-1}}{42c\sqrt{-c^2x^2}}-\frac{bx^2\sqrt{-c^2x^2-1}\,(42c^2d-25e)}{560c^5\sqrt{-c^2x^2}}+\frac{bx^4\sqrt{-c^2x^2-1}\,(42c^2d-25e)}{840c^3\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -1/560*(b*(42*c^2*d - 25*e)*x^2*Sqrt[-1 - c^2*x^2])/(c^5*Sqrt[-(c^2*x^2)])
+ (b*(42*c^2*d - 25*e)*x^4*Sqrt[-1 - c^2*x^2])/(840*c^3*Sqrt[-(c^2*x^2)]) +
 (b*e*x^6*Sqrt[-1 - c^2*x^2])/(42*c*Sqrt[-(c^2*x^2)]) + (d*x^5*(a + b*ArcCs
ch[c*x]))/5 + (e*x^7*(a + b*ArcCsch[c*x]))/7 - (b*(42*c^2*d - 25*e)*x*ArcTa
n[(c*x)/Sqrt[-1 - c^2*x^2]])/(560*c^6*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^4(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{1}{5}dx^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^4(7d\cdots}{35\sqrt{-1\cdots}}}{\sqrt{-c^2x\cdots}}$$

$$=\frac{1}{5}dx^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^4(7d+\cdots}{\sqrt{-1-\cdots}}}{35\sqrt{-c^2\cdots}}$$

$$=\frac{bex^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{5}dx^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\text{csch}^{-1}(c\cdots\right.$$

$$=\frac{b(42c^2d-25e)\,x^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}+\frac{bex^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{5}dx^5\left(a+b\text{c}\cdots\right.$$

$$=-\frac{b(42c^2d-25e)\,x^2\sqrt{-1-c^2x^2}}{560c^5\sqrt{-c^2x^2}}+\frac{b(42c^2d-25e)\,x^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}\cdots$$

$$=-\frac{b(42c^2d-25e)\,x^2\sqrt{-1-c^2x^2}}{560c^5\sqrt{-c^2x^2}}+\frac{b(42c^2d-25e)\,x^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}\cdots$$

$$=-\frac{b(42c^2d-25e)\,x^2\sqrt{-1-c^2x^2}}{560c^5\sqrt{-c^2x^2}}+\frac{b(42c^2d-25e)\,x^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}\cdots$$

Mathematica [A]
time = 0.17, size = 138, normalized size = 0.64

$$\frac{48ac^7x^5(7d+5ex^2)+bc^2\sqrt{1+\frac{1}{c^2x^2}}\,x^2(75e-2c^2(63d+25ex^2)+c^4(84dx^2+40ex^4))+48bc^7x^5(7d+5ex^2)\,\text{csch}^{-1}(cx)+3b(42c^2d-25e)\log\left(\left(1+\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{1680c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (48*a*c^7*x^5*(7*d + 5*e*x^2) + b*c^2*Sqrt[1 + 1/(c^2*x^2)]*x^2*(75*e - 2*c
^2*(63*d + 25*e*x^2) + c^4*(84*d*x^2 + 40*e*x^4)) + 48*b*c^7*x^5*(7*d + 5*e
*x^2)*ArcCsch[c*x] + 3*b*(42*c^2*d - 25*e)*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*
x])/(1680*c^7)
```

Maple [A]
time = 0.35, size = 211, normalized size = 0.99

method	result

derivativedivides	$\frac{\frac{a\left(\frac{1}{5}d\,c^7x^5+\frac{1}{7}e\,c^7x^7\right)}{c^2}+\frac{b\left(\frac{\operatorname{arccsch}(cx)d\,c^7x^5}{5}+\frac{\operatorname{arccsch}(cx)e\,c^7x^7}{7}+\frac{\sqrt{c^2x^2+1}\,\left(84d\,c^5x^3\sqrt{c^2x^2+1}+40e\,c^5x^5\sqrt{c^2x^2}\right.}{}\right.}{}}{c^5}$
default	$\frac{\frac{a\left(\frac{1}{5}d\,c^7x^5+\frac{1}{7}e\,c^7x^7\right)}{c^2}+\frac{b\left(\frac{\operatorname{arccsch}(cx)d\,c^7x^5}{5}+\frac{\operatorname{arccsch}(cx)e\,c^7x^7}{7}+\frac{\sqrt{c^2x^2+1}\,\left(84d\,c^5x^3\sqrt{c^2x^2+1}+40e\,c^5x^5\sqrt{c^2x^2}\right.}{}\right.}{}}{c^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^5*(a/c^2*(1/5*d*c^7*x^5+1/7*e*c^7*x^7)+b/c^2*(1/5*arccsch(c*x)*d*c^7*x^
5+1/7*arccsch(c*x)*e*c^7*x^7+1/1680*(c^2*x^2+1)^(1/2)*(84*d*c^5*x^3*(c^2*x^
2+1)^(1/2)+40*e*c^5*x^5*(c^2*x^2+1)^(1/2)-126*d*c^3*x*(c^2*x^2+1)^(1/2)-50*
e*c^3*x^3*(c^2*x^2+1)^(1/2)+126*d*c^2*arcsinh(c*x)+75*e*c*x*(c^2*x^2+1)^(1/
2)-75*e*arcsinh(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A]
time = 0.26, size = 291, normalized size = 1.36

$$\frac{1}{7}ax^7e+\frac{1}{5}adx^5+\frac{1}{80}\left(16x^5\operatorname{arcsch}(cx)-\frac{\frac{2\left(3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}+1\right)^2-2c^4\left(\frac{1}{c^2x^2}+1\right)+c^4}-\frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bd+\frac{1}{672}\left(96x^7\operatorname{arcsch}(cx)+\frac{\frac{2\left(15\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}+1\right)^3-3c^6\left(\frac{1}{c^2x^2}+1\right)^2+3c^6\left(\frac{1}{c^2x^2}+1\right)-c^6}-\frac{15\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^6}+\frac{15\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^6}}{c}\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*x^7*e + 1/5*a*d*x^5 + 1/80*(16*x^5*arccsch(c*x) - (2*(3*(1/(c^2*x^2)
+ 1)^(3/2) - 5*sqrt(1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) + 1)^2 - 2*c^4*(1/(
c^2*x^2) + 1) + c^4) - 3*log(sqrt(1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(1/
(c^2*x^2) + 1) - 1)/c^4)/c)*b*d + 1/672*(96*x^7*arccsch(c*x) + (2*(15*(1/(c
^2*x^2) + 1)^(5/2) - 40*(1/(c^2*x^2) + 1)^(3/2) + 33*sqrt(1/(c^2*x^2) + 1))
/(c^6*(1/(c^2*x^2) + 1)^3 - 3*c^6*(1/(c^2*x^2) + 1)^2 + 3*c^6*(1/(c^2*x^2)
+ 1) - c^6) - 15*log(sqrt(1/(c^2*x^2) + 1) + 1)/c^6 + 15*log(sqrt(1/(c^2*x^
2) + 1) - 1)/c^6)/c)*b*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 386 vs. 2(191) = 382.
time = 0.40, size = 386, normalized size = 1.80

$$\frac{240\,ac^7x^7\cosh(1)+240\,ac^7x^7\sinh(1)+336\,ac^7dx^5+48\left(7\,bc^7d+5\,bc^7\cosh(1)+5\,bc^7\sinh(1)\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)-3\left(42\,bc^2d-25\,b\cosh(1)-25\,b\sinh(1)\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx\right)-48\left(7\,bc^7d+5\,bc^7\cosh(1)+5\,bc^7\sinh(1)\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)+48\left(7\,bc^7dx^5-7\,bc^7d+5\left(bc^7x^7-bc^7\right)\cosh(1)+5\left(bc^7x^7-bc^7\right)\sinh(1)\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(84\,bc^6dx^4-126\,bc^4dx^2+5\left(8\,bc^6x^6-10\,bc^4x^4+15\,bc^2x^2\right)\cosh(1)+5\left(8\,bc^6x^6-10\,bc^4x^4+15\,bc^2x^2\right)\sinh(1)\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{1680\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*c^7*x^7*cosh(1) + 240*a*c^7*x^7*sinh(1) + 336*a*c^7*d*x^5 + 4
8*(7*b*c^7*d + 5*b*c^7*cosh(1) + 5*b*c^7*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1
)/(c^2*x^2)) - c*x + 1) - 3*(42*b*c^2*d - 25*b*cosh(1) - 25*b*sinh(1))*log(
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 48*(7*b*c^7*d + 5*b*c^7*cosh(1)
+ 5*b*c^7*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 48*(7
*b*c^7*d*x^5 - 7*b*c^7*d + 5*(b*c^7*x^7 - b*c^7)*cosh(1) + 5*(b*c^7*x^7 - b
*c^7)*sinh(1))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (84*b*c
^6*d*x^4 - 126*b*c^4*d*x^2 + 5*(8*b*c^6*x^6 - 10*b*c^4*x^4 + 15*b*c^2*x^2)*
cosh(1) + 5*(8*b*c^6*x^6 - 10*b*c^4*x^4 + 15*b*c^2*x^2)*sinh(1))*sqrt((c^2*
x^2 + 1)/(c^2*x^2)))/c^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4(a + b\,\mathrm{acsch}\,(cx))\left(d + ex^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**4*(a + b*acsch(c*x))*(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x^4*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.77 $\int x^2(d+ex^2)\left(a+b\mathbf{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=167

$$\frac{b(20c^2d-9e)\,x^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+\frac{1}{3}dx^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{b(20c^2}{}$$

```
[Out] 1/3*d*x^3*(a+b*arccsch(c*x))+1/5*e*x^5*(a+b*arccsch(c*x))+1/120*b*(20*c^2*d
-9*e)*x*arctan(c*x/(-c^2*x^2-1)^(1/2))/c^4/(-c^2*x^2)^(1/2)+1/120*b*(20*c^2
*d-9*e)*x^2*(-c^2*x^2-1)^(1/2)/c^3/(-c^2*x^2)^(1/2)+1/20*b*e*x^4*(-c^2*x^2-
1)^(1/2)/c/(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {14, 6437, 12, 470, 327, 223, 209}

$$\frac{1}{3}dx^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx\text{ArcTan}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)(20c^2d-9e)}{120c^4\sqrt{-c^2x^2}}+\frac{bex^4\sqrt{-c^2x^2-1}}{20c\sqrt{-c^2x^2}}+\frac{bx^2\sqrt{-c^2x^2-1}\,(20c^2d-9e)}{120c^3\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(20*c^2*d - 9*e)*x^2*Sqrt[-1 - c^2*x^2])/(120*c^3*Sqrt[-(c^2*x^2)]) + (b
*e*x^4*Sqrt[-1 - c^2*x^2])/(20*c*Sqrt[-(c^2*x^2)]) + (d*x^3*(a + b*ArcCsch[
c*x]))/3 + (e*x^5*(a + b*ArcCsch[c*x]))/5 + (b*(20*c^2*d - 9*e)*x*ArcTan[(c
*x)/Sqrt[-1 - c^2*x^2]])/(120*c^4*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^2(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)dx = \frac{1}{3}dx^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^2(5d+3}{15\sqrt{-1-}}}{\sqrt{-c^2x^2}}$$

$$=\frac{1}{3}dx^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^2(5d+3e}{\sqrt{-1-c}}}{15\sqrt{-c^2x^2}}$$

$$=\frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+\frac{1}{3}dx^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\text{csch}^{-1}(cx)\right.$$

$$=\frac{b(20c^2d-9e)x^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+\frac{1}{3}dx^3\left(a+b\text{csch}\right.$$

$$=\frac{b(20c^2d-9e)x^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+\frac{1}{3}dx^3\left(a+b\text{csch}\right.$$

$$=\frac{b(20c^2d-9e)x^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{bex^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+\frac{1}{3}dx^3\left(a+b\text{csch}\right.$$

Mathematica [A]
time = 0.12, size = 119, normalized size = 0.71

$$\frac{c^2x^2\left(8ac^3x(5d+3ex^2)+b\sqrt{1+\frac{1}{c^2x^2}}\left(-9e+c^2(20d+6ex^2)\right)\right)+8bc^5x^3(5d+3ex^2)\text{csch}^{-1}(cx)+b(-20c^2d+9e)\log\left(\left(1+\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{120c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (c^2*x^2*(8*a*c^3*x*(5*d + 3*e*x^2) + b*Sqrt[1 + 1/(c^2*x^2)]*(-9*e + c^2*(
20*d + 6*e*x^2))) + 8*b*c^5*x^3*(5*d + 3*e*x^2)*ArcCsch[c*x] + b*(-20*c^2*d
 + 9*e)*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(120*c^5)
```

Maple [A]
time = 0.39, size = 171, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{a\left(\frac{1}{3}dc^5x^3+\frac{1}{5}ec^5x^5\right)}{c^2}+\frac{b\left(\frac{\text{arccsch}(cx)dc^5x^3}{3}+\frac{\text{arccsch}(cx)ec^5x^5}{5}-\frac{\sqrt{c^2x^2+1}\left(-20dc^3x\sqrt{c^2x^2+1}-6ec^3x^3\sqrt{c^2x^2}\right.}{120\sqrt{\frac{c^2}{}}}\right.}{c^2}}{c^3}$

default	$\frac{\frac{a\left(\frac{1}{3}d\,c^5x^3+\frac{1}{5}e\,c^5x^5\right)}{c^2}+\frac{b\left(\frac{\operatorname{arccsch}(cx)d\,c^5x^3}{3}+\frac{\operatorname{arccsch}(cx)e\,c^5x^5}{5}-\frac{\sqrt{c^2x^2+1}\,\left(-20d\,c^3x\sqrt{c^2x^2+1}-6e\,c^3x^3\sqrt{c^2x}\right.}{120\sqrt{}}\right.}{c^2}}{c^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a/c^2*(1/3*d*c^5*x^3+1/5*e*c^5*x^5)+b/c^2*(1/3*arccsch(c*x)*d*c^5*x^
3+1/5*arccsch(c*x)*e*c^5*x^5-1/120*(c^2*x^2+1)^(1/2)*(-20*d*c^3*x*(c^2*x^2+
1)^(1/2)-6*e*c^3*x^3*(c^2*x^2+1)^(1/2)+20*d*c^2*arcsinh(c*x)+9*e*c*x*(c^2*x
^2+1)^(1/2)-9*e*arcsinh(c*x))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A]
time = 0.26, size = 229, normalized size = 1.37

$$\frac{1}{5}ax^5e+\frac{1}{3}adx^3+\frac{1}{12}\left(4x^3\operatorname{arcsch}(cx)+\frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}-\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd+\frac{1}{80}\left(16x^5\operatorname{arcsch}(cx)-\frac{\frac{2\left(3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}+1\right)^2-2c^4\left(\frac{1}{c^2x^2}+1\right)+c^4}-\frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*x^5*e + 1/3*a*d*x^3 + 1/12*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2)
+ 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + l
og(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d + 1/80*(16*x^5*arccsch(c*x) - (2*
(3*(1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) + 1
)^2 - 2*c^4*(1/(c^2*x^2) + 1) + c^4) - 3*log(sqrt(1/(c^2*x^2) + 1) + 1)/c^4
 + 3*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(149) = 298$.
time = 0.43, size = 357, normalized size = 2.14

$$\frac{24ac^5x^5\cosh(1)+24ac^5x^5\sinh(1)+40ac^5dx^3+8\left(5bc^5d+3bc^5\cosh(1)+3bc^5\sinh(1)\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)+\left(20bc^2d-9b\cosh(1)-9b\sinh(1)\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx\right)-8\left(5bc^5d+3bc^5\cosh(1)+3bc^5\sinh(1)\right)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)+8\left(5bc^5dx^3-5bc^5d+3\left(bc^5x^5-bc^5\right)\cosh(1)+3\left(bc^5x^5-bc^5\right)\sinh(1)\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(20bc^4dx^2+3\left(2bc^4x^4-3bc^2x^2\right)\cosh(1)+3\left(2bc^4x^4-3bc^2x^2\right)\sinh(1)\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{120c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/120*(24*a*c^5*x^5*cosh(1) + 24*a*c^5*x^5*sinh(1) + 40*a*c^5*d*x^3 + 8*(5*
b*c^5*d + 3*b*c^5*cosh(1) + 3*b*c^5*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^
2*x^2)) - c*x + 1) + (20*b*c^2*d - 9*b*cosh(1) - 9*b*sinh(1))*log(c*x*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 8*(5*b*c^5*d + 3*b*c^5*cosh(1) + 3*b*c^5*
sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 8*(5*b*c^5*d*x^
```

```
3 - 5*b*c^5*d + 3*(b*c^5*x^5 - b*c^5)*cosh(1) + 3*(b*c^5*x^5 - b*c^5)*sinh(
1))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (20*b*c^4*d*x^2 +
3*(2*b*c^4*x^4 - 3*b*c^2*x^2)*cosh(1) + 3*(2*b*c^4*x^4 - 3*b*c^2*x^2)*sinh(
1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a + b\operatorname{acsch}(cx))\left(d + ex^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(a+b*acsch(c*x)),x)
[Out] Integral(x**2*(a + b*acsch(c*x))*(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\left(e\,x^2 + d\right)\left(a + b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)
[Out] int(x^2*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.78 $\int (d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=115

$$\frac{bex^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}}+dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)-\frac{b(6c^2d-e)\,x\text{ArcTan}\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{6c^2\sqrt{-c^2x^2}}$$

```
[Out] d*x*(a+b*arccsch(c*x))+1/3*e*x^3*(a+b*arccsch(c*x))-1/6*b*(6*c^2*d-e)*x*arc
tan(c*x/(-c^2*x^2-1)^(1/2))/c^2/(-c^2*x^2)^(1/2)+1/6*b*e*x^2*(-c^2*x^2-1)^(
1/2)/c/(-c^2*x^2)^(1/2)
```

Rubi [A]

time = 0.04, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6427, 12, 396, 223, 209}

$$dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bx\text{ArcTan}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)(6c^2d-e)}{6c^2\sqrt{-c^2x^2}}+\frac{bex^2\sqrt{-c^2x^2-1}}{6c\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*e*x^2*Sqrt[-1 - c^2*x^2])/(6*c*Sqrt[-(c^2*x^2)]) + d*x*(a + b*ArcCsch[c*
x]) + (e*x^3*(a + b*ArcCsch[c*x]))/3 - (b*(6*c^2*d - e)*x*ArcTan[(c*x)/Sqrt
[-1 - c^2*x^2]])/(6*c^2*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 6427

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x
] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2
*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p +
 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{3d+ex^2}{3\sqrt{-1-c^2x^2}}}{\sqrt{-c^2x^2}}$$

$$=dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{3d+ex^2}{\sqrt{-1-c^2x^2}}}{3\sqrt{-c^2x^2}}$$

$$=\frac{bex^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}}+dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)-$$

$$=\frac{bex^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}}+dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)-$$

$$=\frac{bex^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}}+dx\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\text{csch}^{-1}(cx)\right)-$$

Mathematica [A]
time = 0.37, size = 169, normalized size = 1.47

$$adx+\frac{1}{3}aex^3+\frac{bex^2\sqrt{\frac{1+c^2x^2}{c^2x^2}}}{6c}+bdx\text{csch}^{-1}(cx)+\frac{1}{3}bex^3\text{csch}^{-1}(cx)-\frac{bd\sqrt{1+c^2x^2}\,\log\left(-\sqrt{c^2}\,x+\sqrt{1+c^2x^2}\right)}{c\sqrt{c^2}\,\sqrt{1+\frac{1}{c^2x^2}}\,x}-\frac{be\log\left(x\left(1+\sqrt{\frac{1+c^2x^2}{c^2x^2}}\right)\right)}{6c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] a*d*x + (a*e*x^3)/3 + (b*e*x^2*Sqrt[(1 + c^2*x^2)/(c^2*x^2)])/(6*c) + b*d*x
*ArcCsch[c*x] + (b*e*x^3*ArcCsch[c*x])/3 - (b*d*Sqrt[1 + c^2*x^2]*Log[-(Sqr
t[c^2]*x) + Sqrt[1 + c^2*x^2]])/(c*Sqrt[c^2]*Sqrt[1 + 1/(c^2*x^2)]*x) - (b*
e*Log[x*(1 + Sqrt[(1 + c^2*x^2)/(c^2*x^2)])])/(6*c^3)
```

Maple [A]
time = 0.33, size = 126, normalized size = 1.10

method	result
derivativedivides	$\frac{\frac{a\left(d\,c^{3}x+\frac{1}{3}e\,c^{3}x^{3}\right)}{c^{2}}+\frac{b\left(\operatorname{arccsch}(cx)d\,c^{3}x+\frac{\operatorname{arccsch}(cx)e\,c^{3}x^{3}}{3}+\frac{\sqrt{c^{2}x^{2}+1}\,\left(6d\,c^{2}\operatorname{arcsinh}(cx)+ecx\sqrt{c^{2}x^{2}+1}-e\operatorname{arcsinh}\right.}{6cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}}\right.}{c^{2}}}{c}$
default	$\frac{\frac{a\left(d\,c^{3}x+\frac{1}{3}e\,c^{3}x^{3}\right)}{c^{2}}+\frac{b\left(\operatorname{arccsch}(cx)d\,c^{3}x+\frac{\operatorname{arccsch}(cx)e\,c^{3}x^{3}}{3}+\frac{\sqrt{c^{2}x^{2}+1}\,\left(6d\,c^{2}\operatorname{arcsinh}(cx)+ecx\sqrt{c^{2}x^{2}+1}-e\operatorname{arcsinh}\right.}{6cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}}\right.}{c^{2}}}{c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(a/c^2*(d*c^3*x+1/3*e*c^3*x^3)+b/c^2*(arccsch(c*x)*d*c^3*x+1/3*arccsch(
c*x)*e*c^3*x^3+1/6*(c^2*x^2+1)^(1/2)*(6*d*c^2*arcsinh(c*x)+e*c*x*(c^2*x^2+1
)^(1/2)-e*arcsinh(c*x))/c/x/((c^2*x^2+1)/c^2/x^2)^(1/2)))
```

Maxima [A]
time = 0.27, size = 150, normalized size = 1.30

$$\frac{1}{3}ax^{3}e+adx+\frac{1}{12}\left(4x^{3}\operatorname{arcsch}(cx)+\frac{\frac{2\sqrt{\frac{1}{c^{2}x^{2}}+1}}{c^{2}\left(\frac{1}{c^{2}x^{2}}+1\right)-c^{2}}-\frac{\log\left(\sqrt{\frac{1}{c^{2}x^{2}}+1}+1\right)}{c^{2}}+\frac{\log\left(\sqrt{\frac{1}{c^{2}x^{2}}+1}-1\right)}{c^{2}}}{c}\right)be+\frac{\left(2cx\operatorname{arcsch}(cx)+\log\left(\sqrt{\frac{1}{c^{2}x^{2}}+1}+1\right)-\log\left(\sqrt{\frac{1}{c^{2}x^{2}}+1}-1\right)\right)bd}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3*e + a*d*x + 1/12*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(
c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqr
t(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*e + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/
(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*b*d/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(104) = 208$.
time = 0.38, size = 312, normalized size = 2.71

$$\frac{2ac^{3}x^{3}\cosh(1)+2ac^{3}x^{3}\sinh(1)+6ac^{3}dx+2\left(3bc^{3}d+bc^{3}\cosh(1)+bc^{3}\sinh(1)\right)\log\left(cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}-cx+1\right)-\left(6bc^{2}d-b\cosh(1)-b\sinh(1)\right)\log\left(cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}-cx\right)-2\left(3bc^{3}d+bc^{3}\cosh(1)+bc^{3}\sinh(1)\right)\log\left(cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}-cx-1\right)+2\left(3bc^{3}dx-3bc^{3}d+(bc^{3}x^{3}-bc^{3})\cosh(1)+(bc^{3}x^{3}-bc^{3})\sinh(1)\right)\log\left(\frac{cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+1}{cx}\right)+\left(bc^{2}x^{2}\cosh(1)+bc^{2}x^{2}\sinh(1)\right)\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}}{6c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*x^3*cosh(1) + 2*a*c^3*x^3*sinh(1) + 6*a*c^3*d*x + 2*(3*b*c^3*d
 + b*c^3*cosh(1) + b*c^3*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c
*x + 1) - (6*b*c^2*d - b*cosh(1) - b*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c
^2*x^2)) - c*x) - 2*(3*b*c^3*d + b*c^3*cosh(1) + b*c^3*sinh(1))*log(c*x*sqr
t((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 2*(3*b*c^3*d*x - 3*b*c^3*d + (b*c^3
*x^3 - b*c^3)*cosh(1) + (b*c^3*x^3 - b*c^3)*sinh(1))*log((c*x*sqrt((c^2*x^2
 + 1)/(c^2*x^2)) + 1)/(c*x)) + (b*c^2*x^2*cosh(1) + b*c^2*x^2*sinh(1))*sqrt
((c^2*x^2 + 1)/(c^2*x^2)))/c^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acsch}(cx)) \left(d + ex^2\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x)),x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int((d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.79 $\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=91

$$\frac{bcd\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}}-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{x}+ex\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bex\text{ArcTan}\left(\frac{cx}{\sqrt{-1-c^2x^2}}\right)}{\sqrt{-c^2x^2}}$$

```
[Out] -d*(a+b*arccsch(c*x))/x+e*x*(a+b*arccsch(c*x))-b*e*x*arctan(c*x/(-c^2*x^2-1
)^(1/2))/(-c^2*x^2)^(1/2)+b*c*d*(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {14, 6437, 462, 223, 209}

$$-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{x}+ex\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bex\text{ArcTan}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{\sqrt{-c^2x^2}}+\frac{bcd\sqrt{-c^2x^2-1}}{\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] (b*c*d*Sqrt[-1 - c^2*x^2])/Sqrt[-(c^2*x^2)] - (d*(a + b*ArcCsch[c*x]))/x +
e*x*(a + b*ArcCsch[c*x]) - (b*e*x*ArcTan[(c*x)/Sqrt[-1 - c^2*x^2]])/Sqrt[-(
c^2*x^2)]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx = -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{x}+ex\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{-d+ex^2}{x^2\sqrt{-1-c^2x^2}}}{\sqrt{-c^2x^2}}$$

$$=\frac{bcd\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}}-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{x}+ex\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcex}{}$$

$$=\frac{bcd\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}}-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{x}+ex\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcex}{}$$

$$=\frac{bcd\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}}-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{x}+ex\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bex\,\text{ta}}{}$$

Mathematica [A]
time = 0.16, size = 123, normalized size = 1.35

$$-\frac{ad}{x}+aex+bcd\sqrt{\frac{1+c^2x^2}{c^2x^2}}-\frac{bd\text{csch}^{-1}(cx)}{x}+bex\text{csch}^{-1}(cx)-\frac{be\sqrt{1+c^2x^2}\,\log\left(-\sqrt{c^2}\,x+\sqrt{1+c^2x^2}\right)}{c\sqrt{c^2}\,\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^2,x]
```

[Out] -((a*d)/x) + a*e*x + b*c*d*Sqrt[(1 + c^2*x^2)/(c^2*x^2)] - (b*d*ArcCsch[c*x])/x + b*e*x*ArcCsch[c*x] - (b*e*Sqrt[1 + c^2*x^2]*Log[-(Sqrt[c^2]*x) + Sqrt[1 + c^2*x^2]])/(c*Sqrt[c^2]*Sqrt[1 + 1/(c^2*x^2)]*x)

Maple [A]
time = 0.24, size = 107, normalized size = 1.18

method	result	s
derivativedivides	$c\left(\frac{a\left(ecx-\frac{dc}{x}\right)}{c^{2}}+\frac{b\left(\operatorname{arccsch}(cx)ecx-\frac{\operatorname{arccsch}(cx)dc}{x}+\frac{\sqrt{c^{2}x^{2}+1}\,\left(d\,c^{2}\sqrt{c^{2}x^{2}+1}+e\operatorname{arcsinh}(cx)cx\right)}{c^{2}x^{2}\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}}\right)}{c^{2}}\right)$	1
default	$c\left(\frac{a\left(ecx-\frac{dc}{x}\right)}{c^{2}}+\frac{b\left(\operatorname{arccsch}(cx)ecx-\frac{\operatorname{arccsch}(cx)dc}{x}+\frac{\sqrt{c^{2}x^{2}+1}\,\left(d\,c^{2}\sqrt{c^{2}x^{2}+1}+e\operatorname{arcsinh}(cx)cx\right)}{c^{2}x^{2}\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}}\right)}{c^{2}}\right)$	1

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)*(a+b*arccsch(c*x))/x^2,x,method=_RETURNVERBOSE)

[Out] c*(a/c^2*(e*c*x-d*c/x)+b/c^2*(arccsch(c*x)*e*c*x-arccsch(c*x)*d*c/x+(c^2*x^2+1)^(1/2)*(d*c^2*(c^2*x^2+1)^(1/2)+e*arcsinh(c*x)*c*x)/c^2/x^2/((c^2*x^2+1)/c^2/x^2)^(1/2)))

Maxima [A]
time = 0.26, size = 86, normalized size = 0.95

$$\left(c\sqrt{\frac{1}{c^{2}x^{2}}+1}-\frac{\operatorname{arcsch}\left(cx\right)}{x}\right)bd+axe+\frac{\left(2\,cx\operatorname{arcsch}\left(cx\right)+\log\left(\sqrt{\frac{1}{c^{2}x^{2}}+1}+1\right)-\log\left(\sqrt{\frac{1}{c^{2}x^{2}}+1}-1\right)\right)be}{2\,c}-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^2,x, algorithm="maxima")

[Out] (c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*d + a*x*e + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*b*e/c - a*d/x

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. 2(85) = 170.
time = 0.39, size = 273, normalized size = 3.00

$$\frac{bc^{2}dx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+bc^{2}dx+acx^{2}\cosh\left(1\right)+acx^{2}\sinh\left(1\right)-acd-\left(bcdx-bcx\cosh\left(1\right)-bcx\sinh\left(1\right)\right)\log\left(cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}-cx+1\right)-\left(bx\cosh\left(1\right)+bx\sinh\left(1\right)\right)\log\left(cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}-cx\right)+\left(bcdx-bcx\cosh\left(1\right)-bcx\sinh\left(1\right)\right)\log\left(cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}-cx-1\right)+\left(bcdx-bcd+\left(bcx^{2}-bcx\right)\cosh\left(1\right)+\left(bcx^{2}-bcx\right)\sinh\left(1\right)\right)\log\left(\frac{cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+1}{cx}\right)}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^2,x, algorithm="fricas")

[Out] (b*c^2*d*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + b*c^2*d*x + a*c*x^2*cosh(1) + a*c*x^2*sinh(1) - a*c*d - (b*c*d*x - b*c*x*cosh(1) - b*c*x*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - (b*x*cosh(1) + b*x*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) + (b*c*d*x - b*c*x*cosh(1) - b*c*x*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + (b*c*d*x - b*c*d + (b*c*x^2 - b*c*x)*cosh(1) + (b*c*x^2 - b*c*x)*sinh(1))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/(c*x)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsch}\,(cx))\,(d + ex^2)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)*(a+b*acsch(c*x))/x**2,x)

[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)/x**2, x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^2,x, algorithm="giac")

[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^2, x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x^2 + d)\,\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^2,x)

[Out] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^2, x)

3.80 $\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=109

$$-\frac{bc(2c^2d-9e)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}}+\frac{bcd\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}}-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{x}$$

```
[Out] -1/3*d*(a+b*arccsch(c*x))/x^3-e*(a+b*arccsch(c*x))/x-1/9*b*c*(2*c^2*d-9*e)*
(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/9*b*c*d*(-c^2*x^2-1)^(1/2)/x^2/(-c^2*
x^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {14, 6437, 12, 464, 270}

$$-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{x}-\frac{bc\sqrt{-c^2x^2-1}\,(2c^2d-9e)}{9\sqrt{-c^2x^2}}+\frac{bcd\sqrt{-c^2x^2-1}}{9x^2\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] -1/9*(b*c*(2*c^2*d - 9*e)*Sqrt[-1 - c^2*x^2])/Sqrt[-(c^2*x^2)] + (b*c*d*Sqr
t[-1 - c^2*x^2])/(9*x^2*Sqrt[-(c^2*x^2)]) - (d*(a + b*ArcCsch[c*x]))/(3*x^3
) - (e*(a + b*ArcCsch[c*x]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx &= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}-\frac{(bcx)\int \frac{-d-3ex^2}{3x^4\sqrt{-1-c^2x^2}}\end{aligned}$$
$$\frac{}{\sqrt{-c^2x^2}}$$
$$= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}-\frac{(bcx)\int \frac{-d-3ex^2}{x^4\sqrt{-1-c^2x^2}}\,d}{3\sqrt{-c^2x^2}}$$
$$= \frac{bcd\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}}-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x}-\frac{(bc(2c}{}$$
$$= -\frac{bc(2c^2d-9e)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}}+\frac{bcd\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}}-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}$$

Mathematica [A]
time = 0.06, size = 68, normalized size = 0.62

$$\frac{-3a(d+3ex^2)+bc\sqrt{1+\frac{1}{c^2x^2}}\,x(d-2c^2dx^2+9ex^2)-3b(d+3ex^2)\operatorname{csch}^{-1}(cx)}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] (-3*a*(d + 3*e*x^2) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(d - 2*c^2*d*x^2 + 9*e*x^
2) - 3*b*(d + 3*e*x^2)*ArcCsch[c*x])/(9*x^3)
```

Maple [A]
time = 0.23, size = 122, normalized size = 1.12

method	result	size
derivativedivides	$c^{3}\left(\frac{a\left(-\frac{d}{3c\,x^{3}}-\frac{e}{cx}\right)}{c^{2}}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)d}{3c\,x^{3}}-\frac{\operatorname{arccsch}(cx)e}{cx}-\frac{\left(c^{2}x^{2}+1\right)\left(2c^{4}d\,x^{2}-9c^{2}e\,x^{2}-c^{2}d\right)}{9\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}\,c^{4}x^{4}}\right)}{c^{2}}\right)$	122
default	$c^{3}\left(\frac{a\left(-\frac{d}{3c\,x^{3}}-\frac{e}{cx}\right)}{c^{2}}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)d}{3c\,x^{3}}-\frac{\operatorname{arccsch}(cx)e}{cx}-\frac{\left(c^{2}x^{2}+1\right)\left(2c^{4}d\,x^{2}-9c^{2}e\,x^{2}-c^{2}d\right)}{9\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}\,c^{4}x^{4}}\right)}{c^{2}}\right)$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a/c^2*(-1/3*d/c/x^3-e/c/x)+b/c^2*(-1/3*arccsch(c*x)*d/c/x^3-arccsch(c*
x)*e/c/x-1/9*(c^2*x^2+1)*(2*c^4*d*x^2-9*c^2*e*x^2-c^2*d)/((c^2*x^2+1)/c^2/x
^2)^(1/2)/c^4/x^4))
```

Maxima [A]
time = 0.27, size = 93, normalized size = 0.85

$$\frac{1}{9}\,bd\left(\frac{c^{4}\left(\frac{1}{c^{2}x^{2}}+1\right)^{\frac{3}{2}}-3\,c^{4}\sqrt{\frac{1}{c^{2}x^{2}}+1}}{c}-\frac{3\,\operatorname{arcsch}\left(cx\right)}{x^{3}}\right)+\left(c\sqrt{\frac{1}{c^{2}x^{2}}+1}-\frac{\operatorname{arcsch}\left(cx\right)}{x}\right)be-\frac{ae}{x}-\frac{ad}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*b*d*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*
arccsch(c*x)/x^3) + (c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*e - a*e/x
- 1/3*a*d/x^3
```

Fricas [A]
time = 0.36, size = 133, normalized size = 1.22

$$-\frac{9\,ax^{2}\cosh\left(1\right)+9\,ax^{2}\sinh\left(1\right)+3\,ad+3\left(3\,bx^{2}\cosh\left(1\right)+3\,bx^{2}\sinh\left(1\right)+bd\right)\log\left(\frac{cx\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}+1}{cx}\right)+\left(2\,bc^{3}dx^{3}-9\,bcx^{3}\cosh\left(1\right)-9\,bcx^{3}\sinh\left(1\right)-bcdx\right)\sqrt{\frac{c^{2}x^{2}+1}{c^{2}x^{2}}}}{9\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(9*a*x^2*cosh(1) + 9*a*x^2*sinh(1) + 3*a*d + 3*(3*b*x^2*cosh(1) + 3*b*
x^2*sinh(1) + b*d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (2*
b*c^3*d*x^3 - 9*b*c*x^3*cosh(1) - 9*b*c*x^3*sinh(1) - b*c*d*x)*sqrt((c^2*x^
2 + 1)/(c^2*x^2)))/x^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsch}(cx))\,(d + ex^2)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x^2 + d)\,\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^4,x)
```

```
[Out] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^4, x)
```

3.81 $\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=158

$$\frac{2bc^3(12c^2d-25e)\sqrt{-1-c^2x^2}}{225\sqrt{-c^2x^2}}+\frac{bcd\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}}-\frac{bc(12c^2d-25e)\sqrt{-1-c^2x^2}}{225x^2\sqrt{-c^2x^2}}-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5}-e$$

```
[Out] -1/5*d*(a+b*arccsch(c*x))/x^5-1/3*e*(a+b*arccsch(c*x))/x^3+2/225*b*c^3*(12*
c^2*d-25*e)*(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/25*b*c*d*(-c^2*x^2-1)^(1/
2)/x^4/(-c^2*x^2)^(1/2)-1/225*b*c*(12*c^2*d-25*e)*(-c^2*x^2-1)^(1/2)/x^2/(-
c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {14, 6437, 12, 464, 277, 270}

$$-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{bc\sqrt{-c^2x^2-1}\,(12c^2d-25e)}{225x^2\sqrt{-c^2x^2}}+\frac{bcd\sqrt{-c^2x^2-1}}{25x^4\sqrt{-c^2x^2}}+\frac{2bc^3\sqrt{-c^2x^2-1}\,(12c^2d-25e)}{225\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (2*b*c^3*(12*c^2*d - 25*e)*Sqrt[-1 - c^2*x^2])/(225*Sqrt[-(c^2*x^2)]) + (b*
c*d*Sqrt[-1 - c^2*x^2])/(25*x^4*Sqrt[-(c^2*x^2)]) - (b*c*(12*c^2*d - 25*e)*
Sqrt[-1 - c^2*x^2])/(225*x^2*Sqrt[-(c^2*x^2)]) - (d*(a + b*ArcCsch[c*x]))/(
5*x^5) - (e*(a + b*ArcCsch[c*x]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}-\frac{(bcx)\int\frac{-3d-5ex^2}{15x^6\sqrt{-1-c^2x^2}}}{\sqrt{-c^2x^2}}$$

$$= -\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}-\frac{(bcx)\int\frac{-3d-5ex^2}{x^6\sqrt{-1-c^2x^2}}\,d}{15\sqrt{-c^2x^2}}$$

$$= \frac{bcd\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}}-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3x^3}-\frac{(bc(12}{}$$

$$= \frac{bcd\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}}-\frac{bc(12c^2d-25e)\sqrt{-1-c^2x^2}}{225x^2\sqrt{-c^2x^2}}-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5x^5}$$

$$= \frac{2bc^3(12c^2d-25e)\sqrt{-1-c^2x^2}}{225\sqrt{-c^2x^2}}+\frac{bcd\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}}-\frac{bc(12c^2d-25e)}{225x^2\sqrt{-}}$$

Mathematica [A]

time = 0.09, size = 93, normalized size = 0.59

$$\frac{-15a(3d+5ex^2)+bc\sqrt{1+\frac{1}{c^2x^2}}\,x(25ex^2(1-2c^2x^2)+3d(3-4c^2x^2+8c^4x^4))-15b(3d+5ex^2)\operatorname{csch}^{-1}(cx)}{225x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (-15*a*(3*d + 5*e*x^2) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(25*e*x^2*(1 - 2*c^2*x
^2) + 3*d*(3 - 4*c^2*x^2 + 8*c^4*x^4)) - 15*b*(3*d + 5*e*x^2)*ArcCsch[c*x])
/(225*x^5)
```

Maple [A]
time = 0.24, size = 140, normalized size = 0.89

method	result
derivativedivides	$c^5\left(\frac{a\left(-\frac{e}{3c^3x^3}-\frac{d}{5c^3x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)e}{3c^3x^3}-\frac{\operatorname{arccsch}(cx)d}{5c^3x^5}+\frac{(c^2x^2+1)(24c^6dx^4-50c^4ex^4-12c^4dx^2+25c^2ex^2+9c^2d)}{225\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right)$
default	$c^5\left(\frac{a\left(-\frac{e}{3c^3x^3}-\frac{d}{5c^3x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)e}{3c^3x^3}-\frac{\operatorname{arccsch}(cx)d}{5c^3x^5}+\frac{(c^2x^2+1)(24c^6dx^4-50c^4ex^4-12c^4dx^2+25c^2ex^2+9c^2d)}{225\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^6x^6}\right)}{c^2}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(a/c^2*(-1/3*e/c^3/x^3-1/5*d/c^3/x^5)+b/c^2*(-1/3*arccsch(c*x)*e/c^3/x^
3-1/5*arccsch(c*x)*d/c^3/x^5+1/225*(c^2*x^2+1)*(24*c^6*d*x^4-50*c^4*e*x^4-1
2*c^4*d*x^2+25*c^2*e*x^2+9*c^2*d)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^6/x^6))
```

Maxima [A]
time = 0.25, size = 134, normalized size = 0.85

$$\frac{1}{75}bd\left(\frac{3c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15c^6\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{15\operatorname{arcsch}(cx)}{x^5}\right)+\frac{1}{9}b\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3c^4\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{3\operatorname{arcsch}(cx)}{x^3}\right)e-\frac{ae}{3x^3}-\frac{ad}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] 1/75*b*d*((3*c^6*(1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(1/(c^2*x^2) + 1)^(3/2) +
 15*c^6*sqrt(1/(c^2*x^2) + 1))/c - 15*arccsch(c*x)/x^5) + 1/9*b*((c^4*(1/(c
^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*arccsch(c*x)/x^3)*e
 - 1/3*a*e/x^3 - 1/5*a*d/x^5
```

Fricas [A]
time = 0.36, size = 169, normalized size = 1.07

$$-\frac{75\,ax^2\cosh\left(1\right)+75\,ax^2\sinh\left(1\right)+45\,ad+15\,\left(5\,bx^2\cosh\left(1\right)+5\,bx^2\sinh\left(1\right)+3\,bd\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-\left(24\,bc^5dx^5-12\,bc^3dx^3+9\,bcdx-25\,\left(2\,bc^3x^5-bcx^3\right)\cosh\left(1\right)-25\,\left(2\,bc^3x^5-bcx^3\right)\sinh\left(1\right)\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{225\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] -1/225*(75*a*x^2*cosh(1) + 75*a*x^2*sinh(1) + 45*a*d + 15*(5*b*x^2*cosh(1)
+ 5*b*x^2*sinh(1) + 3*b*d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x
)) - (24*b*c^5*d*x^5 - 12*b*c^3*d*x^3 + 9*b*c*d*x - 25*(2*b*c^3*x^5 - b*c*x
^3)*cosh(1) - 25*(2*b*c^3*x^5 - b*c*x^3)*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x
^2)))/x^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b\operatorname{acsch}\left(cx\right)\right)\left(d + ex^{2}\right)}{x^{6}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x))/x**6,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)/x**6, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^6,x)
[Out] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^6, x)
```

3.82 $\int \frac{(d+ex^2)\left(a+b\,\text{csch}^{-1}(cx)\right)}{x^8}\,dx$

Optimal. Leaf size=205

$$-\frac{8bc^5(30c^2d-49e)\sqrt{-1-c^2x^2}}{3675\sqrt{-c^2x^2}}+\frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}}-\frac{bc(30c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}}+\frac{4bc^3(30c^2d-49e)\sqrt{-}}{3675x^2\sqrt{-c^2x}}$$

```
[Out] -1/7*d*(a+b*arccsch(c*x))/x^7-1/5*e*(a+b*arccsch(c*x))/x^5-8/3675*b*c^5*(30
*c^2*d-49*e)*(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/49*b*c*d*(-c^2*x^2-1)^(1
/2)/x^6/(-c^2*x^2)^(1/2)-1/1225*b*c*(30*c^2*d-49*e)*(-c^2*x^2-1)^(1/2)/x^4/
(-c^2*x^2)^(1/2)+4/3675*b*c^3*(30*c^2*d-49*e)*(-c^2*x^2-1)^(1/2)/x^2/(-c^2*
x^2)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {14, 6437, 12, 464, 277, 270}

$$-\frac{d\left(a+b\,\text{csch}^{-1}(cx)\right)}{7x^7}-\frac{e\left(a+b\,\text{csch}^{-1}(cx)\right)}{5x^5}-\frac{bc\sqrt{-c^2x^2-1}\,(30c^2d-49e)}{1225x^4\sqrt{-c^2x^2}}+\frac{bcd\sqrt{-c^2x^2-1}}{49x^6\sqrt{-c^2x^2}}-\frac{8bc^5\sqrt{-c^2x^2-1}\,(30c^2d-49e)}{3675\sqrt{-c^2x^2}}+\frac{4bc^3\sqrt{-c^2x^2-1}\,(30c^2d-49e)}{3675x^2\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^8,x]
```

```
[Out] (-8*b*c^5*(30*c^2*d - 49*e)*Sqrt[-1 - c^2*x^2])/(3675*Sqrt[-(c^2*x^2)]) + (
b*c*d*Sqrt[-1 - c^2*x^2])/(49*x^6*Sqrt[-(c^2*x^2)]) - (b*c*(30*c^2*d - 49*e
)*Sqrt[-1 - c^2*x^2])/(1225*x^4*Sqrt[-(c^2*x^2)]) + (4*b*c^3*(30*c^2*d - 49
*e)*Sqrt[-1 - c^2*x^2])/(3675*x^2*Sqrt[-(c^2*x^2)]) - (d*(a + b*ArcCsch[c*x
]))/(7*x^7) - (e*(a + b*ArcCsch[c*x]))/(5*x^5)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx = -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{(bcx)\int \frac{-5d-7ex^2}{35x^8\sqrt{-1-c^2x}}}{\sqrt{-c^2x^2}}$$

$$= -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{(bcx)\int \frac{-5d-7ex^2}{x^8\sqrt{-1-c^2x^2}}}{35\sqrt{-c^2x^2}}$$

$$= \frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{(bc(3}{}$$

$$= \frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{bc(30c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}} - \frac{d\left(a+b\text{csch}^{-1}(cx\right.}{7x^7}$$

$$= \frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{bc(30c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}} + \frac{4bc^3(30c^2d-49e}{3675x^2\sqrt{}}$$

$$= -\frac{8bc^5(30c^2d-49e)\sqrt{-1-c^2x^2}}{3675\sqrt{-c^2x^2}} + \frac{bcd\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}} - \frac{bc(30c^2d-49}{1225x^{}}$$

Mathematica [A]
time = 0.11, size = 109, normalized size = 0.53

$$\frac{-105a(5d+7ex^2)+bc\sqrt{1+\frac{1}{c^2x^2}}\,x(49ex^2(3-4c^2x^2+8c^4x^4)-15d(-5+6c^2x^2-8c^4x^4+16c^6x^6))-105b(5d+7ex^2)\operatorname{csch}^{-1}(cx)}{3675x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^8,x]
```

```
[Out] (-105*a*(5*d + 7*e*x^2) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(49*e*x^2*(3 - 4*c^2*
x^2 + 8*c^4*x^4) - 15*d*(-5 + 6*c^2*x^2 - 8*c^4*x^4 + 16*c^6*x^6)) - 105*b*
(5*d + 7*e*x^2)*ArcCsch[c*x])/(3675*x^7)
```

Maple [A]
time = 0.25, size = 158, normalized size = 0.77

method	result
derivativedivides	$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)d}{7c^5x^7}-\frac{\operatorname{arccsch}(cx)e}{5c^5x^5}-\frac{(c^2x^2+1)(240c^8dx^6-392c^6ex^6-120c^6dx^4+196c^4ex^4+90c^4dx}{3675\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^8x^8}\right.}{c^2}\right.$
default	$c^7\left(\frac{a\left(-\frac{d}{7c^5x^7}-\frac{e}{5c^5x^5}\right)}{c^2}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)d}{7c^5x^7}-\frac{\operatorname{arccsch}(cx)e}{5c^5x^5}-\frac{(c^2x^2+1)(240c^8dx^6-392c^6ex^6-120c^6dx^4+196c^4ex^4+90c^4dx}{3675\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,c^8x^8}\right.}{c^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] c^7*(a/c^2*(-1/7*d/c^5/x^7-1/5*e/c^5/x^5)+b/c^2*(-1/7*arccsch(c*x)*d/c^5/x^
7-1/5*arccsch(c*x)*e/c^5/x^5-1/3675*(c^2*x^2+1)*(240*c^8*d*x^6-392*c^6*e*x^
6-120*c^6*d*x^4+196*c^4*e*x^4+90*c^4*d*x^2-147*c^2*e*x^2-75*c^2*d)/((c^2*x^
2+1)/c^2/x^2)^(1/2)/c^8/x^8))
```

Maxima [A]
time = 0.26, size = 167, normalized size = 0.81

$$\frac{1}{245}\,bd\left(\frac{5\,c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21\,c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35\,c^8\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{35\,\operatorname{arcsch}(cx)}{x^7}\right)+\frac{1}{75}\,b\left(\frac{3\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,c^6\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{15\,\operatorname{arcsch}(cx)}{x^5}\right)e-\frac{ae}{5\,x^5}-\frac{ad}{7\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/245*b*d*((5*c^8*(1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(1/(c^2*x^2) + 1)^(5/2)
+ 35*c^8*(1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(1/(c^2*x^2) + 1))/c - 35*arc
csch(c*x)/x^7) + 1/75*b*((3*c^6*(1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(1/(c^2*x^
2) + 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) + 1))/c - 15*arccsch(c*x)/x^5)*e -
1/5*a*e/x^5 - 1/7*a*d/x^7
```

Fricas [A]
time = 0.37, size = 196, normalized size = 0.96

$$-\frac{735\,ax^2\cosh(1)+735\,ax^2\sinh(1)+525\,ad+105\,(7\,bx^2\cosh(1)+7\,bx^2\sinh(1)+5\,bd)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+(240\,bc^7dx^7-120\,bc^5dx^5+90\,bc^3dx^3-75\,bcdx-49\,(8\,bc^5x^7-4\,bc^3x^5+3\,bcx^3)\cosh(1)-49\,(8\,bc^5x^7-4\,bc^3x^5+3\,bcx^3)\sinh(1))\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{3675\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/3675*(735*a*x^2*cosh(1) + 735*a*x^2*sinh(1) + 525*a*d + 105*(7*b*x^2*cos
h(1) + 7*b*x^2*sinh(1) + 5*b*d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)
/(c*x)) + (240*b*c^7*d*x^7 - 120*b*c^5*d*x^5 + 90*b*c^3*d*x^3 - 75*b*c*d*x
- 49*(8*b*c^5*x^7 - 4*b*c^3*x^5 + 3*b*c*x^3)*cosh(1) - 49*(8*b*c^5*x^7 - 4*
b*c^3*x^5 + 3*b*c*x^3)*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/x^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acsch}\,(cx))\,(d+ex^2)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x))/x**8,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)/x**8, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^8,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^8,x)
[Out] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^8, x)
```

3.83 $\int x^5(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=204

$$\frac{b(4c^2d-3e)\,x\sqrt{-1-c^2x^2}}{24c^7\sqrt{-c^2x^2}}+\frac{b(8c^2d-9e)\,x(-1-c^2x^2)^{3/2}}{72c^7\sqrt{-c^2x^2}}+\frac{b(4c^2d-9e)\,x(-1-c^2x^2)^{5/2}}{120c^7\sqrt{-c^2x^2}}-\frac{bex(-1-c^2x^2}{56c^7\sqrt{-c^2x^2}}$$

```
[Out] 1/6*d*x^6*(a+b*arccsch(c*x))+1/8*e*x^8*(a+b*arccsch(c*x))+1/72*b*(8*c^2*d-9
*e)*x*(-c^2*x^2-1)^(3/2)/c^7/(-c^2*x^2)^(1/2)+1/120*b*(4*c^2*d-9*e)*x*(-c^2
*x^2-1)^(5/2)/c^7/(-c^2*x^2)^(1/2)-1/56*b*e*x*(-c^2*x^2-1)^(7/2)/c^7/(-c^2*
x^2)^(1/2)+1/24*b*(4*c^2*d-3*e)*x*(-c^2*x^2-1)^(1/2)/c^7/(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {14, 6437, 12, 457, 78}

$$\frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx(-c^2x^2-1)^{5/2}(4c^2d-9e)}{120c^7\sqrt{-c^2x^2}}+\frac{bx(-c^2x^2-1)^{3/2}(8c^2d-9e)}{72c^7\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}\,(4c^2d-3e)}{24c^7\sqrt{-c^2x^2}}-\frac{bex(-c^2x^2-1)^{7/2}}{56c^7\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(4*c^2*d - 3*e)*x*Sqrt[-1 - c^2*x^2])/(24*c^7*Sqrt[-(c^2*x^2)]) + (b*(8*
c^2*d - 9*e)*x*(-1 - c^2*x^2)^(3/2))/(72*c^7*Sqrt[-(c^2*x^2)]) + (b*(4*c^2*
d - 9*e)*x*(-1 - c^2*x^2)^(5/2))/(120*c^7*Sqrt[-(c^2*x^2)]) - (b*e*x*(-1 -
c^2*x^2)^(7/2))/(56*c^7*Sqrt[-(c^2*x^2)]) + (d*x^6*(a + b*ArcCsch[c*x]))/6
+ (e*x^8*(a + b*ArcCsch[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^5\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^5(4d+3}{24\sqrt{-1-}}}{\sqrt{-c^2x^2}}$$

$$=\frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^5(4d+3e}{\sqrt{-1-c}}}{24\sqrt{-c^2x^2}}$$

$$=\frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\text{Subst}\left(\int-\right.}{48\sqrt{}}$$

$$=\frac{1}{6}dx^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}ex^8\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\text{Subst}\left(\int\left(\right.\right.}{}$$

$$=\frac{b(4c^2d-3e)\,x\sqrt{-1-c^2x^2}}{24c^7\sqrt{-c^2x^2}}+\frac{b(8c^2d-9e)\,x(-1-c^2x^2)^{3/2}}{72c^7\sqrt{-c^2x^2}}+\frac{b(4c^2d}{}$$

Mathematica [A]
time = 0.17, size = 114, normalized size = 0.56

$$\frac{x\left(105ax^5(4d+3ex^2)+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\left(-144e+8c^2(28d+9ex^2)-2c^4(56dx^2+27ex^4)+c^6(84dx^4+45ex^6)\right)}{c^7}+105bx^5(4d+3ex^2)\,\text{csch}^{-1}(cx)\right)}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (x*(105*a*x^5*(4*d + 3*e*x^2) + (b*Sqrt[1 + 1/(c^2*x^2)]*(-144*e + 8*c^2*(2
8*d + 9*e*x^2) - 2*c^4*(56*d*x^2 + 27*e*x^4) + c^6*(84*d*x^4 + 45*e*x^6)))/
c^7 + 105*b*x^5*(4*d + 3*e*x^2)*ArcCsch[c*x]))/2520
```

Maple [A]
time = 0.36, size = 152, normalized size = 0.75

method	result
derivativedivides	$\frac{\frac{a\left(\frac{1}{6}c^8 d\,x^6+\frac{1}{8}e\,c^8x^8\right)}{c^2}+\frac{b\left(\frac{\operatorname{arccsch}(cx)d\,c^8x^6}{6}+\frac{\operatorname{arccsch}(cx)e\,c^8x^8}{8}+\frac{(c^2x^2+1)\left(45c^6e\,x^6+84c^6d\,x^4-54c^4e\,x^4-112c^4d\,x^2+72c^2e\,x^2\right.}{2520\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right.}{c^2}}{c^6}$
default	$\frac{\frac{a\left(\frac{1}{6}c^8 d\,x^6+\frac{1}{8}e\,c^8x^8\right)}{c^2}+\frac{b\left(\frac{\operatorname{arccsch}(cx)d\,c^8x^6}{6}+\frac{\operatorname{arccsch}(cx)e\,c^8x^8}{8}+\frac{(c^2x^2+1)\left(45c^6e\,x^6+84c^6d\,x^4-54c^4e\,x^4-112c^4d\,x^2+72c^2e\,x^2\right.}{2520\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,cx}\right.}{c^2}}{c^6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^6*(a/c^2*(1/6*c^8*d*x^6+1/8*e*c^8*x^8)+b/c^2*(1/6*arccsch(c*x)*d*c^8*x^
6+1/8*arccsch(c*x)*e*c^8*x^8+1/2520*(c^2*x^2+1)*(45*c^6*e*x^6+84*c^6*d*x^4-
54*c^4*e*x^4-112*c^4*d*x^2+72*c^2*e*x^2+224*c^2*d-144*e)/((c^2*x^2+1)/c^2/x
^2)^(1/2)/c/x))
```

Maxima [A]
time = 0.27, size = 178, normalized size = 0.87

$$\frac{1}{8}\,ax^8e+\frac{1}{6}\,adx^6+\frac{1}{90}\left(15\,x^6\operatorname{arcsch}\left(cx\right)+\frac{3\,c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10\,c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^5}\right)bd+\frac{1}{280}\left(35\,x^8\operatorname{arcsch}\left(cx\right)+\frac{5\,c^6x^7\left(\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21\,c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35\,c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^7}\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8*e + 1/6*a*d*x^6 + 1/90*(15*x^6*arccsch(c*x) + (3*c^4*x^5*(1/(c^2*
x^2) + 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2
) + 1))/c^5)*b*d + 1/280*(35*x^8*arccsch(c*x) + (5*c^6*x^7*(1/(c^2*x^2) + 1
)^(7/2) - 21*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(1/(c^2*x^2) + 1)
^(3/2) - 35*x*sqrt(1/(c^2*x^2) + 1))/c^7)*b*e
```

Fricas [A]
time = 0.42, size = 224, normalized size = 1.10

$$\frac{315\,ac^7x^8\cosh\left(1\right)+315\,ac^7x^8\sinh\left(1\right)+420\,ac^7dx^6+105\left(3\,bc^7x^8\cosh\left(1\right)+3\,bc^7x^8\sinh\left(1\right)+4\,bc^7dx^6\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(84\,bc^6dx^5-112\,bc^4dx^3+224\,bc^2dx+9\left(5\,bc^6x^7-6\,bc^4x^5+8\,bc^2x^3-16\,bx\right)\cosh\left(1\right)+9\left(5\,bc^6x^7-6\,bc^4x^5+8\,bc^2x^3-16\,bx\right)\sinh\left(1\right)\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{2520\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*c^7*x^8*cosh(1) + 315*a*c^7*x^8*sinh(1) + 420*a*c^7*d*x^6 + 1
05*(3*b*c^7*x^8*cosh(1) + 3*b*c^7*x^8*sinh(1) + 4*b*c^7*d*x^6)*log((c*x*sqr
t((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (84*b*c^6*d*x^5 - 112*b*c^4*d*x^3
+ 224*b*c^2*d*x + 9*(5*b*c^6*x^7 - 6*b*c^4*x^5 + 8*b*c^2*x^3 - 16*b*x)*cosh
(1) + 9*(5*b*c^6*x^7 - 6*b*c^4*x^5 + 8*b*c^2*x^3 - 16*b*x)*sinh(1))*sqrt((c
^2*x^2 + 1)/(c^2*x^2)))/c^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5(a + b\,\text{acsch}\,(cx))\left(d + ex^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**2+d)*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**5*(a + b*acsch(c*x))*(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x^5, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x^5*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.84 $\int x^3(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=159

$$-\frac{b(3c^2d-2e)\,x\sqrt{-1-c^2x^2}}{12c^5\sqrt{-c^2x^2}}-\frac{b(3c^2d-4e)\,x(-1-c^2x^2)^{3/2}}{36c^5\sqrt{-c^2x^2}}+\frac{bex(-1-c^2x^2)^{5/2}}{30c^5\sqrt{-c^2x^2}}+\frac{1}{4}dx^4\left(a+b\text{csch}^{-1}(cx)\right)-$$

```
[Out] 1/4*d*x^4*(a+b*arccsch(c*x))+1/6*e*x^6*(a+b*arccsch(c*x))-1/36*b*(3*c^2*d-4
*e)*x*(-c^2*x^2-1)^(3/2)/c^5/(-c^2*x^2)^(1/2)+1/30*b*e*x*(-c^2*x^2-1)^(5/2)
/c^5/(-c^2*x^2)^(1/2)-1/12*b*(3*c^2*d-2*e)*x*(-c^2*x^2-1)^(1/2)/c^5/(-c^2*x
^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {14, 6437, 12, 457, 78}

$$\frac{1}{4}dx^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bx(-c^2x^2-1)^{3/2}(3c^2d-4e)}{36c^5\sqrt{-c^2x^2}}-\frac{bx\sqrt{-c^2x^2-1}\,(3c^2d-2e)}{12c^5\sqrt{-c^2x^2}}+\frac{bex(-c^2x^2-1)^{5/2}}{30c^5\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -1/12*(b*(3*c^2*d - 2*e)*x*Sqrt[-1 - c^2*x^2])/(c^5*Sqrt[-(c^2*x^2)]) - (b*
(3*c^2*d - 4*e)*x*(-1 - c^2*x^2)^(3/2))/(36*c^5*Sqrt[-(c^2*x^2)]) + (b*e*x*
(-1 - c^2*x^2)^(5/2))/(30*c^5*Sqrt[-(c^2*x^2)]) + (d*x^4*(a + b*ArcCsch[c*x
]))/4 + (e*x^6*(a + b*ArcCsch[c*x]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
```

```
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^3\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{1}{4}dx^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^3(3d+2}{12\sqrt{-1-}}}{\sqrt{-c^2x^2}}$$

$$=\frac{1}{4}dx^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\int\frac{x^3(3d+2e}{\sqrt{-1-c}}}{12\sqrt{-c^2x^2}}$$

$$=\frac{1}{4}dx^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\text{Subst}\left(\int -\right.}{24}$$

$$=\frac{1}{4}dx^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\text{csch}^{-1}(cx)\right)-\frac{(bcx)\text{Subst}\left(\int\left(\right.\right.}{}$$

$$=-\frac{b(3c^2d-2e)\,x\sqrt{-1-c^2x^2}}{12c^5\sqrt{-c^2x^2}}-\frac{b(3c^2d-4e)\,x(-1-c^2x^2)^{3/2}}{36c^5\sqrt{-c^2x^2}}+\frac{bex}{3}$$

Mathematica [A]
time = 0.15, size = 97, normalized size = 0.61

$$\frac{1}{180}x\left(15ax^3\left(3d+2ex^2\right)+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,\left(16e-2c^2(15d+4ex^2)+3c^4(5dx^2+2ex^4)\right)}{c^5}+15bx^3\left(3d+2ex^2\right)\text{csch}^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (x*(15*a*x^3*(3*d + 2*e*x^2) + (b*Sqrt[1 + 1/(c^2*x^2)]*(16*e - 2*c^2*(15*d
 + 4*e*x^2) + 3*c^4*(5*d*x^2 + 2*e*x^4)))/c^5 + 15*b*x^3*(3*d + 2*e*x^2)*Ar
cCsch[c*x]))/180
```

Maple [A]
time = 0.45, size = 267, normalized size = 1.68

method	result
derivativedivides	$-\frac{a\left(\frac{c^2 d\left(c^2 e x^2+c^2 d\right)^2}{2}-\frac{\left(c^2 e x^2+c^2 d\right)^3}{3}\right)}{2c^2e^2}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)c^6d^3}{12e^2}+\frac{\operatorname{arccsch}(cx)c^6 d x^4}{4}+\frac{e\operatorname{arccsch}(cx)c^6x^6}{6}+\frac{\sqrt{c^2x^2+1}\left(1\right.}{}\right.}{}$
default	$-\frac{a\left(\frac{c^2 d\left(c^2 e x^2+c^2 d\right)^2}{2}-\frac{\left(c^2 e x^2+c^2 d\right)^3}{3}\right)}{2c^2e^2}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)c^6d^3}{12e^2}+\frac{\operatorname{arccsch}(cx)c^6 d x^4}{4}+\frac{e\operatorname{arccsch}(cx)c^6x^6}{6}+\frac{\sqrt{c^2x^2+1}\left(1\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-1/2*a/c^2/e^2*(1/2*c^2*d*(c^2*e*x^2+c^2*d)^2-1/3*(c^2*e*x^2+c^2*d)^
3)+b/c^2*(-1/12/e^2*arccsch(c*x)*c^6*d^3+1/4*arccsch(c*x)*c^6*d*x^4+1/6*e*a
rccsch(c*x)*c^6*x^6+1/180/e^2*(c^2*x^2+1)^(1/2)*(15*c^6*d^3*arctanh(1/(c^2*
x^2+1)^(1/2))+15*c^4*d*e^2*x^2*(c^2*x^2+1)^(1/2)+6*e^3*c^4*x^4*(c^2*x^2+1)^
(1/2)-30*c^2*d*e^2*(c^2*x^2+1)^(1/2)-8*e^3*c^2*x^2*(c^2*x^2+1)^(1/2)+16*e^3
*(c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A]
time = 0.26, size = 139, normalized size = 0.87

$$\frac{1}{6}ax^6e+\frac{1}{4}adx^4+\frac{1}{12}\left(3x^4\operatorname{arcsch}(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)bd+\frac{1}{90}\left(15x^6\operatorname{arcsch}(cx)+\frac{3c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15x\sqrt{\frac{1}{c^2x^2}+1}}{c^5}\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6*e + 1/4*a*d*x^4 + 1/12*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2
) + 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b*d + 1/90*(15*x^6*arccsch(c
*x) + (3*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) + 1)^(3/
2) + 15*x*sqrt(1/(c^2*x^2) + 1))/c^5)*b*e
```

Fricas [A]
time = 0.37, size = 196, normalized size = 1.23

$$\frac{30\,ac^5x^6\cosh\left(1\right) + 30\,ac^5x^6\sinh\left(1\right) + 45\,ac^5dx^4 + 15\left(2\,bc^5x^6\cosh\left(1\right) + 2\,bc^5x^6\sinh\left(1\right) + 3\,bc^5dx^4\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + \left(15\,bc^4dx^3 - 30\,bc^2dx + 2\left(3\,bc^4x^5 - 4\,bc^2x^3 + 8\,bx\right)\cosh\left(1\right) + 2\left(3\,bc^4x^5 - 4\,bc^2x^3 + 8\,bx\right)\sinh\left(1\right)\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{180\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/180*(30*a*c^5*x^6*cosh(1) + 30*a*c^5*x^6*sinh(1) + 45*a*c^5*d*x^4 + 15*(2
*b*c^5*x^6*cosh(1) + 2*b*c^5*x^6*sinh(1) + 3*b*c^5*d*x^4)*log((c*x*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (15*b*c^4*d*x^3 - 30*b*c^2*d*x + 2*(3*b
*c^4*x^5 - 4*b*c^2*x^3 + 8*b*x)*cosh(1) + 2*(3*b*c^4*x^5 - 4*b*c^2*x^3 + 8*
b*x)*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a + b\,\mathrm{acsch}\,(cx))\left(d + ex^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))*(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x^3*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.85 $\int x(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=146

$$\frac{b(2c^2d-e)\,x\sqrt{-1-c^2x^2}}{4c^3\sqrt{-c^2x^2}}-\frac{bex(-1-c^2x^2)^{3/2}}{12c^3\sqrt{-c^2x^2}}+\frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{bcd^2x\text{ArcTan}\left(\sqrt{-1-c^2x^2}\right)}{4e\sqrt{-c^2x^2}}$$

```
[Out] 1/4*(e*x^2+d)^2*(a+b*arccsch(c*x))/e-1/12*b*e*x*(-c^2*x^2-1)^(3/2)/c^3/(-c^
2*x^2)^(1/2)-1/4*b*c*d^2*x*arctan((-c^2*x^2-1)^(1/2))/e/(-c^2*x^2)^(1/2)+1/
4*b*(2*c^2*d-e)*x*(-c^2*x^2-1)^(1/2)/c^3/(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.294$, Rules used = {6435, 457, 90, 65, 211}

$$\frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{bcd^2x\text{ArcTan}\left(\sqrt{-c^2x^2-1}\right)}{4e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}\,(2c^2d-e)}{4c^3\sqrt{-c^2x^2}}-\frac{bex(-c^2x^2-1)^{3/2}}{12c^3\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(2*c^2*d - e)*x*Sqrt[-1 - c^2*x^2])/(4*c^3*Sqrt[-(c^2*x^2)]) - (b*e*x*(-
1 - c^2*x^2)^(3/2))/(12*c^3*Sqrt[-(c^2*x^2)]) + ((d + e*x^2)^2*(a + b*ArcCs
ch[c*x]))/(4*e) - (b*c*d^2*x*ArcTan[Sqrt[-1 - c^2*x^2]])/(4*e*Sqrt[-(c^2*x^
2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6435

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2])), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rubi steps

$$\int x\left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{(bcx)\int\frac{\left(d+ex^2\right)^2}{x\sqrt{-1-c^2x^2}}\,dx}{4e\sqrt{-c^2x^2}}$$

$$=\frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{(bcx)\text{Subst}\left(\int\frac{(d+ex)^2}{x\sqrt{-1-c^2x}}\,dx,x,x^2\right.}{8e\sqrt{-c^2x^2}}$$

$$=\frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{4e}-\frac{(bcx)\text{Subst}\left(\int\left(-\frac{e(-2c^2d+e)}{c^2\sqrt{-1-c^2x}}+\frac{}{x}\right.\right.}{8e}$$

$$=\frac{b(2c^2d-e)\,x\sqrt{-1-c^2x^2}}{4c^3\sqrt{-c^2x^2}}-\frac{bex\left(-1-c^2x^2\right)^{3/2}}{12c^3\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^2\left(a+b\right.}{4e}$$

$$=\frac{b(2c^2d-e)\,x\sqrt{-1-c^2x^2}}{4c^3\sqrt{-c^2x^2}}-\frac{bex\left(-1-c^2x^2\right)^{3/2}}{12c^3\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^2\left(a+b\right.}{4e}$$

$$=\frac{b(2c^2d-e)\,x\sqrt{-1-c^2x^2}}{4c^3\sqrt{-c^2x^2}}-\frac{bex\left(-1-c^2x^2\right)^{3/2}}{12c^3\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^2\left(a+b\right.}{4e}$$

Mathematica [A]
time = 0.08, size = 77, normalized size = 0.53

$$\frac{x\left(3ac^3x(2d+ex^2)+b\sqrt{1+\frac{1}{c^2x^2}}\left(-2e+c^2(6d+ex^2)\right)+3bc^3x(2d+ex^2)\,\text{csch}^{-1}(cx)\right)}{12c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (x*(3*a*c^3*x*(2*d + e*x^2) + b*Sqrt[1 + 1/(c^2*x^2)]*(-2*e + c^2*(6*d + e*
x^2)) + 3*b*c^3*x*(2*d + e*x^2)*ArcCsch[c*x]))/(12*c^3)
```

Maple [A]
time = 0.44, size = 195, normalized size = 1.34

method	result
derivativedivides	$\frac{\frac{\left(c^2 e\,x^2+c^2 d\right)^2 a}{4c^2 e}+\frac{b\left(\frac{\operatorname{arccsch}(cx)c^4d^2}{4e}+\frac{\operatorname{arccsch}(cx)c^4d\,x^2}{2}+\frac{e\operatorname{arccsch}(cx)c^4x^4}{4}-\frac{\sqrt{c^2x^2+1}\left(3c^4d^2\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+}}\right.\right.}{}\right)}{c^2}}{c^2}$
default	$\frac{\frac{\left(c^2 e\,x^2+c^2 d\right)^2 a}{4c^2 e}+\frac{b\left(\frac{\operatorname{arccsch}(cx)c^4d^2}{4e}+\frac{\operatorname{arccsch}(cx)c^4d\,x^2}{2}+\frac{e\operatorname{arccsch}(cx)c^4x^4}{4}-\frac{\sqrt{c^2x^2+1}\left(3c^4d^2\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+}}\right.\right.}{}\right)}{c^2}}{c^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/4*(c^2*e*x^2+c^2*d)^2*a/c^2/e+b/c^2*(1/4/e*arccsch(c*x)*c^4*d^2+1/
2*arccsch(c*x)*c^4*d*x^2+1/4*e*arccsch(c*x)*c^4*x^4-1/12/e*(c^2*x^2+1)^(1/2
)*(3*c^4*d^2*arctanh(1/(c^2*x^2+1)^(1/2))-6*c^2*d*e*(c^2*x^2+1)^(1/2)-e^2*c
^2*x^2*(c^2*x^2+1)^(1/2)+2*e^2*(c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/
2)/c/x))
```

Maxima [A]
time = 0.25, size = 97, normalized size = 0.66

$$\frac{1}{4}\,ax^4e+\frac{1}{2}\,adx^2+\frac{1}{2}\left(x^2\operatorname{arcsch}\left(cx\right)+\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)bd+\frac{1}{12}\left(3\,x^4\operatorname{arcsch}\left(cx\right)+\frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)be$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4*e + 1/2*a*d*x^2 + 1/2*(x^2*arccsch(c*x) + x*sqrt(1/(c^2*x^2) + 1)
/c)*b*d + 1/12*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 3*x
*sqrt(1/(c^2*x^2) + 1))/c^3)*b*e
```

Fricas [A]
time = 0.37, size = 162, normalized size = 1.11

$$\frac{3\,ac^3x^4\cosh\left(1\right)+3\,ac^3x^4\sinh\left(1\right)+6\,ac^3dx^2+3\,\left(bc^3x^4\cosh\left(1\right)+bc^3x^4\sinh\left(1\right)+2\,bc^3dx^2\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(6\,bc^2dx+\left(bc^2x^3-2\,bx\right)\cosh\left(1\right)+\left(bc^2x^3-2\,bx\right)\sinh\left(1\right)\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{12\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^3*x^4*cosh(1) + 3*a*c^3*x^4*sinh(1) + 6*a*c^3*d*x^2 + 3*(b*c^3*
x^4*cosh(1) + b*c^3*x^4*sinh(1) + 2*b*c^3*d*x^2)*log((c*x*sqrt((c^2*x^2 + 1
)/(c^2*x^2)) + 1)/(c*x)) + (6*b*c^2*d*x + (b*c^2*x^3 - 2*b*x)*cosh(1) + (b*
c^2*x^3 - 2*b*x)*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a+b\operatorname{acsch}\left(cx\right))\left(d+ex^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))*(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(e\,x^2+d\right)\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.86 $\int \frac{(d+ex^2)\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=115

$$\frac{be\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c}+\frac{1}{2}bd\text{csch}^{-1}(cx)^2+\frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right)-bd\text{csch}^{-1}(cx)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)+bd\text{csch}^{-1}(c$$

```
[Out] 1/2*b*d*arccsch(c*x)^2+1/2*e*x^2*(a+b*arccsch(c*x))-b*d*arccsch(c*x)*ln(1-(
1/c/x+(1+1/c^2/x^2)^(1/2))^2)+b*d*arccsch(c*x)*ln(1/x)-d*(a+b*arccsch(c*x))
*ln(1/x)-1/2*b*d*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)+1/2*b*e*x*(1+1/c^
2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.20, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.579$, Rules used = {6439, 14, 5822, 6874, 270, 2362, 5775, 3797, 2221, 2317, 2438}

$$-d\log\left(\frac{1}{x}\right)\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bex\sqrt{\frac{1}{c^2x^2}+1}}{2c}-\frac{1}{2}bd\text{Li}_2\left(e^{2\text{csch}^{-1}(cx)}\right)+\frac{1}{2}bd\text{csch}^{-1}(cx)^2-bd\text{csch}^{-1}(cx)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)+bd\log\left(\frac{1}{x}\right)\text{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] (b*e*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*c) + (b*d*ArcCsch[c*x]^2)/2 + (e*x^2*(a +
b*ArcCsch[c*x]))/2 - b*d*ArcCsch[c*x]*Log[1 - E^(2*ArcCsch[c*x])] + b*d*Arc
Csch[c*x]*Log[x^(-1)] - d*(a + b*ArcCsch[c*x])*Log[x^(-1)] - (b*d*PolyLog[2
, E^(2*ArcCsch[c*x])])/2
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2362

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[e, 2]), x]
 - Dist[b*(n/Rt[e, 2]), Int[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5822

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^
2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2*d] &&
IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
```

```
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx = -\text{Subst}\left(\int \frac{(e+dx^2)\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) - d\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{-\frac{\cdot}{2\cdot}}{\sqrt{}}\right)}{}$$

$$= \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) - d\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \left(-\right.\right.}{}$$

$$= \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) - d\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{(bd)\text{Subst}\left(\int\right.}{}$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) + bd\text{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - d$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) + bd\text{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - d$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}bd\text{csch}^{-1}(cx)^2 + \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) + bd\text{csch}^{-}$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}bd\text{csch}^{-1}(cx)^2 + \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) - bd\text{csch}^{-}$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}bd\text{csch}^{-1}(cx)^2 + \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) - bd\text{csch}^{-}$$

$$= \frac{be\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} + \frac{1}{2}bd\text{csch}^{-1}(cx)^2 + \frac{1}{2}ex^2\left(a+b\text{csch}^{-1}(cx)\right) - bd\text{csch}^{-}$$

Mathematica [A]

time = 0.14, size = 93, normalized size = 0.81

$$\frac{be\sqrt{1+\frac{1}{c^2x^2}}\,x+acex^2-bcd\,\mathrm{csch}^{-1}(cx)^2+bc\,\mathrm{csch}^{-1}(cx)\left(ex^2-2d\log\left(1-e^{-2\mathrm{csch}^{-1}(cx)}\right)\right)+2acd\log(x)+bcd\,\mathrm{PolyLog}\left(2,e^{-2\mathrm{csch}^{-1}(cx)}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] (b*e*Sqrt[1 + 1/(c^2*x^2)]*x + a*c*e*x^2 - b*c*d*ArcCsch[c*x]^2 + b*c*ArcCs
ch[c*x]*(e*x^2 - 2*d*Log[1 - E^(-2*ArcCsch[c*x])]) + 2*a*c*d*Log[x] + b*c*d
*PolyLog[2, E^(-2*ArcCsch[c*x])])/(2*c)
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)\,(a+b\,\mathrm{arccsch}(cx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x))/x,x)
```

```
[Out] int((e*x^2+d)*(a+b*arccsch(c*x))/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x,x, algorithm="maxima")
```

```
[Out] 2*b*c^2*d*integrate(1/2*x*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + sqr
t(c^2*x^2 + 1) + 1), x) - 1/2*b*x^2*e*log(c) - 1/2*b*x^2*e*log(x) + 1/2*a*x
^2*e - b*d*log(c)*log(x) - 1/2*b*d*log(x)^2 - 1/4*(2*log(c^2*x^2 + 1)*log(x
) + dilog(-c^2*x^2))*b*d + a*d*log(x) + 1/2*(b*x^2*e + 2*b*d*log(x))*log(sq
rt(c^2*x^2 + 1) + 1) + 1/4*b*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1))*e/c^2
 + 1/4*b*e*log(c^2*x^2 + 1)/c^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccsch(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsch}\,(cx))\,(d + ex^2)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x))/x,x)
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x,x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x^2 + d)\,\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x,x)
[Out] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x, x)
```

3.87 $\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=128

$$\frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x}-\frac{1}{4}bc^2d\text{csch}^{-1}(cx)+\frac{1}{2}be\text{csch}^{-1}(cx)^2-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2}-be\text{csch}^{-1}(cx)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right.$$

```
[Out] -1/4*b*c^2*d*arccsch(c*x)+1/2*b*e*arccsch(c*x)^2-1/2*d*(a+b*arccsch(c*x))/x
^2-b*e*arccsch(c*x)*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))^2)+b*e*arccsch(c*x)*ln
(1/x)-e*(a+b*arccsch(c*x))*ln(1/x)-1/2*b*e*polylog(2,(1/c/x+(1+1/c^2/x^2)^(
1/2))^2)+1/4*b*c*d*(1+1/c^2/x^2)^(1/2)/x
```

Rubi [A]
time = 0.20, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.684$, Rules used = {6439, 14, 5822, 12, 6874, 327, 221, 2362, 5775, 3797, 2221, 2317, 2438}

$$-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2}-e\log\left(\frac{1}{x}\right)\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bcd\sqrt{\frac{1}{c^2x^2}+1}}{4x}-\frac{1}{4}bc^2d\text{csch}^{-1}(cx)-\frac{1}{2}be\text{Li}_2\left(e^{2\text{csch}^{-1}(cx)}\right)+\frac{1}{2}be\text{csch}^{-1}(cx)^2-be\text{csch}^{-1}(cx)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)+be\log\left(\frac{1}{x}\right)\text{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^3,x]
```

```
[Out] (b*c*d*Sqrt[1 + 1/(c^2*x^2)])/(4*x) - (b*c^2*d*ArcCsch[c*x])/4 + (b*e*ArcCs
ch[c*x]^2)/2 - (d*(a + b*ArcCsch[c*x]))/(2*x^2) - b*e*ArcCsch[c*x]*Log[1 -
E^(2*ArcCsch[c*x])] + b*e*ArcCsch[c*x]*Log[x^(-1)] - e*(a + b*ArcCsch[c*x])
*Log[x^(-1)] - (b*e*PolyLog[2, E^(2*ArcCsch[c*x])])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2362

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[e, 2]), x]
 - Dist[b*(n/Rt[e, 2]), Int[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5822

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^
2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2*d] &&
IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex^2)\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = -\text{Subst}\left(\int \frac{(e+dx^2)\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{x}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} - e\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{dx^2+\cdots}{2\sqrt{1\cdots}}\right.}{\cdots}$$

$$= -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} - e\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int \frac{dx^2+\cdots}{\sqrt{1\cdots}}\right.}{2\cdots}$$

$$= -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} - e\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{b\text{Subst}\left(\int\left(\cdots\right.\right.}{\cdots}$$

$$= -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} - e\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{1}{x}\right) + \frac{(bd)\text{Subst}\left(\int -\cdots\right.}{\cdots}$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} + be\text{csch}^{-1}(cx)\log\left(\frac{1}{x}\right) - e\left(a+\cdots\right.$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\text{csch}^{-1}(cx) - \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} + be\text{csch}^{-1}(c\cdots$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\text{csch}^{-1}(cx) + \frac{1}{2}be\text{csch}^{-1}(cx)^2 - \frac{d\left(a+b\text{csch}^{-1}(\cdots\right.}{2x^2}$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\text{csch}^{-1}(cx) + \frac{1}{2}be\text{csch}^{-1}(cx)^2 - \frac{d\left(a+b\text{csch}^{-1}(\cdots\right.}{2x^2}$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\text{csch}^{-1}(cx) + \frac{1}{2}be\text{csch}^{-1}(cx)^2 - \frac{d\left(a+b\text{csch}^{-1}(\cdots\right.}{2x^2}$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{4x} - \frac{1}{4}bc^2d\text{csch}^{-1}(cx) + \frac{1}{2}be\text{csch}^{-1}(cx)^2 - \frac{d\left(a+b\text{csch}^{-1}(\cdots\right.}{2x^2}$$

Mathematica [A]
time = 0.71, size = 138, normalized size = 1.08

$$\frac{1}{4}\left(-\frac{2ad}{x^2}-\frac{2bd\operatorname{csch}^{-1}(cx)}{x^2}-\frac{bd\left(-1-c^2x^2+c^2x^2\sqrt{1+c^2x^2}\,\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)\right)}{c\sqrt{1+\frac{1}{c^2x^2}}\,x^3}-2be\operatorname{csch}^{-1}(cx)\left(\operatorname{csch}^{-1}(cx)+2\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)\right)+4ae\log(x)+2be\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(cx)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCsch[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d)/x^2 - (2*b*d*ArcCsch[c*x])/x^2 - (b*d*(-1 - c^2*x^2 + c^2*x^2*Sqr
t[1 + c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]]))/(c*Sqrt[1 + 1/(c^2*x^2)]*x^3) -
 2*b*e*ArcCsch[c*x]*(ArcCsch[c*x] + 2*Log[1 - E^(-2*ArcCsch[c*x])]) + 4*a*e
*Log[x] + 2*b*e*PolyLog[2, E^(-2*ArcCsch[c*x])])/4
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^2+d\right)\left(a+b\operatorname{arccsch}(cx)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x)
```

```
[Out] int((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*b*d*((2*c^4*x*sqrt(1/(c^2*x^2) + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) - c
^3*log(c*x*sqrt(1/(c^2*x^2) + 1) + 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) -
 1))/c - 4*arccsch(c*x)/x^2) - 1/2*(4*c^2*integrate(x^2*log(x)/(c^2*x^3 + x
), x) - 2*c^2*integrate(x*log(x)/(c^2*x^2 + (c^2*x^2 + 1)^(3/2) + 1), x) -
(log(c^2*x^2 + 1) - 2*log(x))*log(c) + log(c^2*x^2 + 1)*log(c) - 2*log(x)*l
og(sqrt(c^2*x^2 + 1) + 1) + 2*integrate(log(x)/(c^2*x^3 + x), x))*b*e + a*e
*log(x) - 1/2*a*d/x^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccsch(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsch}(cx))(d + ex^2)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acsch(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccsch(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x^2 + d)\,\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^3,x)
```

```
[Out] int(((d + e*x^2)*(a + b*asinh(1/(c*x))))/x^3, x)
```

3.88 $\int x^2(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

Optimal. Leaf size=260

$$\frac{b(280c^4d^2-252c^2de+75e^2)\,x^2\sqrt{-1-c^2x^2}}{1680c^5\sqrt{-c^2x^2}}+\frac{b(84c^2d-25e)\,ex^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}+\frac{be^2x^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{3}d^2x^3$$

```
[Out] 1/3*d^2*x^3*(a+b*arccsch(c*x))+2/5*d*e*x^5*(a+b*arccsch(c*x))+1/7*e^2*x^7*(
a+b*arccsch(c*x))+1/1680*b*(280*c^4*d^2-252*c^2*d*e+75*e^2)*x*arctan(c*x/(-
c^2*x^2-1)^(1/2))/c^6/(-c^2*x^2)^(1/2)+1/1680*b*(280*c^4*d^2-252*c^2*d*e+75
*e^2)*x^2*(-c^2*x^2-1)^(1/2)/c^5/(-c^2*x^2)^(1/2)+1/840*b*(84*c^2*d-25*e)*e
*x^4*(-c^2*x^2-1)^(1/2)/c^3/(-c^2*x^2)^(1/2)+1/42*b*e^2*x^6*(-c^2*x^2-1)^(1
/2)/c/(-c^2*x^2)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {276, 6437, 12, 1281, 470, 327, 223, 209}

$$\frac{1}{3}d^2x^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bx\text{ArcTan}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)(280c^4d^2-252c^2de+75e^2)}{1680c^6\sqrt{-c^2x^2}}+\frac{be^2x^6\sqrt{-c^2x^2-1}}{42c\sqrt{-c^2x^2}}+\frac{bex^4\sqrt{-c^2x^2-1}\,(84c^2d-25e)}{840c^3\sqrt{-c^2x^2}}+\frac{bx^2\sqrt{-c^2x^2-1}\,(280c^4d^2-252c^2de+75e^2)}{1680c^5\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(280*c^4*d^2 - 252*c^2*d*e + 75*e^2)*x^2*Sqrt[-1 - c^2*x^2])/(1680*c^5*S
qrt[-(c^2*x^2)]) + (b*(84*c^2*d - 25*e)*e*x^4*Sqrt[-1 - c^2*x^2])/(840*c^3*
Sqrt[-(c^2*x^2)]) + (b*e^2*x^6*Sqrt[-1 - c^2*x^2])/(42*c*Sqrt[-(c^2*x^2)])
+ (d^2*x^3*(a + b*ArcCsch[c*x]))/3 + (2*d*e*x^5*(a + b*ArcCsch[c*x]))/5 + (
e^2*x^7*(a + b*ArcCsch[c*x]))/7 + (b*(280*c^4*d^2 - 252*c^2*d*e + 75*e^2)*x
*ArcTan[(c*x)/Sqrt[-1 - c^2*x^2]])/(1680*c^6*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^2\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{1}{3}d^2x^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\right.$$

$$= \frac{1}{3}d^2x^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\right.$$

$$= \frac{be^2x^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{3}d^2x^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\text{csch}\right.$$

$$= \frac{b(84c^2d-25e)\,ex^4\sqrt{-1-c^2x^2}}{840c^3\sqrt{-c^2x^2}}+\frac{be^2x^6\sqrt{-1-c^2x^2}}{42c\sqrt{-c^2x^2}}+\frac{1}{3}d^2x^3\left(a\right.$$

$$= \frac{b\left(280c^4d^2-252c^2de+75e^2\right)x^2\sqrt{-1-c^2x^2}}{1680c^5\sqrt{-c^2x^2}}+\frac{b(84c^2d-25e)\,ex^4}{840c^3\sqrt{-}}$$

$$= \frac{b\left(280c^4d^2-252c^2de+75e^2\right)x^2\sqrt{-1-c^2x^2}}{1680c^5\sqrt{-c^2x^2}}+\frac{b(84c^2d-25e)\,ex^4}{840c^3\sqrt{-}}$$

$$= \frac{b\left(280c^4d^2-252c^2de+75e^2\right)x^2\sqrt{-1-c^2x^2}}{1680c^5\sqrt{-c^2x^2}}+\frac{b(84c^2d-25e)\,ex^4}{840c^3\sqrt{-}}$$

Mathematica [A]
time = 0.45, size = 182, normalized size = 0.70

$$\frac{c^2x^2\left(16ac^5x\left(35d^2+42dex^2+15e^2x^4\right)+b\sqrt{1+\frac{1}{c^2x^2}}\left(75e^2-2c^2e\left(126d+25ex^2\right)+8c^4\left(35d^2+21dex^2+5e^2x^4\right)\right)\right)+16bc^7x^3\left(35d^2+42dex^2+15e^2x^4\right)\text{csch}^{-1}(cx)+b\left(-280c^4d^2+252c^2de-75e^2\right)\log\left(\left(1+\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{1680c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (c^2*x^2*(16*a*c^5*x*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) + b*Sqrt[1 + 1/(c^2
*x^2)]*(75*e^2 - 2*c^2*e*(126*d + 25*e*x^2) + 8*c^4*(35*d^2 + 21*d*e*x^2 +
5*e^2*x^4))) + 16*b*c^7*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcCsch[c*x]
 + b*(-280*c^4*d^2 + 252*c^2*d*e - 75*e^2)*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*
x])/(1680*c^7)
```

Maple [A]
time = 0.46, size = 286, normalized size = 1.10

method	result

derivativedivides	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}dc^7ex^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\left(\frac{\operatorname{arccsch}(cx)d^2c^7x^3}{3}+\frac{2\operatorname{arccsch}(cx)dc^7ex^5}{5}+\frac{\operatorname{arccsch}(cx)e^2c^7x^7}{7}-\frac{\sqrt{c^2x^2+1}\,(-280d^2\cdots}{\cdots}\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{a\left(\frac{1}{3}d^2c^7x^3+\frac{2}{5}dc^7ex^5+\frac{1}{7}e^2c^7x^7\right)}{c^4}+\frac{b\left(\frac{\operatorname{arccsch}(cx)d^2c^7x^3}{3}+\frac{2\operatorname{arccsch}(cx)dc^7ex^5}{5}+\frac{\operatorname{arccsch}(cx)e^2c^7x^7}{7}-\frac{\sqrt{c^2x^2+1}\,(-280d^2\cdots}{\cdots}\right.}{\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^2*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(a/c^4*(1/3*d^2*c^7*x^3+2/5*d*c^7*e*x^5+1/7*e^2*c^7*x^7)+b/c^4*(1/3*a
rccsch(c*x)*d^2*c^7*x^3+2/5*arccsch(c*x)*d*c^7*e*x^5+1/7*arccsch(c*x)*e^2*c
^7*x^7-1/1680*(c^2*x^2+1)^(1/2)*(-280*d^2*c^5*x*(c^2*x^2+1)^(1/2)-168*d*c^5
*e*x^3*(c^2*x^2+1)^(1/2)-40*e^2*c^5*x^5*(c^2*x^2+1)^(1/2)+280*d^2*c^4*arcsi
nh(c*x)+252*d*c^3*e*x*(c^2*x^2+1)^(1/2)+50*e^2*c^3*x^3*(c^2*x^2+1)^(1/2)-25
2*d*c^2*e*arcsinh(c*x)-75*e^2*c*x*(c^2*x^2+1)^(1/2)+75*e^2*arcsinh(c*x))/((
c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A]
time = 0.27, size = 396, normalized size = 1.52

$$\frac{1}{7}ax^7e^2+\frac{2}{5}adx^5e+\frac{1}{3}ad^2x^3+\frac{1}{12}\left(4x^3\operatorname{arcsch}(cx)+\frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}-\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)bd^2+\frac{1}{40}\left(16x^5\operatorname{arcsch}(cx)-\frac{\frac{2\left(3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-5\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^4\left(\frac{1}{c^2x^2}+1\right)^2-2c^4\left(\frac{1}{c^2x^2}+1\right)+c^4}-\frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^4}+\frac{3\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^4}}{c}\right)bde+\frac{1}{672}\left(96x^7\operatorname{arcsch}(cx)+\frac{\frac{2\left(15\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-40\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+33\sqrt{\frac{1}{c^2x^2}+1}\right)}{c^6\left(\frac{1}{c^2x^2}+1\right)^3-3c^6\left(\frac{1}{c^2x^2}+1\right)^2+3c^6\left(\frac{1}{c^2x^2}+1\right)-c^6}-\frac{15\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^6}+\frac{15\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^6}}{c}\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*x^7*e^2 + 2/5*a*d*x^5*e + 1/3*a*d^2*x^3 + 1/12*(4*x^3*arccsch(c*x) +
(2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^
2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d^2 + 1/40*(16*
x^5*arccsch(c*x) - (2*(3*(1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(1/(c^2*x^2) + 1))
/(c^4*(1/(c^2*x^2) + 1)^2 - 2*c^4*(1/(c^2*x^2) + 1) + c^4) - 3*log(sqrt(1/(
c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*d*e + 1
/672*(96*x^7*arccsch(c*x) + (2*(15*(1/(c^2*x^2) + 1)^(5/2) - 40*(1/(c^2*x^2
) + 1)^(3/2) + 33*sqrt(1/(c^2*x^2) + 1))/(c^6*(1/(c^2*x^2) + 1)^3 - 3*c^6*(
1/(c^2*x^2) + 1)^2 + 3*c^6*(1/(c^2*x^2) + 1) - c^6) - 15*log(sqrt(1/(c^2*x^
2) + 1) + 1)/c^6 + 15*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^6)/c)*b*e^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 679 vs. 2(231) = 462.
time = 0.48, size = 679, normalized size = 2.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/1680*(240*a*c^7*x^7*cosh(1)^2 + 240*a*c^7*x^7*sinh(1)^2 + 672*a*c^7*d*x^5
*cosh(1) + 560*a*c^7*d^2*x^3 + 16*(35*b*c^7*d^2 + 42*b*c^7*d*cosh(1) + 15*b
*c^7*cosh(1)^2 + 15*b*c^7*sinh(1)^2 + 6*(7*b*c^7*d + 5*b*c^7*cosh(1))*sinh(
1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + (280*b*c^4*d^2 - 252
*b*c^2*d*cosh(1) + 75*b*cosh(1)^2 + 75*b*sinh(1)^2 - 6*(42*b*c^2*d - 25*b*c
osh(1))*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 16*(35*b*c^
7*d^2 + 42*b*c^7*d*cosh(1) + 15*b*c^7*cosh(1)^2 + 15*b*c^7*sinh(1)^2 + 6*(7
*b*c^7*d + 5*b*c^7*cosh(1))*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))
- c*x - 1) + 16*(35*b*c^7*d^2*x^3 - 35*b*c^7*d^2 + 15*(b*c^7*x^7 - b*c^7)*c
osh(1)^2 + 15*(b*c^7*x^7 - b*c^7)*sinh(1)^2 + 42*(b*c^7*d*x^5 - b*c^7*d)*co
sh(1) + 6*(7*b*c^7*d*x^5 - 7*b*c^7*d + 5*(b*c^7*x^7 - b*c^7)*cosh(1))*sinh(
1))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 96*(5*a*c^7*x^7*co
sh(1) + 7*a*c^7*d*x^5)*sinh(1) + (280*b*c^6*d^2*x^2 + 5*(8*b*c^6*x^6 - 10*b
*c^4*x^4 + 15*b*c^2*x^2)*cosh(1)^2 + 5*(8*b*c^6*x^6 - 10*b*c^4*x^4 + 15*b*c
^2*x^2)*sinh(1)^2 + 84*(2*b*c^6*d*x^4 - 3*b*c^4*d*x^2)*cosh(1) + 2*(84*b*c^
6*d*x^4 - 126*b*c^4*d*x^2 + 5*(8*b*c^6*x^6 - 10*b*c^4*x^4 + 15*b*c^2*x^2)*c
osh(1))*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a + b\,\mathrm{acsch}\,(cx))\left(d + ex^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)^2*(a + b*asinh(1/(c*x))),x)
[Out] int(x^2*(d + e*x^2)^2*(a + b*asinh(1/(c*x))), x)
```

3.89 $\int (d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

Optimal. Leaf size=197

$$\frac{b(40c^2d-9e)\,ex^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+d^2x\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}e^2$$

```
[Out] d^2*x*(a+b*arccsch(c*x))+2/3*d*e*x^3*(a+b*arccsch(c*x))+1/5*e^2*x^5*(a+b*ar
ccsch(c*x))-1/120*b*(120*c^4*d^2-40*c^2*d*e+9*e^2)*x*arctan(c*x/(-c^2*x^2-1
)^(1/2))/c^4/(-c^2*x^2)^(1/2)+1/120*b*(40*c^2*d-9*e)*e*x^2*(-c^2*x^2-1)^(1/
2)/c^3/(-c^2*x^2)^(1/2)+1/20*b*e^2*x^4*(-c^2*x^2-1)^(1/2)/c/(-c^2*x^2)^(1/2
)
```

Rubi [A]

time = 0.09, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {200, 6427, 12, 1173, 396, 223, 209}

$$d^2x\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\text{csch}^{-1}(cx)\right)-\frac{bx\text{ArcTan}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)(120c^4d^2-40c^2de+9e^2)}{120c^4\sqrt{-c^2x^2}}+\frac{be^2x^4\sqrt{-c^2x^2-1}}{20c\sqrt{-c^2x^2}}+\frac{bex^2\sqrt{-c^2x^2-1}\,(40c^2d-9e)}{120c^3\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(40*c^2*d - 9*e)*e*x^2*Sqrt[-1 - c^2*x^2])/(120*c^3*Sqrt[-(c^2*x^2)]) +
(b*e^2*x^4*Sqrt[-1 - c^2*x^2])/(20*c*Sqrt[-(c^2*x^2)]) + d^2*x*(a + b*ArcCs
ch[c*x]) + (2*d*e*x^3*(a + b*ArcCsch[c*x]))/3 + (e^2*x^5*(a + b*ArcCsch[c*x
]))/5 - (b*(120*c^4*d^2 - 40*c^2*d*e + 9*e^2)*x*ArcTan[(c*x)/Sqrt[-1 - c^2*
x^2]])/(120*c^4*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1173

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

Rule 6427

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x
] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2
*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p +
 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right) dx = d^2x\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\text{csch}^{-1}\right.$$

$$= d^2x\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\text{csch}^{-1}\right.$$

$$= \frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+d^2x\left(a+b\text{csch}^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\text{csch}^{-1}(cx\right.$$

$$= \frac{b(40c^2d-9e)\,ex^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+d^2x\left(a+b\text{csc}\right.$$

$$= \frac{b(40c^2d-9e)\,ex^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+d^2x\left(a+b\text{csc}\right.$$

$$= \frac{b(40c^2d-9e)\,ex^2\sqrt{-1-c^2x^2}}{120c^3\sqrt{-c^2x^2}}+\frac{be^2x^4\sqrt{-1-c^2x^2}}{20c\sqrt{-c^2x^2}}+d^2x\left(a+b\text{csc}\right.$$

Mathematica [A]
time = 0.13, size = 149, normalized size = 0.76

$$\frac{c^2x\left(8ac^3(15d^2+10dex^2+3e^2x^4)+be\sqrt{1+\frac{1}{c^2x^2}}\,x(-9e+c^2(40d+6ex^2))\right)+8bc^5x(15d^2+10dex^2+3e^2x^4)\,\text{csch}^{-1}(cx)+b(120c^4d^2-40c^2de+9e^2)\log\left(\left(1+\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{120c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (c^2*x*(8*a*c^3*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) + b*e*Sqrt[1 + 1/(c^2*x^2
)]*x*(-9*e + c^2*(40*d + 6*e*x^2))) + 8*b*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^
2*x^4)*ArcCsch[c*x] + b*(120*c^4*d^2 - 40*c^2*d*e + 9*e^2)*Log[(1 + Sqrt[1
+ 1/(c^2*x^2)])*x])/(120*c^5)
```

Maple [A]
time = 0.34, size = 217, normalized size = 1.10

method	result
derivativedivides	$\frac{\frac{a\left(d^2c^5x+\frac{2}{3}dc^5ex^3+\frac{1}{5}e^2c^5x^5\right)}{c^4}+\frac{b\left(\text{arccsch}(cx)d^2c^5x+\frac{2\,\text{arccsch}(cx)dc^5ex^3}{3}+\frac{\text{arccsch}(cx)e^2c^5x^5}{5}+\frac{\sqrt{c^2x^2+1}\,\left(120d^2c^4\,\text{arc}\right.}{}\right.}{}}{c}$

default	$\frac{a\left(d^{2} c^{5} x+\frac{2}{3} d\, c^{5} e\, x^{3}+\frac{1}{5} e^{2} c^{5} x^{5}\right)}{c^{4}}+\frac{b\left(\operatorname{arccsch}(cx) d^{2} c^{5} x+\frac{2 \operatorname{arccsch}(cx) d\, c^{5} e\, x^{3}}{3}+\frac{\operatorname{arccsch}(cx) e^{2} c^{5} x^{5}}{5}+\frac{\sqrt{c^{2} x^{2}+1}\,\left(120 d^{2} c^{4} \operatorname{arcsi}\right.}{}\right.}{c}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^2*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)

[Out] 1/c*(a/c^4*(d^2*c^5*x+2/3*d*c^5*e*x^3+1/5*e^2*c^5*x^5)+b/c^4*(arccsch(c*x)*d^2*c^5*x+2/3*arccsch(c*x)*d*c^5*e*x^3+1/5*arccsch(c*x)*e^2*c^5*x^5+1/120*(c^2*x^2+1)^(1/2)*(120*d^2*c^4*arcsinh(c*x)+40*d*c^3*e*x*(c^2*x^2+1)^(1/2)+6*e^2*c^3*x^3*(c^2*x^2+1)^(1/2)-40*d*c^2*e*arcsinh(c*x)-9*e^2*c*x*(c^2*x^2+1)^(1/2)+9*e^2*arcsinh(c*x))/c/x/((c^2*x^2+1)/c^2/x^2)^(1/2)))

Maxima [A]
time = 0.26, size = 287, normalized size = 1.46

$$\frac{1}{5} a x^{5} e^{2}+\frac{2}{3} a d x^{3} e+a d^{2} x+\frac{1}{6}\left(4 x^{3} \operatorname{arcsch}(cx)+\frac{\frac{2 \sqrt{\frac{1}{c^{2} x^{2}}+1}}{c^{2}\left(\frac{1}{c^{2} x^{2}}+1\right)-c^{2}}-\frac{\log \left(\sqrt{\frac{1}{c^{2} x^{2}}+1}+1\right)}{c^{2}}+\frac{\log \left(\sqrt{\frac{1}{c^{2} x^{2}}+1}-1\right)}{c^{2}}}{c}\right) b d e+\frac{\left(2 c x \operatorname{arcsch}(cx)+\log \left(\sqrt{\frac{1}{c^{2} x^{2}}+1}+1\right)-\log \left(\sqrt{\frac{1}{c^{2} x^{2}}+1}-1\right)\right) b d^{2}}{2 c}+\frac{1}{80}\left(16 x^{5} \operatorname{arcsch}(cx)-\frac{\frac{2\left(3\left(\frac{1}{c^{2} x^{2}}+1\right)^{\frac{3}{2}}-5 \sqrt{\frac{1}{c^{2} x^{2}}+1}\right)}{c^{4}\left(\frac{1}{c^{2} x^{2}}+1\right)^{2}-2 c^{4}\left(\frac{1}{c^{2} x^{2}}+1\right)+c^{4}}-\frac{3 \log \left(\sqrt{\frac{1}{c^{2} x^{2}}+1}+1\right)}{c^{4}}+\frac{3 \log \left(\sqrt{\frac{1}{c^{2} x^{2}}+1}-1\right)}{c^{4}}}{c}\right) b e^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")

[Out] 1/5*a*x^5*e^2 + 2/3*a*d*x^3*e + a*d^2*x + 1/6*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) + 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2) + 1) - 1)/c^2)/c)*b*d*e + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sqrt(1/(c^2*x^2) + 1) - 1))*b*d^2/c + 1/80*(16*x^5*arccsch(c*x) - (2*(3*(1/(c^2*x^2) + 1)^(3/2) - 5*sqrt(1/(c^2*x^2) + 1))/(c^4*(1/(c^2*x^2) + 1)^2 - 2*c^4*(1/(c^2*x^2) + 1) + c^4) - 3*log(sqrt(1/(c^2*x^2) + 1) + 1)/c^4 + 3*log(sqrt(1/(c^2*x^2) + 1) - 1)/c^4)/c)*b*e^2

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(176) = 352$.
time = 0.46, size = 614, normalized size = 3.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")

[Out] 1/120*(24*a*c^5*x^5*cosh(1)^2 + 24*a*c^5*x^5*sinh(1)^2 + 80*a*c^5*d*x^3*cosh(1) + 120*a*c^5*d^2*x + 8*(15*b*c^5*d^2 + 10*b*c^5*d*cosh(1) + 3*b*c^5*cosh(1)^2 + 3*b*c^5*sinh(1)^2 + 2*(5*b*c^5*d + 3*b*c^5*cosh(1))*sinh(1))*log(c

```
*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - (120*b*c^4*d^2 - 40*b*c^2*d*c
osh(1) + 9*b*cosh(1)^2 + 9*b*sinh(1)^2 - 2*(20*b*c^2*d - 9*b*cosh(1))*sinh(
1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) - 8*(15*b*c^5*d^2 + 10*b*c
^5*d*cosh(1) + 3*b*c^5*cosh(1)^2 + 3*b*c^5*sinh(1)^2 + 2*(5*b*c^5*d + 3*b*c
^5*cosh(1))*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 8*(
15*b*c^5*d^2*x - 15*b*c^5*d^2 + 3*(b*c^5*x^5 - b*c^5)*cosh(1)^2 + 3*(b*c^5*
x^5 - b*c^5)*sinh(1)^2 + 10*(b*c^5*d*x^3 - b*c^5*d)*cosh(1) + 2*(5*b*c^5*d*
x^3 - 5*b*c^5*d + 3*(b*c^5*x^5 - b*c^5)*cosh(1))*sinh(1))*log((c*x*sqrt((c^
2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 16*(3*a*c^5*x^5*cosh(1) + 5*a*c^5*d*x^3
)*sinh(1) + (40*b*c^4*d*x^2*cosh(1) + 3*(2*b*c^4*x^4 - 3*b*c^2*x^2)*cosh(1)
^2 + 3*(2*b*c^4*x^4 - 3*b*c^2*x^2)*sinh(1)^2 + 2*(20*b*c^4*d*x^2 + 3*(2*b*c
^4*x^4 - 3*b*c^2*x^2)*cosh(1))*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\operatorname{acsch}(cx))\left(d + ex^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x)),x)
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(e\,x^2 + d\right)^2\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2*(a + b*asinh(1/(c*x))),x)
[Out] int((d + e*x^2)^2*(a + b*asinh(1/(c*x))), x)
```

3.90 $\int \frac{(d+ex^2)^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=170

$$\frac{bcd^2\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}}+\frac{be^2x^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\mathrm{csch}^{-1}(cx)\right)}{x}+2dex\left(a+b\mathrm{csch}^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\mathrm{csch}^{-1}(c\right.$$

```
[Out] -d^2*(a+b*arccsch(c*x))/x+2*d*e*x*(a+b*arccsch(c*x))+1/3*e^2*x^3*(a+b*arccs
ch(c*x))-1/6*b*(12*c^2*d-e)*e*x*arctan(c*x/(-c^2*x^2-1)^(1/2))/c^2/(-c^2*x^
2)^(1/2)+b*c*d^2*(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/6*b*e^2*x^2*(-c^2*x^
2-1)^(1/2)/c/(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {276, 6437, 12, 1279, 396, 223, 209}

$$-\frac{d^2\left(a+b\mathrm{csch}^{-1}(cx)\right)}{x}+2dex\left(a+b\mathrm{csch}^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\mathrm{csch}^{-1}(cx)\right)-\frac{bex\mathrm{ArcTan}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)(12c^2d-e)}{6c^2\sqrt{-c^2x^2}}+\frac{bcd^2\sqrt{-c^2x^2-1}}{\sqrt{-c^2x^2}}+\frac{be^2x^2\sqrt{-c^2x^2-1}}{6c\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] (b*c*d^2*Sqrt[-1 - c^2*x^2])/Sqrt[-(c^2*x^2)] + (b*e^2*x^2*Sqrt[-1 - c^2*x^
2])/(6*c*Sqrt[-(c^2*x^2)]) - (d^2*(a + b*ArcCsch[c*x]))/x + 2*d*e*x*(a + b*
ArcCsch[c*x]) + (e^2*x^3*(a + b*ArcCsch[c*x]))/3 - (b*(12*c^2*d - e)*e*x*Ar
cTan[(c*x)/Sqrt[-1 - c^2*x^2]])/(6*c^2*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^2 (a+b\text{csch}^{-1}(cx))}{x^2} dx = -\frac{d^2(a+b\text{csch}^{-1}(cx))}{x} + 2dex(a+b\text{csch}^{-1}(cx)) + \frac{1}{3}e^2x^3(a+b\text{csch}^{-1}($$

$$= -\frac{d^2(a+b\text{csch}^{-1}(cx))}{x} + 2dex(a+b\text{csch}^{-1}(cx)) + \frac{1}{3}e^2x^3(a+b\text{csch}^{-1}($$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} - \frac{d^2(a+b\text{csch}^{-1}(cx))}{x} + 2dex(a+b\text{csch}^{-1}(cx)) +$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} + \frac{be^2x^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} - \frac{d^2(a+b\text{csch}^{-1}(cx))}{x} + 2de$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} + \frac{be^2x^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} - \frac{d^2(a+b\text{csch}^{-1}(cx))}{x} + 2de$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{\sqrt{-c^2x^2}} + \frac{be^2x^2\sqrt{-1-c^2x^2}}{6c\sqrt{-c^2x^2}} - \frac{d^2(a+b\text{csch}^{-1}(cx))}{x} + 2de$$

Mathematica [A]
time = 0.13, size = 134, normalized size = 0.79

$$\frac{c^2\left(b\sqrt{1+\frac{1}{c^2x^2}}\,x(6c^2d^2+e^2x^2)+2ac(-3d^2+6dex^2+e^2x^4)\right)+2bc^3(-3d^2+6dex^2+e^2x^4)\operatorname{csch}^{-1}(cx)+b(12c^2d-e)ex\log\left(\left(1+\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{6c^3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^2,x]
```

```
[Out] (c^2*(b*Sqrt[1 + 1/(c^2*x^2)]*x*(6*c^2*d^2 + e^2*x^2) + 2*a*c*(-3*d^2 + 6*d
*e*x^2 + e^2*x^4)) + 2*b*c^3*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*ArcCsch[c*x] +
b*(12*c^2*d - e)*e*x*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/(6*c^3*x)
```

Maple [A]
time = 0.34, size = 189, normalized size = 1.11

method	result
derivativedivides	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{b\left(2\,\text{arccsch}(cx)c^3dex+\frac{e^2\,\text{arccsch}(cx)c^3x^3}{3}-\frac{\text{arccsch}(cx)c^3d^2}{x}+\frac{\sqrt{c^2x^2+1}\,\left(6c^4d^2\sqrt{\right.}{}}{}\right.}{c^4}\right.$

default	$c\left(\frac{a\left(2c^3dex+\frac{e^2c^3x^3}{3}-\frac{c^3d^2}{x}\right)}{c^4}+\frac{b\left(2\,\mathrm{arccsch}(cx)c^3dex+\frac{e^2\mathrm{arccsch}(cx)c^3x^3}{3}-\frac{\mathrm{arccsch}(cx)c^3d^2}{x}+\frac{\sqrt{c^2x^2+1}\,\left(6c^4d^2\right.}{}\right.}{c^4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*(a/c^4*(2*c^3*d*e*x+1/3*e^2*c^3*x^3-c^3*d^2/x)+b/c^4*(2*arccsch(c*x)*c^3*
d*e*x+1/3*e^2*arccsch(c*x)*c^3*x^3-arccsch(c*x)*c^3*d^2/x+1/6*(c^2*x^2+1)^(
1/2)*(6*c^4*d^2*(c^2*x^2+1)^(1/2)+12*c^3*d*e*arcsinh(c*x)*x+e^2*c^2*x^2*(c^
2*x^2+1)^(1/2)-arcsinh(c*x)*e^2*c*x)/c^2/x^2/((c^2*x^2+1)/c^2/x^2)^(1/2)))
```

Maxima [A]
time = 0.26, size = 191, normalized size = 1.12

$$\frac{1}{3}\,ax^3e^2+\left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsch}(cx)}{x}\right)bd^2+2\,adxe+\frac{1}{12}\left(4\,x^3\operatorname{arcsch}(cx)+\frac{\frac{2\sqrt{\frac{1}{c^2x^2}+1}}{c^2\left(\frac{1}{c^2x^2}+1\right)-c^2}-\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)}{c^2}+\frac{\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)}{c^2}}{c}\right)be^2+\frac{\left(2\,cx\operatorname{arcsch}(cx)+\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)-\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)bde}{c}-\frac{ad^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3*e^2 + (c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*d^2 + 2*a*d*x*
e + 1/12*(4*x^3*arccsch(c*x) + (2*sqrt(1/(c^2*x^2) + 1)/(c^2*(1/(c^2*x^2) +
 1) - c^2) - log(sqrt(1/(c^2*x^2) + 1) + 1)/c^2 + log(sqrt(1/(c^2*x^2) + 1)
 - 1)/c^2)/c)*b*e^2 + (2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1)
- log(sqrt(1/(c^2*x^2) + 1) - 1))*b*d*e/c - a*d^2/x
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(153) = 306.
time = 0.41, size = 580, normalized size = 3.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*c^3*x^4*cosh(1)^2 + 2*a*c^3*x^4*sinh(1)^2 + 6*b*c^4*d^2*x + 12*a*c
^3*d*x^2*cosh(1) - 6*a*c^3*d^2 - 2*(3*b*c^3*d^2*x - 6*b*c^3*d*x*cosh(1) - b
*c^3*x*cosh(1)^2 - b*c^3*x*sinh(1)^2 - 2*(3*b*c^3*d*x + b*c^3*x*cosh(1))*si
nh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) - (12*b*c^2*d*x*cos
h(1) - b*x*cosh(1)^2 - b*x*sinh(1)^2 + 2*(6*b*c^2*d*x - b*x*cosh(1))*sinh(1
```

```
))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x) + 2*(3*b*c^3*d^2*x - 6*b*c^
3*d*x*cosh(1) - b*c^3*x*cosh(1)^2 - b*c^3*x*sinh(1)^2 - 2*(3*b*c^3*d*x + b*
c^3*x*cosh(1))*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) +
2*(3*b*c^3*d^2*x - 3*b*c^3*d^2 + (b*c^3*x^4 - b*c^3*x)*cosh(1)^2 + (b*c^3*x
^4 - b*c^3*x)*sinh(1)^2 + 6*(b*c^3*d*x^2 - b*c^3*d*x)*cosh(1) + 2*(3*b*c^3*
d*x^2 - 3*b*c^3*d*x + (b*c^3*x^4 - b*c^3*x)*cosh(1))*sinh(1))*log((c*x*sqrt
((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(a*c^3*x^4*cosh(1) + 3*a*c^3*d*x^
2)*sinh(1) + (6*b*c^4*d^2*x + b*c^2*x^3*cosh(1)^2 + 2*b*c^2*x^3*cosh(1)*sin
h(1) + b*c^2*x^3*sinh(1)^2)*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(c^3*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsch}(cx))\left(d + ex^2\right)^2}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**2,x)
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^2,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2 + d\right)^2\,\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^2,x)
[Out] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^2, x)
```

3.91 $\int \frac{(d+ex^2)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=164

$$-\frac{2bcd(c^2d-9e)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}}+\frac{bcd^2\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{x}+e^2x(a+$$

```
[Out] -1/3*d^2*(a+b*arccsch(c*x))/x^3-2*d*e*(a+b*arccsch(c*x))/x+e^2*x*(a+b*arccs
ch(c*x))-b*e^2*x*arctan(c*x/(-c^2*x^2-1)^(1/2))/(-c^2*x^2)^(1/2)-2/9*b*c*d*
(c^2*d-9*e)*(-c^2*x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/9*b*c*d^2*(-c^2*x^2-1)^(1
/2)/x^2/(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {276, 6437, 12, 1279, 462, 223, 209}

$$-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{x}+e^2x\left(a+b\text{csch}^{-1}(cx)\right)-\frac{be^2x\text{ArcTan}\left(\frac{cx}{\sqrt{-c^2x^2-1}}\right)}{\sqrt{-c^2x^2}}+\frac{bcd^2\sqrt{-c^2x^2-1}}{9x^2\sqrt{-c^2x^2}}-\frac{2bcd\sqrt{-c^2x^2-1}\,(c^2d-9e)}{9\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] (-2*b*c*d*(c^2*d - 9*e)*Sqrt[-1 - c^2*x^2])/(9*Sqrt[-(c^2*x^2)]) + (b*c*d^2
*Sqrt[-1 - c^2*x^2])/(9*x^2*Sqrt[-(c^2*x^2)]) - (d^2*(a + b*ArcCsch[c*x]))/
(3*x^3) - (2*d*e*(a + b*ArcCsch[c*x]))/x + e^2*x*(a + b*ArcCsch[c*x]) - (b*
e^2*x*ArcTan[(c*x)/Sqrt[-1 - c^2*x^2]])/Sqrt[-(c^2*x^2)]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 462

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right)}{x^4} dx = -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{x} + e^2x\left(a+b\text{csch}^{-1}(c\right.$$

$$= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{x} + e^2x\left(a+b\text{csch}^{-1}(c\right.$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{x} +$$

$$= -\frac{2bcd(c^2d-9e)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}\right.}{3x^3}$$

$$= -\frac{2bcd(c^2d-9e)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}\right.}{3x^3}$$

$$= -\frac{2bcd(c^2d-9e)\sqrt{-1-c^2x^2}}{9\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-1-c^2x^2}}{9x^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}\right.}{3x^3}$$

Mathematica [A]
time = 0.16, size = 123, normalized size = 0.75

$$\frac{bcd\sqrt{1+\frac{1}{c^2x^2}}\,x(d-2c^2dx^2+18ex^2)-3a(d^2+6dex^2-3e^2x^4)}{9x^3} - \frac{b(d^2+6dex^2-3e^2x^4)\,\text{csch}^{-1}(cx)}{3x^3} + \frac{be^2\log\left(\left(1+\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] (b*c*d*Sqrt[1 + 1/(c^2*x^2)]*x*(d - 2*c^2*d*x^2 + 18*e*x^2) - 3*a*(d^2 + 6*
d*e*x^2 - 3*e^2*x^4))/(9*x^3) - (b*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*ArcCsch[c*
x])/(3*x^3) + (b*e^2*Log[(1 + Sqrt[1 + 1/(c^2*x^2)])*x])/c
```

Maple [A]
time = 0.32, size = 190, normalized size = 1.16

method	result

derivativedivides	$c^3\left(\frac{a\left(e^2cx-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\left(\operatorname{arccsch}(cx)e^2cx-\frac{\operatorname{arccsch}(cx)c\,d^2}{3x^3}-\frac{2\operatorname{arccsch}(cx)cde}{x}+\frac{\sqrt{c^2x^2+1}\,\left(-2\sqrt{c^2x^2+1}\right.}{}\right.}{c^4}\right.$
default	$c^3\left(\frac{a\left(e^2cx-\frac{c\,d^2}{3x^3}-\frac{2cde}{x}\right)}{c^4}+\frac{b\left(\operatorname{arccsch}(cx)e^2cx-\frac{\operatorname{arccsch}(cx)c\,d^2}{3x^3}-\frac{2\operatorname{arccsch}(cx)cde}{x}+\frac{\sqrt{c^2x^2+1}\,\left(-2\sqrt{c^2x^2+1}\right.}{}\right.}{c^4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] c^3*(a/c^4*(e^2*c*x-1/3*c*d^2/x^3-2*c*d*e/x)+b/c^4*(arccsch(c*x)*e^2*c*x-1/
3*arccsch(c*x)*c*d^2/x^3-2*arccsch(c*x)*c*d*e/x+1/9*(c^2*x^2+1)^(1/2)*(-2*(
c^2*x^2+1)^(1/2)*c^6*d^2*x^2+c^4*d^2*(c^2*x^2+1)^(1/2)+18*c^4*d*e*(c^2*x^2+
1)^(1/2)*x^2+9*e^2*arcsinh(c*x)*c^3*x^3)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^4/x^
4))
```

Maxima [A]
time = 0.27, size = 152, normalized size = 0.93

$$\frac{1}{9}\,bd^2\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3\,c^4\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{3\,\operatorname{arcsch}(cx)}{x^3}\right)+2\left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsch}(cx)}{x}\right)bde+axe^2+\frac{\left(2\,cx\,\operatorname{arcsch}(cx)+\log\left(\sqrt{\frac{1}{c^2x^2}+1}+1\right)-\log\left(\sqrt{\frac{1}{c^2x^2}+1}-1\right)\right)be^2}{2\,c}-\frac{2\,ade}{x}-\frac{ad^2}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/9*b*d^2*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c -
3*arccsch(c*x)/x^3) + 2*(c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*d*e +
a*x*e^2 + 1/2*(2*c*x*arccsch(c*x) + log(sqrt(1/(c^2*x^2) + 1) + 1) - log(sq
rt(1/(c^2*x^2) + 1) - 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^2/x^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. 2(146) = 292.
time = 0.49, size = 559, normalized size = 3.41

$$-\frac{2\,bc^4d^2x^3-9\,acx^4\cosh(1)^2-9\,acx^4\sinh(1)^2+3\,acd^2-18\,(bc^2dx^3-acdx^2)\cosh(1)+3\,(bcd^2x^3+6\,bcdx^3\cosh(1)-3\,bcx^3\cosh(1)^2-3\,bcx^3\sinh(1)^2+6\,(bcdx^3-bcx^3\cosh(1))\sinh(1))\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx+1\right)+9\,(bx^3\cosh(1)^2+2\,bx^3\cosh(1)\sinh(1)+bx^3\sinh(1)^2)\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx\right)-3\,(bcd^2x^3+6\,bcdx^3\cosh(1)-3\,bcx^3\cosh(1)^2-3\,bcx^3\sinh(1)^2+6\,(bcdx^3-bcx^3\cosh(1))\sinh(1))\log\left(cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}-cx-1\right)-3\,(bcd^2x^3-bcd^2+3\,(bcx^4-bcx^3)\cosh(1)^2+3\,(bcx^4-bcx^3)\sinh(1)^2+6\,(bcdx^3-bcdx^2)\cosh(1)+6\,(bcdx^3-bcdx^2+(bcx^4-bcx^3)\cosh(1))\sinh(1))\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)-18\,(bc^2dx^3+acx^4\cosh(1)-acdx^2)\sinh(1)+(2\,bc^4d^2x^3-18\,bc^2dx^3\cosh(1)-18\,bc^2dx^3\sinh(1)-bc^2d^2x)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{9\,cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/9*(2*b*c^4*d^2*x^3 - 9*a*c*x^4*cosh(1)^2 - 9*a*c*x^4*sinh(1)^2 + 3*a*c*d
^2 - 18*(b*c^2*d*x^3 - a*c*d*x^2)*cosh(1) + 3*(b*c*d^2*x^3 + 6*b*c*d*x^3*co
sh(1) - 3*b*c*x^3*cosh(1)^2 - 3*b*c*x^3*sinh(1)^2 + 6*(b*c*d*x^3 - b*c*x^3*
cosh(1))*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 9*(b*x
^3*cosh(1)^2 + 2*b*x^3*cosh(1)*sinh(1) + b*x^3*sinh(1)^2)*log(c*x*sqrt((c^2
*x^2 + 1)/(c^2*x^2)) - c*x) - 3*(b*c*d^2*x^3 + 6*b*c*d*x^3*cosh(1) - 3*b*c*
x^3*cosh(1)^2 - 3*b*c*x^3*sinh(1)^2 + 6*(b*c*d*x^3 - b*c*x^3*cosh(1))*sinh(
1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) - 3*(b*c*d^2*x^3 - b*c
*d^2 + 3*(b*c*x^4 - b*c*x^3)*cosh(1)^2 + 3*(b*c*x^4 - b*c*x^3)*sinh(1)^2 +
6*(b*c*d*x^3 - b*c*d*x^2)*cosh(1) + 6*(b*c*d*x^3 - b*c*d*x^2 + (b*c*x^4 - b
*c*x^3)*cosh(1))*sinh(1))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)
) - 18*(b*c^2*d*x^3 + a*c*x^4*cosh(1) - a*c*d*x^2)*sinh(1) + (2*b*c^4*d^2*x
^3 - 18*b*c^2*d*x^3*cosh(1) - 18*b*c^2*d*x^3*sinh(1) - b*c^2*d^2*x)*sqrt((c
^2*x^2 + 1)/(c^2*x^2)))/(c*x^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsch}\,(cx))\,(d + ex^2)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x^2 + d)^2\,\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^4,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^4, x)
```

3.92 $\int \frac{(d+ex^2)^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=189

$$\frac{bc(24c^4d^2-100c^2de+225e^2)\sqrt{-1-c^2x^2}}{225\sqrt{-c^2x^2}}+\frac{bcd^2\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}}-\frac{2bcd(6c^2d-25e)\sqrt{-1-c^2x^2}}{225x^2\sqrt{-c^2x^2}}-\frac{d^2\left(a+bcs\right.}{5x}$$

```
[Out] -1/5*d^2*(a+b*arccsch(c*x))/x^5-2/3*d*e*(a+b*arccsch(c*x))/x^3-e^2*(a+b*arc
csch(c*x))/x+1/225*b*c*(24*c^4*d^2-100*c^2*d*e+225*e^2)*(-c^2*x^2-1)^(1/2)/
(-c^2*x^2)^(1/2)+1/25*b*c*d^2*(-c^2*x^2-1)^(1/2)/x^4/(-c^2*x^2)^(1/2)-2/225
*b*c*d*(6*c^2*d-25*e)*(-c^2*x^2-1)^(1/2)/x^2/(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {276, 6437, 12, 1279, 464, 270}

$$-\frac{d^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{5x^5}-\frac{2de\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3x^3}-\frac{e^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x}+\frac{bcd^2\sqrt{-c^2x^2-1}}{25x^4\sqrt{-c^2x^2}}-\frac{2bcd\sqrt{-c^2x^2-1}\,(6c^2d-25e)}{225x^2\sqrt{-c^2x^2}}+\frac{bc\sqrt{-c^2x^2-1}\,(24c^4d^2-100c^2de+225e^2)}{225\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (b*c*(24*c^4*d^2 - 100*c^2*d*e + 225*e^2)*Sqrt[-1 - c^2*x^2])/(225*Sqrt[-(c
^2*x^2)]) + (b*c*d^2*Sqrt[-1 - c^2*x^2])/(25*x^4*Sqrt[-(c^2*x^2)]) - (2*b*c
*d*(6*c^2*d - 25*e)*Sqrt[-1 - c^2*x^2])/(225*x^2*Sqrt[-(c^2*x^2)]) - (d^2*(
a + b*ArcCsch[c*x]))/(5*x^5) - (2*d*e*(a + b*ArcCsch[c*x]))/(3*x^3) - (e^2*
(a + b*ArcCsch[c*x]))/x
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx = -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} - \frac{e^2\left(a+b\text{csch}^{-1}(cx\right.}{x}$$

$$= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} - \frac{e^2\left(a+b\text{csch}^{-1}(cx\right.}{x}$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5} - \frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{3x^3} -$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{2bcd(6c^2d-25e)\sqrt{-1-c^2x^2}}{225x^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\text{csch}^{-}\right.}{5x^5}$$

$$= \frac{bc(24c^4d^2-100c^2de+225e^2)\sqrt{-1-c^2x^2}}{225\sqrt{-c^2x^2}} + \frac{bcd^2\sqrt{-1-c^2x^2}}{25x^4\sqrt{-c^2x^2}} - \frac{2}{}$$

Mathematica [A]

time = 0.13, size = 126, normalized size = 0.67

$$\frac{-15a(3d^2+10dex^2+15e^2x^4)+bc\sqrt{1+\frac{1}{c^2x^2}}\,x(225e^2x^4-50dex^2(-1+2c^2x^2)+3d^2(3-4c^2x^2+8c^4x^4))-15b(3d^2+10dex^2+15e^2x^4)\operatorname{csch}^{-1}(cx)}{225x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (-15*a*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(225
*e^2*x^4 - 50*d*e*x^2*(-1 + 2*c^2*x^2) + 3*d^2*(3 - 4*c^2*x^2 + 8*c^4*x^4))
 - 15*b*(3*d^2 + 10*d*e*x^2 + 15*e^2*x^4)*ArcCsch[c*x])/(225*x^5)
```

Maple [A]

time = 0.33, size = 191, normalized size = 1.01

method	result
derivativedivides	$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{2de}{3c\,x^3}-\frac{d^2}{5c\,x^5}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)e^2}{cx}-\frac{2\operatorname{arccsch}(cx)de}{3c\,x^3}-\frac{\operatorname{arccsch}(cx)d^2}{5c\,x^5}+\frac{(c^2x^2+1)\left(24c^8d^2x^4-100c^6de\,x^4-12c^6 \ldots\right.}{225\sqrt{\frac{c^2x^2 \ldots}{c^2x \ldots}}}\right.}{c^4}\right.$
default	$c^5\left(\frac{a\left(-\frac{e^2}{cx}-\frac{2de}{3c\,x^3}-\frac{d^2}{5c\,x^5}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)e^2}{cx}-\frac{2\operatorname{arccsch}(cx)de}{3c\,x^3}-\frac{\operatorname{arccsch}(cx)d^2}{5c\,x^5}+\frac{(c^2x^2+1)\left(24c^8d^2x^4-100c^6de\,x^4-12c^6 \ldots\right.}{225\sqrt{\frac{c^2x^2 \ldots}{c^2x \ldots}}}\right.}{c^4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] c^5*(a/c^4*(-e^2/c/x-2/3/c*d*e/x^3-1/5/c*d^2/x^5)+b/c^4*(-arccsch(c*x)*e^2/
c/x-2/3*arccsch(c*x)/c*d*e/x^3-1/5*arccsch(c*x)/c*d^2/x^5+1/225*(c^2*x^2+1)
*(24*c^8*d^2*x^4-100*c^6*d*e*x^4-12*c^6*d^2*x^2+225*c^4*e^2*x^4+50*c^4*d*e*
x^2+9*c^4*d^2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^6/x^6))
```

Maxima [A]

time = 0.26, size = 175, normalized size = 0.93

$$\frac{1}{75}bd^2\left(\frac{3c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15c^6\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{15\operatorname{arcsch}(cx)}{x^5}\right)+\frac{2}{9}bd\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3c^4\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{3\operatorname{arcsch}(cx)}{x^3}\right)e+\left(c\sqrt{\frac{1}{c^2x^2}+1}-\frac{\operatorname{arcsch}(cx)}{x}\right)be^2-\frac{ae^2}{x}-\frac{2ade}{3x^3}-\frac{ad^2}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x, algorithm="maxima")

[Out] 1/75*b*d^2*((3*c^6*(1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(1/(c^2*x^2) + 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) + 1))/c - 15*arccsch(c*x)/x^5) + 2/9*b*d*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c - 3*arccsch(c*x)/x^ 3)*e + (c*sqrt(1/(c^2*x^2) + 1) - arccsch(c*x)/x)*b*e^2 - a*e^2/x - 2/3*a*d *e/x^3 - 1/5*a*d^2/x^5

Fricas [A]
time = 0.42, size = 278, normalized size = 1.47

$$-\frac{225\,ax^4\cosh(1)^2 + 225\,ax^4\sinh(1)^2 + 150\,adx^2\cosh(1) + 45\,ad^2 + 15\,(15\,bx^4\cosh(1)^2 + 15\,bx^4\sinh(1)^2 + 10\,bdx^2\cosh(1) + 3\,bd^2 + 10\,(3\,bx^4\cosh(1) + bdx^2)\sinh(1))\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + 150\,(3\,ax^4\cosh(1) + adx^2)\sinh(1) - (24\,bc^5d^2x^5 - 12\,bc^3d^2x^3 + 225\,bcx^5\cosh(1)^2 + 225\,bcx^5\sinh(1)^2 + 9\,bcd^2x - 50\,(2\,bc^3dx^5 - bcdx^3)\cosh(1) - 50\,(2\,bc^3dx^5 - 9\,bcx^5\cosh(1) - bcdx^3)\sinh(1))\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{225\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x, algorithm="fricas")

[Out] -1/225*(225*a*x^4*cosh(1)^2 + 225*a*x^4*sinh(1)^2 + 150*a*d*x^2*cosh(1) + 4 5*a*d^2 + 15*(15*b*x^4*cosh(1)^2 + 15*b*x^4*sinh(1)^2 + 10*b*d*x^2*cosh(1) + 3*b*d^2 + 10*(3*b*x^4*cosh(1) + b*d*x^2)*sinh(1))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 150*(3*a*x^4*cosh(1) + a*d*x^2)*sinh(1) - (24 *b*c^5*d^2*x^5 - 12*b*c^3*d^2*x^3 + 225*b*c*x^5*cosh(1)^2 + 225*b*c*x^5*sin h(1)^2 + 9*b*c*d^2*x - 50*(2*b*c^3*d*x^5 - b*c*d*x^3)*cosh(1) - 50*(2*b*c^3 *d*x^5 - 9*b*c*x^5*cosh(1) - b*c*d*x^3)*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^ 2)))/x^5

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsch}(cx))\,(d + ex^2)^2}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**6,x)

[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**6, x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^6,x, algorithm="giac")

[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^6, x)

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,x^2+d\right)^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^6} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^6,x)
[Out] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^6, x)
```

3.93 $\int \frac{(d+ex^2)^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^8}\,dx$

Optimal. Leaf size=249

$$-\frac{2bc^3(360c^4d^2-1176c^2de+1225e^2)\sqrt{-1-c^2x^2}}{11025\sqrt{-c^2x^2}}+\frac{bcd^2\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}}-\frac{2bcd(15c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}}+$$

```
[Out] -1/7*d^2*(a+b*arccsch(c*x))/x^7-2/5*d*e*(a+b*arccsch(c*x))/x^5-1/3*e^2*(a+b
*arccsch(c*x))/x^3-2/11025*b*c^3*(360*c^4*d^2-1176*c^2*d*e+1225*e^2)*(-c^2*
x^2-1)^(1/2)/(-c^2*x^2)^(1/2)+1/49*b*c*d^2*(-c^2*x^2-1)^(1/2)/x^6/(-c^2*x^2
)^(1/2)-2/1225*b*c*d*(15*c^2*d-49*e)*(-c^2*x^2-1)^(1/2)/x^4/(-c^2*x^2)^(1/2
)+1/11025*b*c*(360*c^4*d^2-1176*c^2*d*e+1225*e^2)*(-c^2*x^2-1)^(1/2)/x^2/(-
c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {276, 6437, 12, 1279, 464, 277, 270}

$$-\frac{d^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3x^3}+\frac{bcd^2\sqrt{-c^2x^2-1}}{49x^6\sqrt{-c^2x^2}}-\frac{2bcd\sqrt{-c^2x^2-1}\,(15c^2d-49e)}{1225x^4\sqrt{-c^2x^2}}+\frac{bc\sqrt{-c^2x^2-1}\,(360c^4d^2-1176c^2de+1225e^2)}{11025x^2\sqrt{-c^2x^2}}-\frac{2bc^3\sqrt{-c^2x^2-1}\,(360c^4d^2-1176c^2de+1225e^2)}{11025\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^8,x]
```

```
[Out] (-2*b*c^3*(360*c^4*d^2 - 1176*c^2*d*e + 1225*e^2)*Sqrt[-1 - c^2*x^2])/(1102
5*Sqrt[-(c^2*x^2)]) + (b*c*d^2*Sqrt[-1 - c^2*x^2])/(49*x^6*Sqrt[-(c^2*x^2)]
) - (2*b*c*d*(15*c^2*d - 49*e)*Sqrt[-1 - c^2*x^2])/(1225*x^4*Sqrt[-(c^2*x^2
)]) + (b*c*(360*c^4*d^2 - 1176*c^2*d*e + 1225*e^2)*Sqrt[-1 - c^2*x^2])/(110
25*x^2*Sqrt[-(c^2*x^2)]) - (d^2*(a + b*ArcCsch[c*x]))/(7*x^7) - (2*d*e*(a +
 b*ArcCsch[c*x]))/(5*x^5) - (e^2*(a + b*ArcCsch[c*x]))/(3*x^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x)
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx = -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\text{csch}^{-1}(cx\right.}{3x^3}$$

$$= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5}-\frac{e^2\left(a+b\text{csch}^{-1}(cx\right.}{3x^3}$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{7x^7}-\frac{2de\left(a+b\text{csch}^{-1}(cx)\right)}{5x^5}-$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}}-\frac{2bcd(15c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\text{csch}\right.}{7x^7}$$

$$= \frac{bcd^2\sqrt{-1-c^2x^2}}{49x^6\sqrt{-c^2x^2}}-\frac{2bcd(15c^2d-49e)\sqrt{-1-c^2x^2}}{1225x^4\sqrt{-c^2x^2}}+\frac{bc(360c^4d^2-}{}$$

$$= -\frac{2bc^3(360c^4d^2-1176c^2de+1225e^2)\sqrt{-1-c^2x^2}}{11025\sqrt{-c^2x^2}}+\frac{bcd^2\sqrt{-1-c^2}}{49x^6\sqrt{-c^2x^2}}$$

Mathematica [A]

time = 0.16, size = 152, normalized size = 0.61

$$\frac{-105a(15d^2+42dex^2+35e^2x^4)+bc\sqrt{1+\frac{1}{c^2x^2}}\,x(1225e^2x^4(1-2c^2x^2)+294dex^2(3-4c^2x^2+8c^4x^4)-45d^2(-5+6c^2x^2-8c^4x^4+16c^6x^6))-105b(15d^2+42dex^2+35e^2x^4)\,\text{csch}^{-1}(cx)}{11025x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^8,x]
```

```
[Out] (-105*a*(15*d^2 + 42*d*e*x^2 + 35*e^2*x^4) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(1
225*e^2*x^4*(1 - 2*c^2*x^2) + 294*d*e*x^2*(3 - 4*c^2*x^2 + 8*c^4*x^4) - 45*
d^2*(-5 + 6*c^2*x^2 - 8*c^4*x^4 + 16*c^6*x^6)) - 105*b*(15*d^2 + 42*d*e*x^2
 + 35*e^2*x^4)*ArcCsch[c*x])/(11025*x^7)
```

Maple [A]

time = 0.34, size = 223, normalized size = 0.90

method	result
derivativedivides	$c^7\left(\frac{a\left(-\frac{e^2}{3c^3x^3}-\frac{2de}{5c^3x^5}-\frac{d^2}{7c^3x^7}\right)}{c^4}+\frac{b\left(-\frac{\text{arccsch}(cx)e^2}{3c^3x^3}-\frac{2\,\text{arccsch}(cx)de}{5c^3x^5}-\frac{\text{arccsch}(cx)d^2}{7c^3x^7}-\frac{(c^2x^2+1)(720c^{10}d^2x^6-2352c^8d\right.}{}\right.$

default	$c^7\left(\frac{a\left(-\frac{e^2}{3c^3x^3}-\frac{2de}{5c^3x^5}-\frac{d^2}{7c^3x^7}\right)}{c^4}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)e^2}{3c^3x^3}-\frac{2\operatorname{arccsch}(cx)de}{5c^3x^5}-\frac{\operatorname{arccsch}(cx)d^2}{7c^3x^7}-\frac{(c^2x^2+1)(720c^{10}d^2x^6-2352c^8de\,x\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x,method=_RETURNVERBOSE)
```

```
[Out] c^7*(a/c^4*(-1/3*e^2/c^3/x^3-2/5/c^3*d*e/x^5-1/7/c^3*d^2/x^7)+b/c^4*(-1/3*a
rccsch(c*x)*e^2/c^3/x^3-2/5*arccsch(c*x)/c^3*d*e/x^5-1/7*arccsch(c*x)/c^3*d
^2/x^7-1/11025*(c^2*x^2+1)*(720*c^10*d^2*x^6-2352*c^8*d*e*x^6-360*c^8*d^2*x
^4+2450*c^6*e^2*x^6+1176*c^6*d*e*x^4+270*c^6*d^2*x^2-1225*c^4*e^2*x^4-882*c
^4*d*e*x^2-225*c^4*d^2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/c^8/x^8))
```

Maxima [A]
time = 0.26, size = 232, normalized size = 0.93

$$\frac{1}{245}bd^2\left(\frac{5c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35c^8\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35c^8\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{35\operatorname{arcsch}(cx)}{x^7}\right)+\frac{2}{75}bd\left(\frac{3c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10c^6\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15c^6\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{15\operatorname{arcsch}(cx)}{x^5}\right)e+\frac{1}{9}b\left(\frac{c^4\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3c^4\sqrt{\frac{1}{c^2x^2}+1}}{c}-\frac{3\operatorname{arcsch}(cx)}{x^3}\right)e^2-\frac{ae^2}{3x^3}-\frac{2ade}{5x^5}-\frac{ad^2}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/245*b*d^2*((5*c^8*(1/(c^2*x^2) + 1)^(7/2) - 21*c^8*(1/(c^2*x^2) + 1)^(5/2
) + 35*c^8*(1/(c^2*x^2) + 1)^(3/2) - 35*c^8*sqrt(1/(c^2*x^2) + 1))/c - 35*a
rccsch(c*x)/x^7) + 2/75*b*d*((3*c^6*(1/(c^2*x^2) + 1)^(5/2) - 10*c^6*(1/(c^
2*x^2) + 1)^(3/2) + 15*c^6*sqrt(1/(c^2*x^2) + 1))/c - 15*arccsch(c*x)/x^5)*
e + 1/9*b*((c^4*(1/(c^2*x^2) + 1)^(3/2) - 3*c^4*sqrt(1/(c^2*x^2) + 1))/c -
3*arccsch(c*x)/x^3)*e^2 - 1/3*a*e^2/x^3 - 2/5*a*d*e/x^5 - 1/7*a*d^2/x^7
```

Fricas [A]
time = 0.35, size = 347, normalized size = 1.39

$$-\frac{3675\,ax^4\cosh(1)^2+3675\,ax^4\sinh(1)^2+4410\,adx^2\cosh(1)+1575\,ad^2+105\left(35\,bx^4\cosh(1)^2+35\,bx^4\sinh(1)^2+42\,bdx^2\cosh(1)+15\,bd^2+14\left(5\,bx^4\cosh(1)+3\,bdx^2\right)\sinh(1)\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+1470\left(5\,ax^4\cosh(1)+3\,adx^2\right)\sinh(1)+\left(720\,bc^7d^2x^7-360\,bc^5d^2x^5+270\,bc^3d^2x^3-225\,bcd^2x+1225\left(2\,bc^3x^7-bcx^5\right)\cosh(1)^2+1225\left(2\,bc^3x^7-bcx^5\right)\sinh(1)^2-294\left(8\,bc^5dx^7-4\,bc^3dx^5+3\,bcdx^3\right)\cosh(1)-98\left(24\,bc^5dx^7-12\,bc^3dx^5+9\,bcdx^3-25\left(2\,bc^3x^7-bcx^5\right)\cosh(1)\right)\sinh(1)\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{11025\,x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] -1/11025*(3675*a*x^4*cosh(1)^2 + 3675*a*x^4*sinh(1)^2 + 4410*a*d*x^2*cosh(1
) + 1575*a*d^2 + 105*(35*b*x^4*cosh(1)^2 + 35*b*x^4*sinh(1)^2 + 42*b*d*x^2*
cosh(1) + 15*b*d^2 + 14*(5*b*x^4*cosh(1) + 3*b*d*x^2)*sinh(1))*log((c*x*sqr
t((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 1470*(5*a*x^4*cosh(1) + 3*a*d*x^2)
*sinh(1) + (720*b*c^7*d^2*x^7 - 360*b*c^5*d^2*x^5 + 270*b*c^3*d^2*x^3 - 225
```

```
*b*c*d^2*x + 1225*(2*b*c^3*x^7 - b*c*x^5)*cosh(1)^2 + 1225*(2*b*c^3*x^7 - b
*c*x^5)*sinh(1)^2 - 294*(8*b*c^5*d*x^7 - 4*b*c^3*d*x^5 + 3*b*c*d*x^3)*cosh(
1) - 98*(24*b*c^5*d*x^7 - 12*b*c^3*d*x^5 + 9*b*c*d*x^3 - 25*(2*b*c^3*x^7 -
b*c*x^5)*cosh(1))*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/x^7
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsch}(cx))(d + ex^2)^2}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**8,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**8, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^8,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x^2 + d)^2\,\left(a + b\,\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^8,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^8, x)
```

3.94 $\int x^3(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=250

$$-\frac{b(6c^4d^2-8c^2de+3e^2)\,x\sqrt{-1-c^2x^2}}{24c^7\sqrt{-c^2x^2}}-\frac{b(6c^4d^2-16c^2de+9e^2)\,x(-1-c^2x^2)^{3/2}}{72c^7\sqrt{-c^2x^2}}+\frac{b(8c^2d-9e)\,ex(-1-c^2}{120c^7\sqrt{-c^2x^2}}$$

```
[Out] 1/4*d^2*x^4*(a+b*arccsch(c*x))+1/3*d*e*x^6*(a+b*arccsch(c*x))+1/8*e^2*x^8*(
a+b*arccsch(c*x))-1/72*b*(6*c^4*d^2-16*c^2*d*e+9*e^2)*x*(-c^2*x^2-1)^(3/2)/
c^7/(-c^2*x^2)^(1/2)+1/120*b*(8*c^2*d-9*e)*e*x*(-c^2*x^2-1)^(5/2)/c^7/(-c^2
*x^2)^(1/2)-1/56*b*e^2*x*(-c^2*x^2-1)^(7/2)/c^7/(-c^2*x^2)^(1/2)-1/24*b*(6*
c^4*d^2-8*c^2*d*e+3*e^2)*x*(-c^2*x^2-1)^(1/2)/c^7/(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {272, 45, 6437, 12, 1265, 785}

$$\frac{1}{4}d^2x^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bex(-c^2x^2-1)^{5/2}(8c^2d-9e)}{120c^7\sqrt{-c^2x^2}}-\frac{be^2x(-c^2x^2-1)^{7/2}}{56c^7\sqrt{-c^2x^2}}-\frac{bx(-c^2x^2-1)^{3/2}(6c^4d^2-16c^2de+9e^2)}{72c^7\sqrt{-c^2x^2}}-\frac{bx\sqrt{-c^2x^2-1}\,(6c^4d^2-8c^2de+3e^2)}{24c^7\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -1/24*(b*(6*c^4*d^2 - 8*c^2*d*e + 3*e^2)*x*Sqrt[-1 - c^2*x^2])/(c^7*Sqrt[-(
c^2*x^2)]) - (b*(6*c^4*d^2 - 16*c^2*d*e + 9*e^2)*x*(-1 - c^2*x^2)^(3/2))/(7
2*c^7*Sqrt[-(c^2*x^2)]) + (b*(8*c^2*d - 9*e)*e*x*(-1 - c^2*x^2)^(5/2))/(120
*c^7*Sqrt[-(c^2*x^2)]) - (b*e^2*x*(-1 - c^2*x^2)^(7/2))/(56*c^7*Sqrt[-(c^2*
x^2)]) + (d^2*x^4*(a + b*ArcCsch[c*x]))/4 + (d*e*x^6*(a + b*ArcCsch[c*x]))/
3 + (e^2*x^8*(a + b*ArcCsch[c*x]))/8
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^3\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{1}{4}d^2x^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\right.$$

$$= \frac{1}{4}d^2x^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\right.$$

$$= \frac{1}{4}d^2x^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\right.$$

$$= \frac{1}{4}d^2x^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\right.$$

$$= -\frac{b(6c^4d^2-8c^2de+3e^2)\,x\sqrt{-1-c^2x^2}}{24c^7\sqrt{-c^2x^2}}-\frac{b(6c^4d^2-16c^2de+9e^2)}{72c^7\sqrt{-c^2}}$$

Mathematica [A]
time = 0.23, size = 159, normalized size = 0.64

$$\frac{x\left(105ax^3(6d^2+8dex^2+3e^2x^4)+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,(-144e^2+8c^2e(56d+9ex^2)-2c^4(210d^2+112dex^2+27e^2x^4)+3c^6(70d^2x^2+56dex^4+15e^2x^6))}{c^7}+105bx^3(6d^2+8dex^2+3e^2x^4)\operatorname{csch}^{-1}(cx)\right)}{2520}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (x*(105*a*x^3*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) + (b*Sqrt[1 + 1/(c^2*x^2)]*(-
144*e^2 + 8*c^2*e*(56*d + 9*e*x^2) - 2*c^4*(210*d^2 + 112*d*e*x^2 + 27*e^2*
x^4) + 3*c^6*(70*d^2*x^2 + 56*d*e*x^4 + 15*e^2*x^6)))/c^7 + 105*b*x^3*(6*d^
2 + 8*d*e*x^2 + 3*e^2*x^4)*ArcCsch[c*x]))/2520
```

Maple [A]
time = 0.57, size = 377, normalized size = 1.51

method	result
derivativedivides	$-\frac{a\left(\frac{c^2d(c^2ex^2+c^2d)^3}{3}-\frac{(c^2ex^2+c^2d)^4}{4}\right)}{2c^4e^2}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)c^8d^4}{24e^2}+\frac{\operatorname{arccsch}(cx)c^8d^2x^4}{4}+\frac{e\operatorname{arccsch}(cx)c^8dx^6}{3}+\frac{e^2\operatorname{arccsch}(cx)c^8x^8}{8}+\right.}{}$
default	$-\frac{a\left(\frac{c^2d(c^2ex^2+c^2d)^3}{3}-\frac{(c^2ex^2+c^2d)^4}{4}\right)}{2c^4e^2}+\frac{b\left(-\frac{\operatorname{arccsch}(cx)c^8d^4}{24e^2}+\frac{\operatorname{arccsch}(cx)c^8d^2x^4}{4}+\frac{e\operatorname{arccsch}(cx)c^8dx^6}{3}+\frac{e^2\operatorname{arccsch}(cx)c^8x^8}{8}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^2*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(-1/2*a/c^4/e^2*(1/3*c^2*d*(c^2*e*x^2+c^2*d)^3-1/4*(c^2*e*x^2+c^2*d)^
4)+b/c^4*(-1/24/e^2*arccsch(c*x)*c^8*d^4+1/4*arccsch(c*x)*c^8*d^2*x^4+1/3*e
*arccsch(c*x)*c^8*d*x^6+1/8*e^2*arccsch(c*x)*c^8*x^8+1/2520/e^2*(c^2*x^2+1)
^(1/2)*(105*c^8*d^4*arctanh(1/(c^2*x^2+1)^(1/2))+210*c^6*d^2*e^2*x^2*(c^2*x
^2+1)^(1/2)+168*c^6*d*e^3*x^4*(c^2*x^2+1)^(1/2)+45*e^4*c^6*x^6*(c^2*x^2+1)^
(1/2)-420*c^4*d^2*e^2*(c^2*x^2+1)^(1/2)-224*c^4*d*e^3*x^2*(c^2*x^2+1)^(1/2)
-54*e^4*c^4*x^4*(c^2*x^2+1)^(1/2)+448*c^2*d*e^3*(c^2*x^2+1)^(1/2)+72*e^4*c^
2*x^2*(c^2*x^2+1)^(1/2)-144*e^4*(c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1
/2)/c/x))
```

Maxima [A]
time = 0.27, size = 244, normalized size = 0.98

$$\frac{1}{8}ax^8e^2+\frac{1}{3}adx^6e+\frac{1}{4}ad^2x^4+\frac{1}{12}\left(3x^4\operatorname{arcsch}(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)bd^2+\frac{1}{45}\left(15x^6\operatorname{arcsch}(cx)+\frac{3c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15x\sqrt{\frac{1}{c^2x^2}+1}}{c^5}\right)bde+\frac{1}{280}\left(35x^8\operatorname{arcsch}(cx)+\frac{5c^6x^7\left(\frac{1}{c^2x^2}+1\right)^{\frac{7}{2}}-21c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}+35c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-35x\sqrt{\frac{1}{c^2x^2}+1}}{c^7}\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*x^8*e^2 + 1/3*a*d*x^6*e + 1/4*a*d^2*x^4 + 1/12*(3*x^4*arccsch(c*x) +
(c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b*d^2 +
1/45*(15*x^6*arccsch(c*x) + (3*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2) - 10*c^2*x^3
*(1/(c^2*x^2) + 1)^(3/2) + 15*x*sqrt(1/(c^2*x^2) + 1))/c^5)*b*d*e + 1/280*(
35*x^8*arccsch(c*x) + (5*c^6*x^7*(1/(c^2*x^2) + 1)^(7/2) - 21*c^4*x^5*(1/(c
^2*x^2) + 1)^(5/2) + 35*c^2*x^3*(1/(c^2*x^2) + 1)^(3/2) - 35*x*sqrt(1/(c^2*
x^2) + 1))/c^7)*b*e^2
```

Fricas [**A**]
time = 0.38, size = 412, normalized size = 1.65

$$\frac{315\,ac^7x^8\cosh\left(1\right)^2+315\,ac^7x^8\sinh\left(1\right)^2+840\,ac^7dx^6\cosh\left(1\right)+630\,ac^7d^2x^4+105\left(3\,bc^7x^8\cosh\left(1\right)^2+3\,bc^7x^8\sinh\left(1\right)^2+8\,bc^7dx^6\cosh\left(1\right)+6\,bc^7d^2x^4+2\left(3\,bc^7x^8\cosh\left(1\right)+4\,bc^7dx^6\right)\sinh\left(1\right)\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+210\left(3\,ac^7x^8\cosh\left(1\right)+4\,ac^7dx^6\right)\sinh\left(1\right)+\left(210\,bc^6d^2x^3-420\,bc^4d^2x+9\left(5\,bc^6x^7-6\,bc^4x^5+8\,bc^2x^3-16\,bx\right)\cosh\left(1\right)^2+9\left(5\,bc^6x^7-6\,bc^4x^5+8\,bc^2x^3-16\,bx\right)\sinh\left(1\right)^2+56\left(3\,bc^6dx^5-4\,bc^4dx^3+8\,bc^2dx\right)\cosh\left(1\right)+2\left(84\,bc^6dx^5-112\,bc^4dx^3+224\,bc^2dx+9\left(5\,bc^6x^7-6\,bc^4x^5+8\,bc^2x^3-16\,bx\right)\cosh\left(1\right)\right)\sinh\left(1\right)\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{2520\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/2520*(315*a*c^7*x^8*cosh(1)^2 + 315*a*c^7*x^8*sinh(1)^2 + 840*a*c^7*d*x^6
*cosh(1) + 630*a*c^7*d^2*x^4 + 105*(3*b*c^7*x^8*cosh(1)^2 + 3*b*c^7*x^8*sin
h(1)^2 + 8*b*c^7*d*x^6*cosh(1) + 6*b*c^7*d^2*x^4 + 2*(3*b*c^7*x^8*cosh(1) +
 4*b*c^7*d*x^6)*sinh(1))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))
 + 210*(3*a*c^7*x^8*cosh(1) + 4*a*c^7*d*x^6)*sinh(1) + (210*b*c^6*d^2*x^3 -
 420*b*c^4*d^2*x + 9*(5*b*c^6*x^7 - 6*b*c^4*x^5 + 8*b*c^2*x^3 - 16*b*x)*cos
h(1)^2 + 9*(5*b*c^6*x^7 - 6*b*c^4*x^5 + 8*b*c^2*x^3 - 16*b*x)*sinh(1)^2 + 5
6*(3*b*c^6*d*x^5 - 4*b*c^4*d*x^3 + 8*b*c^2*d*x)*cosh(1) + 2*(84*b*c^6*d*x^5
 - 112*b*c^4*d*x^3 + 224*b*c^2*d*x + 9*(5*b*c^6*x^7 - 6*b*c^4*x^5 + 8*b*c^2
*x^3 - 16*b*x)*cosh(1))*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/c^7
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a+b\operatorname{acsch}(cx))\left(d+ex^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(e\,x^2+d\right)^2 \left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)^2*(a + b*asinh(1/(c*x))),x)
[Out] int(x^3*(d + e*x^2)^2*(a + b*asinh(1/(c*x))), x)
```

3.95 $\int x(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

Optimal. Leaf size=203

$$\frac{b(3c^4d^2-3c^2de+e^2)\,x\sqrt{-1-c^2x^2}}{6c^5\sqrt{-c^2x^2}}-\frac{b(3c^2d-2e)\,ex(-1-c^2x^2)^{3/2}}{18c^5\sqrt{-c^2x^2}}+\frac{be^2x(-1-c^2x^2)^{5/2}}{30c^5\sqrt{-c^2x^2}}+\frac{(d+ex^2)^3\,(a-}{6}$$

```
[Out] 1/6*(e*x^2+d)^3*(a+b*arccsch(c*x))/e-1/18*b*(3*c^2*d-2*e)*e*x*(-c^2*x^2-1)^
(3/2)/c^5/(-c^2*x^2)^(1/2)+1/30*b*e^2*x*(-c^2*x^2-1)^(5/2)/c^5/(-c^2*x^2)^(
1/2)-1/6*b*c*d^3*x*arctan((-c^2*x^2-1)^(1/2))/e/(-c^2*x^2)^(1/2)+1/6*b*(3*c
^4*d^2-3*c^2*d*e+e^2)*x*(-c^2*x^2-1)^(1/2)/c^5/(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6435, 457, 90, 65, 211}

$$\frac{(d+ex^2)^3\,(a+b\text{csch}^{-1}(cx))}{6e}-\frac{bcd^3x\text{ArcTan}\left(\sqrt{-c^2x^2-1}\right)}{6e\sqrt{-c^2x^2}}-\frac{bex(-c^2x^2-1)^{3/2}\,(3c^2d-2e)}{18c^5\sqrt{-c^2x^2}}+\frac{be^2x(-c^2x^2-1)^{5/2}}{30c^5\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}\,(3c^4d^2-3c^2de+e^2)}{6c^5\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(3*c^4*d^2 - 3*c^2*d*e + e^2)*x*Sqrt[-1 - c^2*x^2])/(6*c^5*Sqrt[-(c^2*x^
2)]) - (b*(3*c^2*d - 2*e)*e*x*(-1 - c^2*x^2)^(3/2))/(18*c^5*Sqrt[-(c^2*x^2)
]) + (b*e^2*x*(-1 - c^2*x^2)^(5/2))/(30*c^5*Sqrt[-(c^2*x^2)]) + ((d + e*x^2
)^3*(a + b*ArcCsch[c*x]))/(6*e) - (b*c*d^3*x*ArcTan[Sqrt[-1 - c^2*x^2]])/(6
*e*Sqrt[-(c^2*x^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6435

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2])), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rubi steps

$$
\begin{aligned}
\int x\left(d+ex^2\right)^2\left(a+b\operatorname{csch}^{-1}(cx)\right)dx &= \frac{\left(d+ex^2\right)^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6e} - \frac{(bcx)\int\frac{\left(d+ex^2\right)^3}{x\sqrt{-1-c^2x^2}}\,dx}{6e\sqrt{-c^2x^2}}\\
&= \frac{\left(d+ex^2\right)^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6e} - \frac{(bcx)\text{Subst}\left(\int\frac{(d+ex)^3}{x\sqrt{-1-c^2x}}\,dx,x,x^{2}\right.}{12e\sqrt{-c^2x^2}}\\
&= \frac{\left(d+ex^2\right)^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{6e} - \frac{(bcx)\text{Subst}\left(\int\left(\frac{e\left(3c^4d^2-3c^2de+e^2\right)}{c^4\sqrt{-1-c^2x}}+\cdots\right.\right.}{\cdots}\\
&= \frac{b\left(3c^4d^2-3c^2de+e^2\right)x\sqrt{-1-c^2x^2}}{6c^5\sqrt{-c^2x^2}} - \frac{b\left(3c^2d-2e\right)ex\left(-1-c^2x^2\right)^{3/\cdots}}{18c^5\sqrt{-c^2x^2}}\\
&= \frac{b\left(3c^4d^2-3c^2de+e^2\right)x\sqrt{-1-c^2x^2}}{6c^5\sqrt{-c^2x^2}} - \frac{b\left(3c^2d-2e\right)ex\left(-1-c^2x^2\right)^{3/\cdots}}{18c^5\sqrt{-c^2x^2}}\\
&= \frac{b\left(3c^4d^2-3c^2de+e^2\right)x\sqrt{-1-c^2x^2}}{6c^5\sqrt{-c^2x^2}} - \frac{b\left(3c^2d-2e\right)ex\left(-1-c^2x^2\right)^{3/\cdots}}{18c^5\sqrt{-c^2x^2}}
\end{aligned}
$$

Mathematica [**A**]

time = 0.18, size = 123, normalized size = 0.61

$$\frac{1}{90}x\left(15ax\left(3d^2+3dex^2+e^2x^4\right)+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\left(8e^2-2c^2e(15d+2ex^2)+3c^4(15d^2+5dex^2+e^2x^4)\right)}{c^5}+15bx\left(3d^2+3dex^2+e^2x^4\right)\operatorname{csch}^{-1}(cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (x*(15*a*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) + (b*Sqrt[1 + 1/(c^2*x^2)]*(8*e^2
- 2*c^2*e*(15*d + 2*e*x^2) + 3*c^4*(15*d^2 + 5*d*e*x^2 + e^2*x^4)))/c^5 + 1
5*b*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcCsch[c*x]))/90
```

Maple [A]
time = 0.55, size = 280, normalized size = 1.38

method	result
derivativedivides	$\frac{\frac{\left(c^2 e x^2+c^2 d\right)^3 a}{6c^4 e}+\frac{b\left(\frac{\operatorname{arccsch}(cx)c^6d^3}{6e}+\frac{\operatorname{arccsch}(cx)c^6d^2x^2}{2}+\frac{e\operatorname{arccsch}(cx)c^6dx^4}{2}+\frac{e^2\operatorname{arccsch}(cx)c^6x^6}{6}-\frac{\sqrt{c^2x^2+1}\left(15c^6d^3\right.}{}\right.}{}}{}$
default	$\frac{\frac{\left(c^2 e x^2+c^2 d\right)^3 a}{6c^4 e}+\frac{b\left(\frac{\operatorname{arccsch}(cx)c^6d^3}{6e}+\frac{\operatorname{arccsch}(cx)c^6d^2x^2}{2}+\frac{e\operatorname{arccsch}(cx)c^6dx^4}{2}+\frac{e^2\operatorname{arccsch}(cx)c^6x^6}{6}-\frac{\sqrt{c^2x^2+1}\left(15c^6d^3\right.}{}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^2*(a+b*arccsch(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/6*(c^2*e*x^2+c^2*d)^3*a/c^4/e+b/c^4*(1/6/e*arccsch(c*x)*c^6*d^3+1/
2*arccsch(c*x)*c^6*d^2*x^2+1/2*e*arccsch(c*x)*c^6*d*x^4+1/6*e^2*arccsch(c*x
)*c^6*x^6-1/90/e*(c^2*x^2+1)^(1/2)*(15*c^6*d^3*arctanh(1/(c^2*x^2+1)^(1/2))
-45*c^4*d^2*e*(c^2*x^2+1)^(1/2)-15*c^4*d*e^2*x^2*(c^2*x^2+1)^(1/2)-3*e^3*c^
4*x^4*(c^2*x^2+1)^(1/2)+30*c^2*d*e^2*(c^2*x^2+1)^(1/2)+4*e^3*c^2*x^2*(c^2*x
^2+1)^(1/2)-8*e^3*(c^2*x^2+1)^(1/2))/((c^2*x^2+1)/c^2/x^2)^(1/2)/c/x))
```

Maxima [A]
time = 0.27, size = 183, normalized size = 0.90

$$\frac{1}{6}ax^6e^2+\frac{1}{2}adx^4e+\frac{1}{2}ad^2x^2+\frac{1}{2}\left(x^2\operatorname{arcsch}(cx)+\frac{x\sqrt{\frac{1}{c^2x^2}+1}}{c}\right)bd^2+\frac{1}{6}\left(3x^4\operatorname{arcsch}(cx)+\frac{c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}-3x\sqrt{\frac{1}{c^2x^2}+1}}{c^3}\right)bde+\frac{1}{90}\left(15x^6\operatorname{arcsch}(cx)+\frac{3c^4x^5\left(\frac{1}{c^2x^2}+1\right)^{\frac{5}{2}}-10c^2x^3\left(\frac{1}{c^2x^2}+1\right)^{\frac{3}{2}}+15x\sqrt{\frac{1}{c^2x^2}+1}}{c^5}\right)be^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*x^6*e^2 + 1/2*a*d*x^4*e + 1/2*a*d^2*x^2 + 1/2*(x^2*arccsch(c*x) + x*s
qrt(1/(c^2*x^2) + 1)/c)*b*d^2 + 1/6*(3*x^4*arccsch(c*x) + (c^2*x^3*(1/(c^2*
x^2) + 1)^(3/2) - 3*x*sqrt(1/(c^2*x^2) + 1))/c^3)*b*d*e + 1/90*(15*x^6*arcc
sch(c*x) + (3*c^4*x^5*(1/(c^2*x^2) + 1)^(5/2) - 10*c^2*x^3*(1/(c^2*x^2) + 1
)^(3/2) + 15*x*sqrt(1/(c^2*x^2) + 1))/c^5)*b*e^2
```

Fricas [A]
time = 0.37, size = 346, normalized size = 1.70

$$\frac{15\,ac^5x^6\cosh(1)^2 + 15\,ac^5x^6\sinh(1)^2 + 45\,ac^5dx^4\cosh(1) + 45\,ac^5d^2x^2 + 15\left(bc^5x^6\cosh(1)^2 + bc^5x^6\sinh(1)^2 + 3\,bc^5dx^4\cosh(1) + 3\,bc^5d^2x^2 + \left(2\,bc^5x^6\cosh(1) + 3\,bc^5dx^4\right)\sinh(1)\right)\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right) + 15\left(2\,ac^5x^6\cosh(1) + 3\,ac^5dx^4\right)\sinh(1) + \left(45\,bc^4d^2x + \left(3\,bc^4x^5 - 4\,bc^2x^3 + 8\,bx\right)\cosh(1)^2 + \left(3\,bc^4x^5 - 4\,bc^2x^3 + 8\,bx\right)\sinh(1)^2 + 15\left(bc^4dx^3 - 2\,bc^2dx\right)\cosh(1) + \left(15\,bc^4dx^3 - 30\,bc^2dx + 2\left(3\,bc^4x^5 - 4\,bc^2x^3 + 8\,bx\right)\cosh(1)\right)\sinh(1)\right)\sqrt{\frac{c^2x^2+1}{c^2x^2}}}{90\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] 1/90*(15*a*c^5*x^6*cosh(1)^2 + 15*a*c^5*x^6*sinh(1)^2 + 45*a*c^5*d*x^4*cosh
(1) + 45*a*c^5*d^2*x^2 + 15*(b*c^5*x^6*cosh(1)^2 + b*c^5*x^6*sinh(1)^2 + 3*
b*c^5*d*x^4*cosh(1) + 3*b*c^5*d^2*x^2 + (2*b*c^5*x^6*cosh(1) + 3*b*c^5*d*x^
4)*sinh(1))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 15*(2*a*c^
5*x^6*cosh(1) + 3*a*c^5*d*x^4)*sinh(1) + (45*b*c^4*d^2*x + (3*b*c^4*x^5 - 4
*b*c^2*x^3 + 8*b*x)*cosh(1)^2 + (3*b*c^4*x^5 - 4*b*c^2*x^3 + 8*b*x)*sinh(1)
^2 + 15*(b*c^4*d*x^3 - 2*b*c^2*d*x)*cosh(1) + (15*b*c^4*d*x^3 - 30*b*c^2*d*
x + 2*(3*b*c^4*x^5 - 4*b*c^2*x^3 + 8*b*x)*cosh(1))*sinh(1))*sqrt((c^2*x^2 +
 1)/(c^2*x^2)))/c^5
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b\,\mathrm{acsch}\,(cx))\left(d + ex^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(e\,x^2+d\right)^2\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)^2*(a + b*asinh(1/(c*x))),x)
[Out] int(x*(d + e*x^2)^2*(a + b*asinh(1/(c*x))), x)
```

3.96 $\int \frac{(d+ex^2)^2\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=178

$$\frac{b(6c^2d-e)\,e\sqrt{1+\frac{1}{c^2x^2}}\,x}{6c^3}+\frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x^3}{12c}+\frac{1}{2}bd^2\text{csch}^{-1}(cx)^2+dex^2\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\text{csch}^{-1}\right.$$

```
[Out] 1/2*b*d^2*arccsch(c*x)^2+d*e*x^2*(a+b*arccsch(c*x))+1/4*e^2*x^4*(a+b*arccsc
h(c*x))-b*d^2*arccsch(c*x)*ln(1-(1/c/x+(1+1/c^2/x^2)^(1/2))^2)+b*d^2*arccsc
h(c*x)*ln(1/x)-d^2*(a+b*arccsch(c*x))*ln(1/x)-1/2*b*d^2*polylog(2,(1/c/x+(1
+1/c^2/x^2)^(1/2))^2)+1/6*b*(6*c^2*d-e)*e*x*(1+1/c^2/x^2)^(1/2)/c^3+1/12*b*
e^2*x^3*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.29, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {6439, 272, 45, 5822, 6874, 464, 270, 2362, 5775, 3797, 2221, 2317, 2438}

$$-d^2\log\left(\frac{1}{x}\right)\left(a+b\text{csch}^{-1}(cx)\right)+dex^2\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\text{csch}^{-1}(cx)\right)+\frac{be^2x^3\sqrt{\frac{1}{c^2x^2}+1}}{12c}+\frac{bex\sqrt{\frac{1}{c^2x^2}+1}\,(6c^2d-e)}{6c^3}-\frac{1}{2}bd^2\text{Li}_2\left(e^{2\text{csch}^{-1}(cx)}\right)+\frac{1}{2}bd^2\text{csch}^{-1}(cx)^2-bd^2\text{csch}^{-1}(cx)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)+bd^2\log\left(\frac{1}{x}\right)\text{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] (b*(6*c^2*d - e)*e*Sqrt[1 + 1/(c^2*x^2)]*x)/(6*c^3) + (b*e^2*Sqrt[1 + 1/(c^
2*x^2)]*x^3)/(12*c) + (b*d^2*ArcCsch[c*x]^2)/2 + d*e*x^2*(a + b*ArcCsch[c*x
]) + (e^2*x^4*(a + b*ArcCsch[c*x]))/4 - b*d^2*ArcCsch[c*x]*Log[1 - E^(2*Arc
Csch[c*x])] + b*d^2*ArcCsch[c*x]*Log[x^(-1)] - d^2*(a + b*ArcCsch[c*x])*Log
[x^(-1)] - (b*d^2*PolyLog[2, E^(2*ArcCsch[c*x])])/2
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2362

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[e, 2]), x]
 - Dist[b*(n/Rt[e, 2]), Int[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5822

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^
2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2*d] &&
IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right)}{x} \, dx = -\text{Subst}\left(\int \frac{(e+dx^2)^2 \left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{x^5} \, dx, x, \frac{1}{x}\right)$$

$$= dex^2\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\text{csch}^{-1}(cx)\right) - d^2\left(a+b\text{csch}^{-1}(c\right.$$

$$= dex^2\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\text{csch}^{-1}(cx)\right) - d^2\left(a+b\text{csch}^{-1}(c\right.$$

$$= dex^2\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\text{csch}^{-1}(cx)\right) - d^2\left(a+b\text{csch}^{-1}(c\right.$$

$$= \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\, x^3}{12c} + dex^2\left(a+b\text{csch}^{-1}(cx)\right) + \frac{1}{4}e^2x^4\left(a+b\text{csch}^{-1}(cx\right.$$

$$= \frac{b(6c^2d-e)\, e\sqrt{1+\frac{1}{c^2x^2}}\, x}{6c^3} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\, x^3}{12c} + dex^2\left(a+b\text{csch}^{-1}(\right.$$

$$= \frac{b(6c^2d-e)\, e\sqrt{1+\frac{1}{c^2x^2}}\, x}{6c^3} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\, x^3}{12c} + \frac{1}{2}bd^2\text{csch}^{-1}(cx)^2 +$$

$$= \frac{b(6c^2d-e)\, e\sqrt{1+\frac{1}{c^2x^2}}\, x}{6c^3} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\, x^3}{12c} + \frac{1}{2}bd^2\text{csch}^{-1}(cx)^2 +$$

$$= \frac{b(6c^2d-e)\, e\sqrt{1+\frac{1}{c^2x^2}}\, x}{6c^3} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\, x^3}{12c} + \frac{1}{2}bd^2\text{csch}^{-1}(cx)^2 +$$

$$= \frac{b(6c^2d-e)\, e\sqrt{1+\frac{1}{c^2x^2}}\, x}{6c^3} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\, x^3}{12c} + \frac{1}{2}bd^2\text{csch}^{-1}(cx)^2 +$$

Mathematica [A]
time = 0.25, size = 148, normalized size = 0.83

$$adex^2+\frac{1}{4}ae^2x^4+\frac{bdex\left(\sqrt{1+\frac{1}{c^2x^2}}+cx\operatorname{csch}^{-1}(cx)\right)}{c}+\frac{be^2x\left(\sqrt{1+\frac{1}{c^2x^2}}\,(-2+c^2x^2)+3c^3x^3\operatorname{csch}^{-1}(cx)\right)}{12c^3}+ad^2\log(x)+\frac{1}{2}bd^2\left(-\operatorname{csch}^{-1}(cx)\left(\operatorname{csch}^{-1}(cx)+2\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)\right)+\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(cx)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x,x]
```

```
[Out] a*d*e*x^2 + (a*e^2*x^4)/4 + (b*d*e*x*(Sqrt[1 + 1/(c^2*x^2)] + c*x*ArcCsch[c
*x]))/c + (b*e^2*x*(Sqrt[1 + 1/(c^2*x^2)]*(-2 + c^2*x^2) + 3*c^3*x^3*ArcCsc
h[c*x]))/(12*c^3) + a*d^2*Log[x] + (b*d^2*(-(ArcCsch[c*x]*(ArcCsch[c*x] + 2
*Log[1 - E^(-2*ArcCsch[c*x])])) + PolyLog[2, E^(-2*ArcCsch[c*x])]))/2
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^2+d\right)^2\left(a+b\,\mathrm{arccsch}(cx)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x)
```

```
[Out] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4*e^2 + 4*b*c^2*d^2*integrate(1/4*x*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x
^2 + c^2*x^2 + sqrt(c^2*x^2 + 1) + 1), x) + a*d*x^2*e - b*d^2*log(c)*log(x)
 - 1/4*(2*log(c^2*x^2 + 1)*log(x) + dilog(-c^2*x^2))*b*d^2 + a*d^2*log(x) +
 1/2*b*d*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1))*e/c^2 - 1/8*(2*b*c^2*x^4*
e^2*log(c) + 4*b*c^2*d^2*log(x)^2 + (8*b*c^2*d*e*log(c) - b*e^2)*x^2 + 2*(b
*c^2*x^4*e^2 + 4*b*c^2*d*x^2*e)*log(x) - 2*(b*c^2*x^4*e^2 + 4*b*c^2*d*x^2*e
 + 4*b*c^2*d^2*log(x))*log(sqrt(c^2*x^2 + 1) + 1))/c^2 - 1/24*(3*c^2*x^2 -
2*(c^2*x^2 + 1)^(3/2) + 6*sqrt(c^2*x^2 + 1) - 3*log(c^2*x^2 + 1) + 3)*b*e^2
/c^4 + 1/8*(4*b*c^2*d*e - b*e^2)*log(c^2*x^2 + 1)/c^4
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arccsch(c*x))/x, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsch}\,(cx))\,(d + ex^2)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x^2 + d)^2\,\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x, x)
```

3.97 $\int \frac{(d+ex^2)^2 \left(a+b\text{csch}^{-1}(cx)\right)}{x^3} \, dx$

Optimal. Leaf size=178

$$\frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x}+\frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c}-\frac{1}{4}bc^2d^2\text{csch}^{-1}(cx)+bde\,\text{csch}^{-1}(cx)^2-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^2x^2\left(a+b\right.$$

```
[Out] -1/4*b*c^2*d^2*arccsch(c*x)+b*d*e*arccsch(c*x)^2-1/2*d^2*(a+b*arccsch(c*x))
/x^2+1/2*e^2*x^2*(a+b*arccsch(c*x))-2*b*d*e*arccsch(c*x)*ln(1-(1/c/x+(1+1/c
^2/x^2)^(1/2))^2)+2*b*d*e*arccsch(c*x)*ln(1/x)-2*d*e*(a+b*arccsch(c*x))*ln(
1/x)-b*d*e*polylog(2,(1/c/x+(1+1/c^2/x^2)^(1/2))^2)+1/4*b*c*d^2*(1+1/c^2/x^
2)^(1/2)/x+1/2*b*e^2*x*(1+1/c^2/x^2)^(1/2)/c
```

Rubi [A]
time = 0.29, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6439, 272, 45, 5822, 12, 6874, 270, 327, 221, 2362, 5775, 3797, 2221, 2317, 2438}

$$-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2}-2de\log\left(\frac{1}{x}\right)\left(a+b\text{csch}^{-1}(cx)\right)+\frac{1}{2}e^2x^2\left(a+b\text{csch}^{-1}(cx)\right)+\frac{bcd^2\sqrt{\frac{1}{c^2x^2}+1}}{4x}-\frac{1}{4}bc^2d^2\text{csch}^{-1}(cx)+\frac{be^2x\sqrt{\frac{1}{c^2x^2}+1}}{2c}-bde\,\text{Li}_2\left(e^{2\text{csch}^{-1}(cx)}\right)+bde\,\text{csch}^{-1}(cx)^2-2bde\,\text{csch}^{-1}(cx)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)+2bde\log\left(\frac{1}{x}\right)\text{csch}^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^3,x]
```

```
[Out] (b*c*d^2*Sqrt[1 + 1/(c^2*x^2)])/(4*x) + (b*e^2*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*
c) - (b*c^2*d^2*ArcCsch[c*x])/4 + b*d*e*ArcCsch[c*x]^2 - (d^2*(a + b*ArcCsc
h[c*x]))/(2*x^2) + (e^2*x^2*(a + b*ArcCsch[c*x]))/2 - 2*b*d*e*ArcCsch[c*x]*
Log[1 - E^(2*ArcCsch[c*x])] + 2*b*d*e*ArcCsch[c*x]*Log[x^(-1)] - 2*d*e*(a +
 b*ArcCsch[c*x])*Log[x^(-1)] - b*d*e*PolyLog[2, E^(2*ArcCsch[c*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2362

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]*((a + b*Log[c*x^n])/Rt[e, 2]), x]
 - Dist[b*(n/Rt[e, 2]), Int[ArcSinh[Rt[e, 2]*(x/Sqrt[d])]/x, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && GtQ[d, 0] && PosQ[e]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5822

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcSinh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/Sqrt[1 + c^
2*x^2], x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[e, c^2*d] &&
IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m + p, 0]))
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx = -\text{Subst}\left(\int \frac{\left(e+dx^2\right)^2\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{x^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{csch}^{-1}(cx)\right) - 2de\left(a+b\text{csch}^{-1}\right.$$

$$= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{csch}^{-1}(cx)\right) - 2de\left(a+b\text{csch}^{-1}\right.$$

$$= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{csch}^{-1}(cx)\right) - 2de\left(a+b\text{csch}^{-1}\right.$$

$$= -\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2\left(a+b\text{csch}^{-1}(cx)\right) - 2de\left(a+b\text{csch}^{-1}\right.$$

$$= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} - \frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{2x^2} + \frac{1}{2}e^2x^2$$

$$= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} - \frac{1}{4}bc^2d^2\text{csch}^{-1}(cx) - \frac{d^2\left(a+b\right.}{}$$

$$= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} - \frac{1}{4}bc^2d^2\text{csch}^{-1}(cx) + bde\text{csch}^{-}$$

$$= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} - \frac{1}{4}bc^2d^2\text{csch}^{-1}(cx) + bde\text{csch}^{-}$$

$$= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} - \frac{1}{4}bc^2d^2\text{csch}^{-1}(cx) + bde\text{csch}^{-}$$

$$= \frac{bcd^2\sqrt{1+\frac{1}{c^2x^2}}}{4x} + \frac{be^2\sqrt{1+\frac{1}{c^2x^2}}\,x}{2c} - \frac{1}{4}bc^2d^2\text{csch}^{-1}(cx) + bde\text{csch}^{-}$$

Mathematica [A]
time = 0.59, size = 187, normalized size = 1.05

$$\frac{1}{4}\left(-\frac{2ad^2}{x^2}+2ae^2x^2-\frac{2bd^2\operatorname{csch}^{-1}(cx)}{x^2}+\frac{2be^2x\left(\sqrt{1+\frac{1}{c^2x^2}}+cx\operatorname{csch}^{-1}(cx)\right)}{c}-\frac{bd^2\left(-1-c^2x^2+c^2x^2\sqrt{1+c^2x^2}\tanh^{-1}\left(\sqrt{1+c^2x^2}\right)\right)}{c\sqrt{1+\frac{1}{c^2x^2}}\,x^3}-4bde\operatorname{csch}^{-1}(cx)\left(\operatorname{csch}^{-1}(cx)+2\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)\right)+8ade\log(x)+4bde\operatorname{PolyLog}\left(2,e^{-2\operatorname{csch}^{-1}(cx)}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCsch[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d^2)/x^2 + 2*a*e^2*x^2 - (2*b*d^2*ArcCsch[c*x])/x^2 + (2*b*e^2*x*(Sq
rt[1 + 1/(c^2*x^2)] + c*x*ArcCsch[c*x]))/c - (b*d^2*(-1 - c^2*x^2 + c^2*x^2
*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 + c^2*x^2]]))/(c*Sqrt[1 + 1/(c^2*x^2)]*x^
3) - 4*b*d*e*ArcCsch[c*x]*(ArcCsch[c*x] + 2*Log[1 - E^(-2*ArcCsch[c*x])]) +
 8*a*d*e*Log[x] + 4*b*d*e*PolyLog[2, E^(-2*ArcCsch[c*x])])/4
```

Maple [F]
time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^2+d\right)^2\left(a+b\operatorname{arccsch}(cx)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x)
```

```
[Out] int((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 4*b*c^2*d*e*integrate(1/2*x*log(x)/(sqrt(c^2*x^2 + 1)*c^2*x^2 + c^2*x^2 + s
qrt(c^2*x^2 + 1) + 1), x) - 1/2*b*x^2*e^2*log(c) - 1/2*b*x^2*e^2*log(x) - 2
*b*d*e*log(c)*log(x) - b*d*e*log(x)^2 + 1/8*b*d^2*((2*c^4*x*sqrt(1/(c^2*x^2
) + 1)/(c^2*x^2*(1/(c^2*x^2) + 1) - 1) - c^3*log(c*x*sqrt(1/(c^2*x^2) + 1)
+ 1) + c^3*log(c*x*sqrt(1/(c^2*x^2) + 1) - 1))/c - 4*arccsch(c*x)/x^2) + 1/
2*a*x^2*e^2 - 1/2*(2*log(c^2*x^2 + 1)*log(x) + dilog(-c^2*x^2))*b*d*e + 2*a
*d*e*log(x) + 1/2*(b*x^2*e^2 + 4*b*d*e*log(x))*log(sqrt(c^2*x^2 + 1) + 1) -
 1/2*a*d^2/x^2 + 1/4*b*(2*sqrt(c^2*x^2 + 1) - log(c^2*x^2 + 1))*e^2/c^2 + 1
/4*b*e^2*log(c^2*x^2 + 1)/c^2
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arccsch(c*x))/x^3, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acsch}\,(cx))\,(d + ex^2)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acsch(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**2/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccsch(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x^2 + d)^2\,\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^3,x)
```

```
[Out] int(((d + e*x^2)^2*(a + b*asinh(1/(c*x))))/x^3, x)
```

3.98 $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=512

$$\frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{e}+\frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce}+\frac{\sqrt{-d}\,\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e^{3/2}}-\frac{\sqrt{-}}{}$$

```
[Out] x*(a+b*arccsch(c*x))/e+b*arctanh((1+1/c^2/x^2)^(1/2))/c/e+1/2*(a+b*arccsch(
c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/
2)))*(-d)^(1/2)/e^(3/2)-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^
(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*(a+b*a
rccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d
+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^
2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2
*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^
(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*b*polylog(2,-c*
(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*(-d)^(1/
2)/e^(3/2)+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(3/2)
```

Rubi [A]
time = 0.84, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6439, 5823, 5776, 272, 65, 214, 5793, 5827, 5680, 2221, 2317, 2438}

$$\frac{\sqrt{-d}\,\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\,\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2e^{3/2}}+\frac{\sqrt{-d}\,\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\,\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2e^{3/2}}+\frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{e}-\frac{b\sqrt{-d}\,\text{Li}_2\left(-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^{3/2}}+\frac{b\sqrt{-d}\,\text{Li}_2\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^{3/2}}-\frac{b\sqrt{-d}\,\text{Li}_2\left(-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e^{3/2}}+\frac{b\sqrt{-d}\,\text{Li}_2\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e^{3/2}}+\frac{b\tanh^{-1}\left(\sqrt{\frac{1}{c^2x^2}+1}\right)}{ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

```
[Out] (x*(a + b*ArcCsch[c*x]))/e + (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(c*e) + (Sq
rt[-d]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] -
Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1 +
(c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) +
(Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
 + Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1
 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^(3/2))
 - (b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c
^2*d) + e]))])/(2*e^(3/2)) + (b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c
*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2,
-((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(2*e^(3/2))
```

```
 + (b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2
*d) + e])])/(2*e^(3/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
```

```
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{x^2(a+b\text{csch}^{-1}(cx))}{d+ex^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x^2\,(e+dx^2)}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{ex^2} - \frac{d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e\,(e+dx^2)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x^2}\,dx, x, \frac{1}{x}\right)}{e} + \frac{d\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e}$$

$$= \frac{x(a+b\text{csch}^{-1}(cx))}{e} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{ce} + \frac{d\text{Subst}\left(\int \left(\frac{a\cdots}{2\sqrt{e}\,(\cdots}\right.\right.}{\cdots}$$

$$= \frac{x(a+b\text{csch}^{-1}(cx))}{e} + \frac{d\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2e^{3/2}} + \frac{d\text{Subst}\left(\int \frac{a+b\sinh\cdots}{\sqrt{e}+\sqrt{\cdots}}\right.}{2e^{3/\cdots}}$$

$$= \frac{x(a+b\text{csch}^{-1}(cx))}{e} + \frac{d\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\,\sinh(x)}\,dx, x, \text{csch}^{-1}(cx)\right)}{2e^{3/2}} + \frac{d\text{Su}\cdots}{\cdots}$$

$$= \frac{x(a+b\text{csch}^{-1}(cx))}{e} + \frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce} + \frac{d\text{Subst}\left(\int \frac{e^x(a+\cdots}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+\cdots}}{c}}\right.}{\cdots}$$

$$= \frac{x(a+b\text{csch}^{-1}(cx))}{e} + \frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce} + \frac{\sqrt{-d}\,(a+b\text{csch}^{-1}(cx))\,\text{lo}\cdots}{2e\cdots}$$

$$= \frac{x(a+b\text{csch}^{-1}(cx))}{e} + \frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce} + \frac{\sqrt{-d}\,(a+b\text{csch}^{-1}(cx))\,\text{lo}\cdots}{2e\cdots}$$

$$= \frac{x(a+b\text{csch}^{-1}(cx))}{e} + \frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce} + \frac{\sqrt{-d}\,(a+b\text{csch}^{-1}(cx))\,\text{lo}\cdots}{2e\cdots}$$

Mathematica [C] Result contains complex when optimal does not.

time = 1.13, size = 1221, normalized size = 2.38

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

```
[Out] (4*a*c*Sqrt[e]*x + 4*b*c*Sqrt[e]*x*ArcCsch[c*x] - 4*a*c*Sqrt[d]*ArcTan[(Sqr
t[e]*x)/Sqrt[d]] - (8*I)*b*c*Sqrt[d]*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/S
qrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqr
t[-(c^2*d) + e]] - (8*I)*b*c*Sqrt[d]*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/S
qrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqr
t[-(c^2*d) + e]] + b*c*Sqrt[d]*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e]
)*E^ArcCsch[c*x])/(c*Sqrt[d])] - (2*I)*b*c*Sqrt[d]*ArcCsch[c*x]*Log[1 - (I*
(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*c*Sqrt[d
]*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt
[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - b*c*Sqrt[d]*Pi*Log[1 + (I*(-
Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*b*c*Sqrt
[d]*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])
/(c*Sqrt[d])] - 4*b*c*Sqrt[d]*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]
*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] -
b*c*Sqrt[d]*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])] + (2*I)*b*c*Sqrt[d]*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^
2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*c*Sqrt[d]*ArcSin[Sqrt[1 - Sqr
t[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcC
sch[c*x])/(c*Sqrt[d])] + b*c*Sqrt[d]*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d)
 + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (2*I)*b*c*Sqrt[d]*ArcCsch[c*x]*Log[1
+ (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - 4*b*c*Sq
rt[d]*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + S
qrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + b*c*Sqrt[d]*Pi*Log[Sqrt[e
] - (I*Sqrt[d])/x] - b*c*Sqrt[d]*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] - 4*b*Sqrt
[e]*Log[Tanh[ArcCsch[c*x]/2]] + (2*I)*b*c*Sqrt[d]*PolyLog[2, ((-I)*(-Sqrt[e
] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (2*I)*b*c*Sqrt[d]*Po
lyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] -
(2*I)*b*c*Sqrt[d]*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch
[c*x])/(c*Sqrt[d])] + (2*I)*b*c*Sqrt[d]*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2
*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/(4*c*e^(3/2))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\,\mathrm{arccsch}(cx))}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -(sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2) - x*e^(-1))*a + b*integrate(x^
2*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(x^2*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)/(x^2*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\operatorname{acsch}(cx))}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))/(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \left(a+b \operatorname{asinh}\left(\frac{1}{c x}\right)\right)}{e x^2+d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2),x)
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2), x)
```

3.99 $\int \frac{x\left(a+b\mathbf{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=467

$$-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)}{e}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}}}{\sqrt{e}-\sqrt{-c^2d}}\right.}{2e}$$

```
[Out] -(a+b*arccsch(c*x))^2/b/e-(a+b*arccsch(c*x))*ln(1-1/(1/c/x+(1+1/c^2/x^2)^(1
/2))^2)/e+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1
/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e+1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1
/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e+1/2*(a+b*arccsch(
c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/
2)))/e+1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)+(-c^2*d+e)^(1/2)))/e+1/2*b*polylog(2,1/(1/c/x+(1+1/c^2/x^2)^(1/2)
)^2)/e+1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(
-c^2*d+e)^(1/2)))/e+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e+1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/
2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e+1/2*b*polylog(2,c*(1/c/x+(1+1/
c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e
```

Rubi [A]
time = 0.79, antiderivative size = 467, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {6439, 5823, 5775, 3797, 2221, 2317, 2438, 5827, 5680}

$$\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2e}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2e}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{be}-\frac{\log\left(1-e^{-2\operatorname{csch}^{-1}(cx)}\right)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e}+\frac{b\operatorname{Li}_2\left(-\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e}+\frac{b\operatorname{Li}_2\left(\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e}+\frac{b\operatorname{Li}_2\left(-\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e}+\frac{b\operatorname{Li}_2\left(\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e}+\frac{b\operatorname{Li}_2\left(e^{-2\operatorname{csch}^{-1}(cx)}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

```
[Out] -((a + b*ArcCsch[c*x])^2/(b*e)) - ((a + b*ArcCsch[c*x])*Log[1 - E^(-2*ArcCs
ch[c*x])])/e + ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(S
qrt[e] - Sqrt[-(c^2*d) + e])])/(2*e) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqr
t[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e) + ((a + b*ArcC
sch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]
)])/(2*e) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt
[e] + Sqrt[-(c^2*d) + e])])/(2*e) + (b*PolyLog[2, E^(-2*ArcCsch[c*x])])/(2*
e) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) +
 e]))])/(2*e) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-
(c^2*d) + e])])/(2*e) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e
] + Sqrt[-(c^2*d) + e]))])/(2*e) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x]
)/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x\,(e+dx^2)}\,dx,x,\frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{ex}-\frac{dx\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e\,(e+dx^2)}\right)dx,x,\frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x}\,dx,x,\frac{1}{x}\right)}{e}+\frac{d\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx,x,\frac{1}{x}\right)}{e}$$

$$= -\frac{\text{Subst}\left(\int (a+bx)\coth(x)\,dx,x,\text{csch}^{-1}(cx)\right)}{e}+\frac{d\text{Subst}\left(\int\left(-\frac{\sqrt{-d}\,(a+b\sinh^{-1}(\ldots}{2d\left(\sqrt{e}-\sqrt{-d}\,x\ldots}\right.\right.}{\ldots}$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2be}+\frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1-e^{2x}}\,dx,x,\text{csch}^{-1}(cx)\right)}{e}-\frac{\sqrt{-d}\,\text{Subst}\left(\int \ldots\right.}{\ldots}$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2be}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)}{e}+\frac{b\text{Subst}\left(\int \log\left(1\ldots\right.\right.}{\ldots}$$

$$= -\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{2\text{csch}^{-1}(cx)}\right)}{e}+\frac{b\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2\text{csch}^{-1}(cx)}\right)\ldots}{2e}$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\ldots\right.}{2e}$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\ldots\right.}{2e}$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2e}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\ldots\right.}{2e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.33, size = 1103, normalized size = 2.36

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

```
[Out] (b*Pi^2 - (4*I)*b*Pi*ArcCsch[c*x] - 8*b*ArcCsch[c*x]^2 + 16*b*ArcSin[Sqrt[1
 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2
*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 16*b*ArcSin[Sqrt[1 - Sqrt[e]/(c
*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c
*x])/4])/Sqrt[-(c^2*d) + e]] - 8*b*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])
] + (2*I)*b*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E
^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/
Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt
[d])] + (2*I)*b*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x
])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e
])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d
])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*
Sqrt[d])] + (2*I)*b*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[
c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) +
 e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt
[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])] + (2*I)*b*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch
[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d)
+ e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])] - (2*I)*b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - (2*I)*b*Pi*Log[Sqrt
[e] + (I*Sqrt[d])/x] + 4*a*Log[d + e*x^2] + 4*b*PolyLog[2, E^(-2*ArcCsch[c*
x])] + 4*b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])
/(c*Sqrt[d])] + 4*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch
[c*x])/(c*Sqrt[d])] + 4*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E
^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/(8*e)
```

Maple [**F**]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\,\mathrm{arccsch}(cx))}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

```
[Out] int(x*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*e^(-1)*log(x^2*e + d) + b*integrate(x*log(sqrt(1/(c^2*x^2) + 1) + 1/(
c*x))/(x^2*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccsch(c*x) + a*x)/(x^2*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\,\mathrm{acsch}\,(cx))}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))/(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2),x)
```

```
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2), x)
```

3.100 $\int \frac{a+b\,\mathrm{csch}^{-1}(cx)}{d+ex^2}\,dx$

Optimal. Leaf size=477

$$\frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}\,e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{(a+b\mathrm{cs}}{}$$

```
[Out] 1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c
/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e
^(1/2)+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccsch(c*x))*ln(1
+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^
(1/2)/e^(1/2)-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,c*(1/c/x+(1+1/c
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/
2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)
^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(
-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(1/2)/e^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {6429, 5793, 5827, 5680, 2221, 2317, 2438}

$$\frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\,\mathrm{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{b\,\mathrm{Li}_2\left(-\frac{c\sqrt{-d}\,e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{b\,\mathrm{Li}_2\left(\frac{c\sqrt{-d}\,e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{b\,\mathrm{Li}_2\left(-\frac{c\sqrt{-d}\,e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{b\,\mathrm{Li}_2\left(\frac{c\sqrt{-d}\,e^{\mathrm{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-
(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqr
t[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e])
 + ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqr
t[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCsch[c*x])*Log[1 + (c*
Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[
e]) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d)
+ e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(
Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((c*S
qrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(2*Sqrt[-d]*Sqrt[
e]) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) +
e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6429

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x^(2*(p + 1)
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{d+ex^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right) dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{e}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\,\sinh(x)}\,dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\,\sinh(x)}\,dx\right.}{2\sqrt{e}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}\,e^x}\,dx, x, \text{csch}^{-1}(cx)\right)}{2\sqrt{e}} - \frac{\text{Subst}\left(\int \frac{}{\frac{\sqrt{e}}{c}+\frac{\sqrt{-}}{}}\right.}{}$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{}}{\sqrt{e}}\right.}{2\sqrt{-d}\,\sqrt{e}}$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{}}{\sqrt{e}}\right.}{2\sqrt{-d}\,\sqrt{e}}$$

$$= \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2\sqrt{-d}\,\sqrt{e}} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{}}{\sqrt{e}}\right.}{2\sqrt{-d}\,\sqrt{e}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.36, size = 1055, normalized size = 2.21

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2),x]
```

```
[Out] (4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[
d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4
```

```
])/Sqrt[-(c^2*d) + e]] + (8*I)*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[
2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(
c^2*d) + e]] - b*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*
x])/(c*Sqrt[d])] + (2*I)*b*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d
) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - 4*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[
d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])] + b*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x]
)/(c*Sqrt[d])] - (2*I)*b*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d)
+ e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d]
)]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*S
qrt[d])] + b*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])] - (2*I)*b*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e]
)*E^ArcCsch[c*x])/(c*Sqrt[d])] - 4*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/S
qrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d
])] - b*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])] + (2*I)*b*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^A
rcCsch[c*x])/(c*Sqrt[d])] + 4*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2
]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] -
 b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + b*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] - (2
*I)*b*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*S
qrt[d])] + (2*I)*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[
c*x])/(c*Sqrt[d])] + (2*I)*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e]
)*E^ArcCsch[c*x])/(c*Sqrt[d])] - (2*I)*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^
2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/(4*Sqrt[d]*Sqrt[e])
```

Maple [**F**]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arccsch}(cx)}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x^2+d),x)
```

```
[Out] int((a+b*arccsch(c*x))/(e*x^2+d),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] a*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/sqrt(d) + b*integrate(log(sqrt(1/(c^2*
x^2) + 1) + 1/(c*x))/(x^2*e + d), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*arccsch(c*x) + a)/(x^2*e + d), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acsch}(cx)}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d),x)
[Out] Integral((a + b*acsch(c*x))/(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/(e*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x^2),x)
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x^2), x)
```

3.101 $\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex^2)}\,dx$

Optimal. Leaf size=425

$$\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2bd}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1+\frac{c\sqrt{-d}\,e^{\text{cs}}}{\sqrt{e}-\sqrt{-}}\right.}{2d}$$

```
[Out] 1/2*(a+b*arccsch(c*x))^2/b/d-1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/
x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d-1/2*(a+b*arccsch(c*x))
*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/
d-1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(
1/2)+(-c^2*d+e)^(1/2)))/d-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d-1/2*b*polylog(2,-c*(1/c/x
+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d-1/2*b*polylo
g(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d-
1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+
e)^(1/2)))/d-1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1
/2)+(-c^2*d+e)^(1/2)))/d
```

Rubi [A]
time = 0.59, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6439, 5823, 5827, 5680, 2221, 2317, 2438}

$$-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2d}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2d}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2d}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2bd}-\frac{b\text{Li}_2\left(-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d}-\frac{b\text{Li}_2\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d}-\frac{b\text{Li}_2\left(-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2d}-\frac{b\text{Li}_2\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] (a + b*ArcCsch[c*x])^2/(2*b*d) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*
E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d) - ((a + b*ArcCsch[c*
x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2
*d) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
Sqrt[-(c^2*d) + e])])/(2*d) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^A
rcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d) - (b*PolyLog[2, -((c*Sq
rt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*d) - (b*PolyLog
[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d) - (b
*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])
/(2*d) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d)
 + e])])/(2*d)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)}\,dx = -\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}\,x\right)}+\frac{\sqrt{-d}\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{-d}}+\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\,\sinh(x)}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2\sqrt{-d}}+\frac{\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}+\sqrt{-d}\,\sinh(x)}\,dx,\right.}{2\sqrt{-d}}$$

$$= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd}-\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}\,e^x}\,dx, x, \operatorname{csch}^{-1}(cx)\right)}{2\sqrt{-d}} - \cdots$$

$$= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d}-\frac{(a+b\operatorname{csch}}{}$$

$$= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d}-\frac{(a+b\operatorname{csch}}{}$$

$$= \frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2d}-\frac{(a+b\operatorname{csch}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.63, size = 387, normalized size = 0.91

$$\frac{4a\log(x)-2a\log\left(d+ex^2\right)+b\left(-2\left(\operatorname{csch}^{-1}(cx)^2+i\text{ArcSin}\left(\sqrt{\frac{e}{c^2d}}\right)\left(2\tanh^{-1}\left(\frac{\sqrt{e\left(-c^2d+e\right)}}{ce\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)-\log\left(\frac{e^{-2\operatorname{csch}^{-1}(cx)}\left(2e-2\sqrt{e\left(-c^2d+e\right)}+c^2d\left(-1+e^{2\operatorname{csch}^{-1}(cx)}\right)\right)}{c^2d}\right)+\log\left(\frac{e^{-2\operatorname{csch}^{-1}(cx)}\left(2\left(e+\sqrt{e\left(-c^2d+e\right)}\right)+c^2d\left(-1+e^{2\operatorname{csch}^{-1}(cx)}\right)\right)}{c^2d}\right)\right)+\operatorname{csch}^{-1}(cx)\left(\log\left(\frac{e^{-2\operatorname{csch}^{-1}(cx)}\left(2e-2\sqrt{e\left(-c^2d+e\right)}+c^2d\left(-1+e^{2\operatorname{csch}^{-1}(cx)}\right)\right)}{c^2d}\right)+\log\left(\frac{e^{-2\operatorname{csch}^{-1}(cx)}\left(2\left(e+\sqrt{e\left(-c^2d+e\right)}\right)+c^2d\left(-1+e^{2\operatorname{csch}^{-1}(cx)}\right)\right)}{c^2d}\right)\right)\right)+\text{PolyLog}\left(2,\frac{\left(c^2d-2e+2\sqrt{e\left(-c^2d+e\right)}\right)e^{-2\operatorname{csch}^{-1}(cx)}}{c^2d}\right)+\text{PolyLog}\left(2,\frac{\left(c^2d-2\left(e+\sqrt{e\left(-c^2d+e\right)}\right)\right)e^{-2\operatorname{csch}^{-1}(cx)}}{c^2d}\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] (4*a*Log[x] - 2*a*Log[d + e*x^2] + b*(-2*(ArcCsch[c*x]^2 + I*ArcSin[Sqrt[e/
(c^2*d)]]*(2*ArcTanh[Sqrt[e*(-(c^2*d) + e)]/(c*e*Sqrt[1 + 1/(c^2*x^2)]*x)]
- Log[(2*e - 2*Sqrt[e*(-(c^2*d) + e)] + c^2*d*(-1 + E^(2*ArcCsch[c*x])))/(c
```

```
^2*d*E^(2*ArcCsch[c*x]))] + Log[(2*(e + Sqrt[e*(-(c^2*d) + e)]) + c^2*d*(-1
 + E^(2*ArcCsch[c*x])))/(c^2*d*E^(2*ArcCsch[c*x]))]) + ArcCsch[c*x]*(Log[(2
*e - 2*Sqrt[e*(-(c^2*d) + e)] + c^2*d*(-1 + E^(2*ArcCsch[c*x])))/(c^2*d*E^(
2*ArcCsch[c*x]))] + Log[(2*(e + Sqrt[e*(-(c^2*d) + e)]) + c^2*d*(-1 + E^(2*
ArcCsch[c*x])))/(c^2*d*E^(2*ArcCsch[c*x]))]) + PolyLog[2, (c^2*d - 2*e + 2
*Sqrt[e*(-(c^2*d) + e)])/(c^2*d*E^(2*ArcCsch[c*x]))] + PolyLog[2, (c^2*d -
2*(e + Sqrt[e*(-(c^2*d) + e)]))/(c^2*d*E^(2*ArcCsch[c*x]))]))/(4*d)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsch}(cx)}{x\,(e\,x^2+d)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(e*x^2+d),x)
[Out] int((a+b*arccsch(c*x))/x/(e*x^2+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d),x, algorithm="maxima")
[Out] -1/2*a*(log(x^2*e + d)/d - 2*log(x)/d) + b*integrate(log(sqrt(1/(c^2*x^2) +
 1) + 1/(c*x))/(x^3*e + d*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*arccsch(c*x) + a)/(x^3*e + d*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{acsch}\,(cx)}{x\,(d+ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x*(d + e*x**2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)), x)
```

3.102 $\int \frac{a+b\,\text{csch}^{-1}(cx)}{x^2(d+ex^2)}\,dx$

Optimal. Leaf size=518

$$\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d}-\frac{a}{dx}-\frac{b\text{csch}^{-1}(cx)}{dx}+\frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{-c^2d+e}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\left(a+b\text{csch}^{-1}(\right.}{}$$

```
[Out] -a/d/x-b*arccsch(c*x)/d/x+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*(a+b
*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2
*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/
c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1
/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/
x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*b
*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/
2)))*e^(1/2)/(-d)^(3/2)-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)
^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*b*polylog(2,c*(1/
c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*e^(1/2)/(-d
)^(3/2)+b*c*(1+1/c^2/x^2)^(1/2)/d
```

Rubi [A]
time = 0.73, antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6439, 5823, 5772, 267, 5793, 5827, 5680, 2221, 2317, 2438}

$$\frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2(-d)^{3/2}}+\frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2(-d)^{3/2}}-\frac{a}{dx}-\frac{b\sqrt{e}\,\text{Li}_2\left(-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\,\text{Li}_2\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}-\frac{b\sqrt{e}\,\text{Li}_2\left(-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\,\text{Li}_2\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2(-d)^{3/2}}+\frac{bc\sqrt{\frac{1}{c^2x^2}+1}}{d}-\frac{b\text{csch}^{-1}(cx)}{dx}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/d - a/(d*x) - (b*ArcCsch[c*x])/(d*x) + (Sqrt[e]
*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-
(c^2*d) + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1 + (c*S
qrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/2)) + (
Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
 Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1
 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/
2)) - (b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-
(c^2*d) + e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcCs
ch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyL
og[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(2*(-
```

```
d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sq
rt[-(c^2*d) + e])])/(2*(-d)^(3/2))
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
```

```
 0 || IGtQ[n, 0])
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = -\text{Subst}\left(\int \frac{x^2\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{d} - \frac{e\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{a}{dx} - \frac{b\text{Subst}\left(\int \sinh^{-1}\left(\frac{x}{c}\right)dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{a+}{2\sqrt{e}\left(\right.}\right.\right.}{d}$$

$$= -\frac{a}{dx} - \frac{b\operatorname{csch}^{-1}(cx)}{dx} + \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cd} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\right.}{2d}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\operatorname{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{(a+bx)\cosh(x)}{\frac{\sqrt{e}}{c}-\sqrt{-d}\sinh(x)}\,dx, x, \operatorname{csch}^{-1}\right.}{2d}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\operatorname{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{e^x(a+bx)}{\frac{\sqrt{e}}{c}-\frac{\sqrt{-c^2d+e}}{c}-\sqrt{-d}\,e^x}\,dx\right.}{2d}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\operatorname{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-}}}{\sqrt{e}-\sqrt{-c^2d}}\right.}{2(-d)^{3/2}}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\operatorname{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-}}}{\sqrt{e}-\sqrt{-c^2d}}\right.}{2(-d)^{3/2}}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d} - \frac{a}{dx} - \frac{b\operatorname{csch}^{-1}(cx)}{dx} + \frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-}}}{\sqrt{e}-\sqrt{-c^2d}}\right.}{2(-d)^{3/2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.15, size = 1211, normalized size = 2.34

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] -(a/(d*x)) - (a*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/d^(3/2) + b*((c*Sqrt[1
 + 1/(c^2*x^2)] - ArcCsch[c*x]/x)/d - ((I/16)*Sqrt[e]*(Pi^2 - (4*I)*Pi*ArcC
sch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[
2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(
c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1
 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcC
sch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt
[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-S
qrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1
+ (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsc
h[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d]
)] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[
e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e
] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, (I*(-
Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (
(-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]))/d^(3/2)
+ ((I/16)*Sqrt[e]*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 - 32*Arc
Sin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Co
t[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1
- E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e]
)*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt
[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 - Sqrt[
e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs
ch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*
E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(
c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 - Sqrt[e]/
(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c
*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 4*PolyLog[2, E^
(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^A
rcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*
E^ArcCsch[c*x])/(c*Sqrt[d])]))/d^(3/2))
```

Maple [**F**]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{a + b \,\mathrm{arccsch}(cx)}{x^2 \left(e\, x^2 + d\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x^2+d),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^2/(e*x^2+d),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
[Out] -a*(arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/d^(3/2) + 1/(d*x)) + b*integrate(log(
sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(x^4*e + d*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*arccsch(c*x) + a)/(x^4*e + d*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}\left(cx\right)}{x^{2}\left(d + ex^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x**2+d),x)
[Out] Integral((a + b*acsch(c*x))/(x**2*(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x^2\,\left(e\,x^2+d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)), x)
```

3.103 $\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=591

$$\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{2ce^2}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)}+\frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)^2}{be^3}-\frac{bd\text{ArcTan}\left(\frac{\sqrt{c^2d}}{c\sqrt{e}\sqrt{1-}}\right.}{2\sqrt{c^2d-e}\,e^5}$$

```
[Out] 1/2*d*(a+b*arccsch(c*x))/e^2/(e+d/x^2)+1/2*x^2*(a+b*arccsch(c*x))/e^2+2*d*(
a+b*arccsch(c*x))^2/b/e^3+2*d*(a+b*arccsch(c*x))*ln(1-1/(1/c/x+(1+1/c^2/x^2
)^(1/2))^2)/e^3-d*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3-d*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+
(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3-d*(a+b*arcc
sch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)
^(1/2)))/e^3-d*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(
1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3-b*d*polylog(2,1/(1/c/x+(1+1/c^2/x^2)^(
1/2))^2)/e^3-b*d*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)-(-c^2*d+e)^(1/2)))/e^3-b*d*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^
(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3-b*d*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)
^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3-b*d*polylog(2,c*(1/c/x+(
1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3-1/2*b*d*arct
an((c^2*d-e)^(1/2)/c/x/e^(1/2)/(1+1/c^2/x^2)^(1/2))/e^(5/2)/(c^2*d-e)^(1/2)
+1/2*b*x*(1+1/c^2/x^2)^(1/2)/c/e^2
```

Rubi [A]
time = 0.91, antiderivative size = 591, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 14, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6439, 5823, 5776, 270, 5775, 3797, 2221, 2317, 2438, 5821, 385, 211, 5827, 5680}

$$-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{e^3}-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{e^3}-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{e^3}-\frac{d\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{e^3}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)^2}{be^3}+\frac{2d\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2\left(\frac{d}{x^2}+e\right)}+\frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2}-\frac{bd\text{ArcTan}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\,x\sqrt{\frac{1}{c^2x^2}+1}}\right)}{2e^{5/2}\sqrt{c^2d-e}}-\frac{bd\text{Li}_2\left(-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{e^3}-\frac{bd\text{Li}_2\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{e^3}-\frac{bd\text{Li}_2\left(-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{e^3}-\frac{bd\text{Li}_2\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{e^3}+\frac{bx\sqrt{\frac{1}{c^2x^2}+1}}{2ce^2}-\frac{bd\text{Li}_2\left(e^{-2\text{csch}^{-1}(cx)}\right)}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (b*Sqrt[1 + 1/(c^2*x^2)]*x)/(2*c*e^2) + (d*(a + b*ArcCsch[c*x]))/(2*e^2*(e
+ d/x^2)) + (x^2*(a + b*ArcCsch[c*x]))/(2*e^2) + (2*d*(a + b*ArcCsch[c*x])^
2)/(b*e^3) - (b*d*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x
)])/(2*Sqrt[c^2*d - e]*e^(5/2)) + (2*d*(a + b*ArcCsch[c*x])*Log[1 - E^(-2*A
rcCsch[c*x])])/e^3 - (d*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[
c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/e^3 - (d*(a + b*ArcCsch[c*x])*Log[1
```

```
+ (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/e^3 - (d*(a
+ b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2
*d) + e])])/e^3 - (d*(a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x
])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/e^3 - (b*d*PolyLog[2, E^(-2*ArcCsch[c*x
])])/e^3 - (b*d*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(
c^2*d) + e]))])/e^3 - (b*d*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
- Sqrt[-(c^2*d) + e])])/e^3 - (b*d*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])
/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/e^3 - (b*d*PolyLog[2, (c*Sqrt[-d]*E^ArcC
sch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/e^3
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 + c^2*x^2], x
], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x^3\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e^2x^3} - \frac{2d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e^3x} + \frac{d^2x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right.\right.}{e^2\left(e+dx^2\right)^2}\right.\right.$$

$$= \frac{(2d)\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^3} - \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^3}$$

$$= \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2} + \frac{(2d)\text{Subst}\left(\int (a+bx)\coth(x)\,d\right.}{e^3}$$

$$= \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2} - \frac{d\left(a+b\text{csch}^{-}\right.}{be^3}$$

$$= \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2} - \frac{d\left(a+b\text{csch}^{-}\right.}{be^3}$$

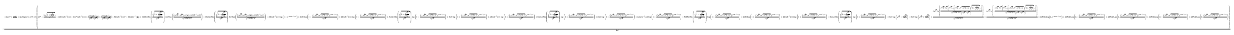
$$= \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2} - \frac{bd\tan^{-1}\left(\frac{-}{c\sqrt{}}\right.}{2\sqrt{c^2}}$$

$$= \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2} - \frac{bd\tan^{-1}\left(\frac{-}{c\sqrt{}}\right.}{2\sqrt{c^2}}$$

$$= \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2} - \frac{bd\tan^{-1}\left(\frac{-}{c\sqrt{}}\right.}{2\sqrt{c^2}}$$

$$= \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x}{2ce^2} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{2e^2} - \frac{bd\tan^{-1}\left(\frac{-}{c\sqrt{}}\right.}{2\sqrt{c^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.91, size = 1447, normalized size = 2.45



Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(-2*a*e*x^2 + (2*a*d^2)/(d + e*x^2) + 4*a*d*Log[d + e*x^2] + b*(d*Pi^2
 - (2*e*Sqrt[1 + 1/(c^2*x^2)]*x)/c - (4*I)*d*Pi*ArcCsch[c*x] - 2*e*x^2*ArcC
sch[c*x] + (d^(3/2)*ArcCsch[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (d^(3/2)*ArcCsc
h[c*x])/(Sqrt[d] + I*Sqrt[e]*x) - 8*d*ArcCsch[c*x]^2 - 2*d*ArcSinh[1/(c*x)]
 + 16*d*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] -
Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 16*d*ArcSi
n[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[
(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*d*ArcCsch[c*x]*Log[1
- E^(-2*ArcCsch[c*x])] + (2*I)*d*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*d*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] +
Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*d*ArcSin[Sqrt[1 +
Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^
ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*d*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d
) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*d*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e
] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*d*ArcSin[Sqrt[
1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e]
)*E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*d*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^
2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*d*ArcCsch[c*x]*Log[1 - (I*(Sqrt
[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*d*ArcSin[Sqr
t[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e
])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (2*I)*d*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c
^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*d*ArcCsch[c*x]*Log[1 + (I*(Sqr
t[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*d*ArcSin[Sq
rt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (2*I)*d*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x]
- (2*I)*d*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + (d*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[
e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*
x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]])/Sqrt[-(c^2*d) + e] + (d*
Sqrt[e]*Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) +
 e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))
]])/Sqrt[-(c^2*d) + e] + 4*d*PolyLog[2, E^(-2*ArcCsch[c*x])] + 4*d*PolyLog[2
, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 4*d*
PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]
+ 4*d*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sq
rt[d])] + 4*d*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/
(c*Sqrt[d])]))/e^3
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x^5(a+b\,\mathrm{arccsch}(cx))}{\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(x^2*e^(-2) - 2*d*e^(-3)*log(x^2*e + d) - d^2/(x^2*e^4 + d*e^3))*a + b*
integrate(x^5*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(x^4*e^2 + 2*d*x^2*e + d
^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^5*arccsch(c*x) + a*x^5)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^5/(e*x^2 + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.104 $\int \frac{x^3\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=553

$$-\frac{a+b\text{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{be^2}+\frac{b\text{ArcTan}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2\sqrt{c^2d-e}\,e^{3/2}}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-e^{-2\text{cs}}\right.}{e^2}$$

```
[Out] 1/2*(-a-b*arccsch(c*x))/e/(e+d/x^2)-(a+b*arccsch(c*x))^2/b/e^2-(a+b*arccsch
(c*x))*ln(1-1/(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e^2+1/2*(a+b*arccsch(c*x))*ln(
1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^2+
1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)-(-c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2
)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^2+1/2*(a+b*arccsch(c*x))*
ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e
^2+1/2*b*polylog(2,1/(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e^2+1/2*b*polylog(2,-c*
(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^2+1/2*
b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1
/2)))/e^2+1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(-c^2*d+e)^(1/2)))/e^2+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)
^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^2+1/2*b*arctan((c^2*d-e)^(1/2)/c/x/e^(
1/2)/(1+1/c^2/x^2)^(1/2))/e^(3/2)/(c^2*d-e)^(1/2)
```

Rubi [A]
time = 0.84, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6439, 5823, 5775, 3797, 2221, 2317, 2438, 5821, 385, 211, 5827, 5680}

$$\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^2}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2e^2}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2e^2}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2e^2}-\frac{a+b\text{csch}^{-1}(cx)}{2e\left(\frac{d}{x^2}+e\right)}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{be^2}-\frac{\log\left(1-e^{-2\text{csch}^{-1}(cx)}\right)\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{b\text{ArcTan}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\,x\sqrt{\frac{1}{c^2x^2}+1}}\right)}{2e^{3/2}\sqrt{c^2d-e}}+\frac{b\text{Li}_2\left(-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^2}+\frac{b\text{Li}_2\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2e^2}+\frac{b\text{Li}_2\left(-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e^2}+\frac{b\text{Li}_2\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2e^2}+\frac{b\text{Li}_2\left(e^{-2\text{csch}^{-1}(cx)}\right)}{2e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(a + b*ArcCsch[c*x])/(e*(e + d/x^2)) - (a + b*ArcCsch[c*x])^2/(b*e^2)
+ (b*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])/(2*Sqrt[c
^2*d - e]*e^(3/2)) - ((a + b*ArcCsch[c*x])*Log[1 - E^(-2*ArcCsch[c*x])])/e^
2 + ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sq
rt[-(c^2*d) + e])])/(2*e^2) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^A
rcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^2) + ((a + b*ArcCsch[c*x
])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*
e^2) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
```

```
 Sqrt[-(c^2*d) + e])])/(2*e^2) + (b*PolyLog[2, E^(-2*ArcCsch[c*x])])/(2*e^2
) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) +
e]))])/(2*e^2) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[
-(c^2*d) + e])])/(2*e^2) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqr
t[e] + Sqrt[-(c^2*d) + e]))])/(2*e^2) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch
[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 + c^2*x^2], x
], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e^2x} - \frac{dx\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^2} - \frac{dx\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)}\right.\right.$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subs}}{}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int (a+bx)\coth(x)\,dx, x, \text{csch}^{-1}(cx)\right)}{e^2} + \frac{d\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2be^2} + \frac{2\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1-e^{2x}}\,dx, x, \text{csch}^{-1}(cx\right.}{e^2}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2be^2} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2\sqrt{c^2d-e}\,e^{3/2}} - \frac{(a}{}$$

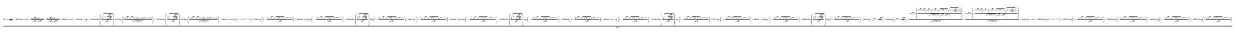
$$= -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2\sqrt{c^2d-e}\,e^{3/2}} - \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1\right.}{e^2}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2\sqrt{c^2d-e}\,e^{3/2}} + \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1\right.}{2e}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2\sqrt{c^2d-e}\,e^{3/2}} + \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1\right.}{2e}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(e+\frac{d}{x^2}\right)} + \frac{b\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2\sqrt{c^2d-e}\,e^{3/2}} + \frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1\right.}{2e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.56, size = 1410, normalized size = 2.55



Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (b*Pi^2 + (4*a*d)/(d + e*x^2) - (4*I)*b*Pi*ArcCsch[c*x] + (2*b*Sqrt[d]*ArcC
sch[c*x])/(Sqrt[d] - I*Sqrt[e]*x) + (2*b*Sqrt[d]*ArcCsch[c*x])/(Sqrt[d] + I
*Sqrt[e]*x) - 8*b*ArcCsch[c*x]^2 - 4*b*ArcSinh[1/(c*x)] + 16*b*ArcSin[Sqrt[
1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (
2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 16*b*ArcSin[Sqrt[1 - Sqrt[e]/(
c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[
c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*b*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x]
)] + (2*I)*b*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/
(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*
E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]
/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])] + (2*I)*b*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*
x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (8*I)*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[
d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])] + (2*I)*b*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch
[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d)
+ e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])] + (2*I)*b*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsc
h[c*x])/(c*Sqrt[d])] + 4*b*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d)
 + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (8*I)*b*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/
(c*Sqrt[d])] - (2*I)*b*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] - (2*I)*b*Pi*Log[Sqr
t[e] + (I*Sqrt[d])/x] + (2*b*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(I*Sqrt[e] + c*
(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d)
 + e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) + e] + (2*b*Sqrt[e]*Log[(-2*
Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(
c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d)
 + e] + 4*a*Log[d + e*x^2] + 4*b*PolyLog[2, E^(-2*ArcCsch[c*x])] + 4*b*Poly
Log[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] +
 4*b*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[
d])] + 4*b*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/
(c*Sqrt[d])] + 4*b*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c
*x])/(c*Sqrt[d])])/(8*e^2)
```

Maple [F]

time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\,\mathrm{arccsch}(cx))}{\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(e^(-2)*log(x^2*e + d) + d/(x^2*e^3 + d*e^2))*a + b*integrate(x^3*log(s
qrt(1/(c^2*x^2) + 1) + 1/(c*x))/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccsch(c*x) + a*x^3)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\,\mathrm{acsch}\,(cx))}{\left(d+ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))/(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x^2 + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.105 $\int \frac{x\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=139

$$-\frac{a+b\mathrm{csch}^{-1}(cx)}{2e\,(d+ex^2)}+\frac{bcx\mathrm{ArcTan}\left(\sqrt{-1-c^2x^2}\right)}{2de\sqrt{-c^2x^2}}+\frac{bcx\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{-1-c^2x^2}}{\sqrt{c^2d-e}}\right)}{2d\sqrt{c^2d-e}\,\sqrt{e}\,\sqrt{-c^2x^2}}$$

```
[Out] 1/2*(-a-b*arccsch(c*x))/e/(e*x^2+d)+1/2*b*c*x*arctan((-c^2*x^2-1)^(1/2))/d/
e/(-c^2*x^2)^(1/2)+1/2*b*c*x*arctanh(e^(1/2)*(-c^2*x^2-1)^(1/2)/(c^2*d-e)^(
1/2))/d/(c^2*d-e)^(1/2)/e^(1/2)/(-c^2*x^2)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6435, 457, 88, 65, 211, 214}

$$-\frac{a+b\mathrm{csch}^{-1}(cx)}{2e\,(d+ex^2)}+\frac{bcx\mathrm{ArcTan}\left(\sqrt{-c^2x^2-1}\right)}{2de\sqrt{-c^2x^2}}+\frac{bcx\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{2d\sqrt{e}\,\sqrt{-c^2x^2}\,\sqrt{c^2d-e}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(a + b*ArcCsch[c*x])/(e*(d + e*x^2)) + (b*c*x*ArcTan[Sqrt[-1 - c^2*x^2
]])/(2*d*e*Sqrt[-(c^2*x^2)]) + (b*c*x*ArcTanh[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/
Sqrt[c^2*d - e]])/(2*d*Sqrt[c^2*d - e]*Sqrt[e]*Sqrt[-(c^2*x^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 88

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :> Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d
/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f,
 p}, x] &&  !IntegerQ[p]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6435

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2])), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rubi steps

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{(bcx)\int \frac{1}{x\sqrt{-1-c^2x^2}\left(d+ex^2\right)}\,dx}{2e\sqrt{-c^2x^2}}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{(bcx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\,(d+ex)}\,dx,x,x^2\right)}{4e\sqrt{-c^2x^2}}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(d+ex^2\right)}-\frac{(bcx)\text{Subst}\left(\int \frac{1}{\sqrt{-1-c^2x}\,(d+ex)}\,dx,x,x^2\right)}{4d\sqrt{-c^2x^2}}+\frac{(bcx)\text{Subst}\cdots}{\cdots}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{(bx)\text{Subst}\left(\int \frac{1}{d-\frac{e}{c^2}-\frac{ex^2}{c^2}}\,dx,x,\sqrt{-1-c^2x^2}\right)}{2cd\sqrt{-c^2x^2}}-\frac{(bx)\text{Subst}\cdots}{\cdots}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{bcx\tan^{-1}\left(\sqrt{-1-c^2x^2}\right)}{2de\sqrt{-c^2x^2}}+\frac{bcx\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{-1-c^2\cdots}}{\sqrt{c^2d-e}}\right)}{2d\sqrt{c^2d-e}\,\sqrt{e}\,\sqrt{-c^2x^2}}\cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.54, size = 271, normalized size = 1.95

$$-\frac{\frac{2a}{d+ex^2}+\frac{2b\operatorname{csch}^{-1}(cx)}{d+ex^2}-\frac{2b\sinh^{-1}\left(\frac{1}{cx}\right)}{d}+\frac{b\sqrt{e}\log\left(-\frac{4\left(ide+cd\sqrt{e}\left(c\sqrt{d}+i\sqrt{-c^2d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{b\sqrt{-c^2d+e}\left(\sqrt{d}-i\sqrt{e}x\right)}\right)}{d\sqrt{-c^2d+e}}+\frac{b\sqrt{e}\log\left(\frac{4i\left(de+cd\sqrt{e}\left(ic\sqrt{d}+\sqrt{-c^2d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{b\sqrt{-c^2d+e}\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{d\sqrt{-c^2d+e}}}{4e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*((2*a)/(d + e*x^2) + (2*b*ArcCsch[c*x])/(d + e*x^2) - (2*b*ArcSinh[1/(
c*x)])/d + (b*Sqrt[e]*Log[(-4*(I*d*e + c*d*Sqrt[e]*(c*Sqrt[d] + I*Sqrt[-(c^
2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*Sqrt[-(c^2*d) + e]*(Sqrt[d] - I*Sqr
t[e]*x))]])/(d*Sqrt[-(c^2*d) + e]) + (b*Sqrt[e]*Log[((4*I)*(d*e + c*d*Sqrt[e
]*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*Sqrt[-(c^
2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))]])/(d*Sqrt[-(c^2*d) + e]))/e
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(120) = 240$.
time = 5.20, size = 356, normalized size = 2.56

method	result
derivativedivides	$-\frac{a c^4}{2e\left(c^2 e x^2+c^2 d\right)}-\frac{b c^4 \operatorname{arccsch}(cx)}{2e\left(c^2 e x^2+c^2 d\right)}+\frac{bc\sqrt{c^2x^2+1}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)}{2e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,xd}-\frac{bc\sqrt{c^2x^2+1}\,\ln\left(\frac{2\sqrt{c^2x^2+1}\ldots}{\ldots}\right)}{4e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\ldots}\ldots$
default	$-\frac{a c^4}{2e\left(c^2 e x^2+c^2 d\right)}-\frac{b c^4 \operatorname{arccsch}(cx)}{2e\left(c^2 e x^2+c^2 d\right)}+\frac{bc\sqrt{c^2x^2+1}\,\operatorname{arctanh}\left(\frac{1}{\sqrt{c^2x^2+1}}\right)}{2e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\,xd}-\frac{bc\sqrt{c^2x^2+1}\,\ln\left(\frac{2\sqrt{c^2x^2+1}\ldots}{\ldots}\right)}{4e\sqrt{\frac{c^2x^2+1}{c^2x^2}}\ldots}\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/2*a*c^4/e/(c^2*e*x^2+c^2*d)-1/2*b*c^4/e/(c^2*e*x^2+c^2*d)*arccsch
(c*x)+1/2*b*c/e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d*arctanh(1
/(c^2*x^2+1)^(1/2))-1/4*b*c/e*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)
/x/d/(-(c^2*d-e)/e)^(1/2)*ln(2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e-(-
c^2*d*e)^(1/2)*c*x+e)/(e*c*x+(-c^2*d*e)^(1/2)))-1/4*b*c/e*(c^2*x^2+1)^(1/2)
/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/(-(c^2*d-e)/e)^(1/2)*ln(-2*((c^2*x^2+1)^(1
```

```
/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-e*c*x+(-c^2*d*e)^(1/2)
)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*c^2*integrate(1/2*x/(c^2*x^4*e^2 + (c^2*d*e + e^2)*x^2 + d*e + (c^2
*x^4*e^2 + (c^2*d*e + e^2)*x^2 + d*e)*sqrt(c^2*x^2 + 1)), x) - (2*c^2*d^2*l
og(c) - 2*(c^2*d*e - e^2)*x^2*log(x) - 2*d*e*log(c) + (c^2*d*x^2*e + c^2*d^
2)*log(c^2*x^2 + 1) - 2*(c^2*d^2 - d*e)*log(sqrt(c^2*x^2 + 1) + 1))/(c^2*d^
3*e + (c^2*d^2*e^2 - d*e^3)*x^2 - d^2*e^2) + log(x^2*e + d)/(c^2*d^2 - d*e)
)*b - 1/2*a/(x^2*e^2 + d*e)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(116) = 232$.
time = 0.39, size = 1060, normalized size = 7.63

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*c^2*d^2 - 2*a*d*cosh(1) - 2*a*d*sinh(1) + (b*x^2*cosh(1) + b*x^2
*sinh(1) + b*d)*sqrt(-(c^2*d - cosh(1) - sinh(1))/(cosh(1) - sinh(1)))*log(
-(c^2*d + 2*c*x*sqrt(-(c^2*d - cosh(1) - sinh(1))/(cosh(1) - sinh(1)))*sqrt
((c^2*x^2 + 1)/(c^2*x^2)) - (c^2*x^2 + 2)*cosh(1) - (c^2*x^2 + 2)*sinh(1))/
(x^2*cosh(1) + x^2*sinh(1) + d)) - 2*(b*c^2*d^2 - b*x^2*cosh(1)^2 - b*x^2*s
inh(1)^2 + (b*c^2*d*x^2 - b*d)*cosh(1) + (b*c^2*d*x^2 - 2*b*x^2*cosh(1) - b
*d)*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 2*(b*c^2*d^
2 - b*x^2*cosh(1)^2 - b*x^2*sinh(1)^2 + (b*c^2*d*x^2 - b*d)*cosh(1) + (b*c^
2*d*x^2 - 2*b*x^2*cosh(1) - b*d)*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x
^2)) - c*x - 1) + 2*(b*c^2*d^2 - b*d*cosh(1) - b*d*sinh(1))*log((c*x*sqrt((
c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^3*cosh(1) - d*x^2*cosh(1)^3 - d
*x^2*sinh(1)^3 + (c^2*d^2*x^2 - d^2)*cosh(1)^2 + (c^2*d^2*x^2 - 3*d*x^2*cos
h(1) - d^2)*sinh(1)^2 + (c^2*d^3 - 3*d*x^2*cosh(1)^2 + 2*(c^2*d^2*x^2 - d^2
)*cosh(1))*sinh(1)), -1/2*(a*c^2*d^2 - a*d*cosh(1) - a*d*sinh(1) + (b*x^2*c
osh(1) + b*x^2*sinh(1) + b*d)*sqrt((c^2*d - cosh(1) - sinh(1))/(cosh(1) - s
inh(1)))*arctan(-c*x*sqrt((c^2*d - cosh(1) - sinh(1))/(cosh(1) - sinh(1)))*
sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d - cosh(1) - sinh(1))) - (b*c^2*d^2 - b
*x^2*cosh(1)^2 - b*x^2*sinh(1)^2 + (b*c^2*d*x^2 - b*d)*cosh(1) + (b*c^2*d*x
^2 - 2*b*x^2*cosh(1) - b*d)*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))
```

```
- c*x + 1) + (b*c^2*d^2 - b*x^2*cosh(1)^2 - b*x^2*sinh(1)^2 + (b*c^2*d*x^2
- b*d)*cosh(1) + (b*c^2*d*x^2 - 2*b*x^2*cosh(1) - b*d)*sinh(1))*log(c*x*sqr
t((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + (b*c^2*d^2 - b*d*cosh(1) - b*d*sinh
(1))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)))/(c^2*d^3*cosh(1) -
 d*x^2*cosh(1)^3 - d*x^2*sinh(1)^3 + (c^2*d^2*x^2 - d^2)*cosh(1)^2 + (c^2*d
^2*x^2 - 3*d*x^2*cosh(1) - d^2)*sinh(1)^2 + (c^2*d^3 - 3*d*x^2*cosh(1)^2 +
2*(c^2*d^2*x^2 - d^2)*cosh(1))*sinh(1))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\operatorname{acsch}(cx))}{(d + ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))/(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\left(a + b\operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{\left(e\,x^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.106 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x(d+ex^2)^2}\,dx$

Optimal. Leaf size=515

$$-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd^2}+\frac{b\sqrt{e}\operatorname{ArcTan}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d-e}}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1\right.}{2d^2}$$

```
[Out] -1/2*e*(a+b*arccsch(c*x))/d^2/(e+d/x^2)+1/2*(a+b*arccsch(c*x))^2/b/d^2-1/2*
(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(
-c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1
/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d^2-1/2*(a+b*arccsch(c*x))*ln(1
-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d^2-1
/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2
)+(-c^2*d+e)^(1/2)))/d^2-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d^2-1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x
^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d^2-1/2*b*polylog(2,-c*(1
/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d^2-1/2*b*
polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2
)))/d^2+1/2*b*arctan((c^2*d-e)^(1/2)/c/x/e^(1/2)/(1+1/c^2/x^2)^(1/2))*e^(1/
2)/d^2/(c^2*d-e)^(1/2)
```

Rubi [A]
time = 0.77, antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6439, 5823, 5821, 385, 211, 5827, 5680, 2221, 2317, 2438}

$$-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^2}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{2d^2}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{2d^2}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{2d^2}-\frac{e\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2d^2\left(\frac{d}{x^2}+e\right)}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)^2}{2bd^2}+\frac{b\sqrt{e}\operatorname{ArcTan}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\,x\sqrt{\frac{1}{c^2x^2}+1}}\right)}{2d^2\sqrt{c^2d-e}}-\frac{b\operatorname{Li}_2\left(-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^2}-\frac{b\operatorname{Li}_2\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{2d^2}-\frac{b\operatorname{Li}_2\left(-\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2d^2}-\frac{b\operatorname{Li}_2\left(\frac{c\sqrt{-d}e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] -1/2*(e*(a + b*ArcCsch[c*x]))/(d^2*(e + d/x^2)) + (a + b*ArcCsch[c*x])^2/(2
*b*d^2) + (b*Sqrt[e]*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)
]*x)])/(2*d^2*Sqrt[c^2*d - e]) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*
E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d^2) - ((a + b*ArcCsch[
c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/
(2*d^2) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e
] + Sqrt[-(c^2*d) + e])])/(2*d^2) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-
d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d^2) - (b*PolyLog[2,
 -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*d^2) -
```

```
(b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/
(2*d^2) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2
*d) + e]))])/(2*d^2) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
 Sqrt[-(c^2*d) + e])])/(2*d^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])/(2*e*(p + 1))),
```

```
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 + c^2*x^2], x
], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{x^3\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{ex\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^2} + \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int \left(-\frac{\sqrt{-d}\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{\sqrt{-d}\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right) d\right.}{d}$$

$$= -\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2(-d)^{3/2}} - \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}+\sqrt{-d}\,x}\,d\right.}{2(-d)^{3/2}}$$

$$= -\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{b\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d-e}} + \frac{\text{Subst}\left(\int \frac{(a+bx)\cosh}{\frac{\sqrt{e}}{c}-\sqrt{-d}}\right.}{2(}$$

$$= -\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2bd^2} + \frac{b\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d-e}} +$$

$$= -\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2bd^2} + \frac{b\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d-e}} -$$

$$= -\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2bd^2} + \frac{b\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d-e}} -$$

$$= -\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{2d^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2bd^2} + \frac{b\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{c^2d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{2d^2\sqrt{c^2d-e}} -$$

Mathematica [F]
time = 40.57, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^2),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^2), x]
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsch}(cx)}{x\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x)
[Out] int((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
[Out] 1/2*a*(1/(d*x^2*e + d^2) - log(x^2*e + d)/d^2 + 2*log(x)/d^2) + b*integrate
(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(x^5*e^2 + 2*d*x^3*e + d^2*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccsch(c*x) + a)/(x^5*e^2 + 2*d*x^3*e + d^2*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{acsch}\,(cx)}{x\left(d+ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d)**2,x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x*(d + e*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^2*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^2), x)
```

3.107 $\int \frac{x^4\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=756

$$-\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}\,-\frac{d}{x}\right)}+\frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}\,+\frac{d}{x}\right)}+\frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}+\frac{b\tanh^{-1}\left(\sqrt{1+\frac{1}{c^2x^2}}\right)}{ce^2}+\frac{b\sqrt{d}\,\tanh}{}$$

```
[Out] x*(a+b*arccsch(c*x))/e^2+b*arctanh((1+1/c^2/x^2)^(1/2))/c/e^2+3/4*(a+b*arcc
sch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)
^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x
^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*(a
+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c
^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+
1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)
-3/4*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d
+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2)
)*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*b*polylog(2
,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*(-d)
^(1/2)/e^(5/2)+3/4*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)+(-c^2*d+e)^(1/2)))*(-d)^(1/2)/e^(5/2)+1/4*b*arctanh((c^2*d-(-d)^(1/2)
*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))*d^(1/2)/e^2/(c^2
*d-e)^(1/2)+1/4*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^
(1/2)/(1+1/c^2/x^2)^(1/2))*d^(1/2)/e^2/(c^2*d-e)^(1/2)-1/4*d*(a+b*arccsch(c
*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*d*(a+b*arccsch(c*x))/e^2/(d/x+(-d)^(
1/2)*e^(1/2))
```

Rubi [A]
time = 1.67, antiderivative size = 756, normalized size of antiderivative = 1.00, number of steps used = 51, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6439, 5823, 5776, 272, 65, 214, 5793, 5828, 739, 212, 5827, 5680, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(d*(a + b*ArcCsch[c*x]))/(e^2*(Sqrt[-d]*Sqrt[e] - d/x)) + (d*(a + b*Ar
cCsch[c*x]))/(4*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) + (x*(a + b*ArcCsch[c*x]))/e^
2 + (b*ArcTanh[Sqrt[1 + 1/(c^2*x^2)]])/(c*e^2) + (b*Sqrt[d]*ArcTanh[(c^2*d
```

```
- (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])
/(4*Sqrt[c^2*d - e]*e^2) + (b*Sqrt[d]*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x
)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*Sqrt[c^2*d - e]*e^
2) + (3*Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(
Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcCsch[c*
x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4
*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[
c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*Arc
Csch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e
])])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(
Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (c*
Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*e^(5/2)) - (3*
b*Sqrt[-d]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d
) + e]))])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x
])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*e^(5/2))
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5776

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(d*(m + 1))), x] - Dist[b*c*
(n/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1 + c
^2*x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
```

```
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x^2\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e^2x^2} - \frac{d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^2} - \frac{d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)} \cdots\right.\right.$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^2} + \frac{d\text{Subst}\cdots}{\cdots}$$

$$= \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{ce^2} + \frac{d\text{Subst}\left(\int \left(\frac{a\cdots}{2\sqrt{e}\,(\cdots}\right.\right.}{\cdots}$$

$$= \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{d\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{2e^{5/2}} + \frac{d\text{Subst}\left(\int \frac{a+b\sinh\cdots}{\sqrt{e}+\sqrt{\cdots}}\right.}{2e^{5/\cdots}}$$

$$= -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{d\text{Su}\cdots}{\cdots}$$

$$= -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{b\tan\cdots}{\cdots}$$

$$= -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{b\tan\cdots}{\cdots}$$

$$= -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{4e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{b\tan\cdots}{\cdots}$$

$$= -\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{\cdots} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{\cdots} + \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{b\tan\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.04, size = 1583, normalized size = 2.09

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a*x)/e^2 + (a*d*x)/(2*e^2*(d + e*x^2)) - (3*a*Sqrt[d]*ArcTan[(Sqrt[e]*x)/S
qrt[d]])/(2*e^(5/2)) + b*(-1/4*(d*(-(ArcCsch[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)
) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(I*Sqrt[e] + c*(c
*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) +
 e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d]))/e^2 - (d*(-(Ar
cCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) + (I*(ArcSinh[1/(c*x)]/Sqrt[e] - L
og[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[
1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(
c^2*d) + e]))/Sqrt[d]))/(4*e^2) - (((3*I)/32)*Sqrt[d]*(Pi^2 - (4*I)*Pi*ArcC
sch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[
2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(
c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1
 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcC
sch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt
[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-S
qrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1
+ (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsc
h[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d]
)] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[
e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e
] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, (I*(-
Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (
(-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]))/e^(5/2)
+ (((3*I)/32)*Sqrt[d]*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 - 32
*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e]
)*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Lo
g[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d)
+ e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] +
Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 - S
qrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^A
rcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqr
t[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 - Sqrt
[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs
ch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 4*PolyLog[2
, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])
```

```
*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])]))/e^(5/2) + ((ArcCsch[c*x]*Coth[ArcCsch[c*
x]/2])/2 - Log[Tanh[ArcCsch[c*x]/2]] - (ArcCsch[c*x]*Tanh[ArcCsch[c*x]/2])/
2)/(c*e^2))
```

Maple [**F**]
time = 13.29, size = 0, normalized size = 0.00

$$\int \frac{x^4(a + b\,\mathrm{arccsch}(cx))}{(e\,x^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(3*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2) - 2*x*e^(-2) - d*x/(x^2*
e^3 + d*e^2))*a + b*integrate(x^4*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(x^4
*e^2 + 2*d*x^2*e + d^2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccsch(c*x) + a*x^4)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a + b\,\mathrm{acsch}\,(cx))}{(d + ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x**4*(a + b*acsch(c*x))/(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^4/(e*x^2 + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.108 $\int \frac{x^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=719

$$\frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}-\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d-e}\,\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4\sqrt{d}\,\sqrt{c^2d-e}\,e}-\frac{b\tanh^{-1}\left(\frac{c^2 \cdots}{c\sqrt{d}\,\sqrt{c \cdots}}\right)}{4\sqrt{d}\,\sqrt{\cdots}}$$

```
[Out] 1/4*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)-(-c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*(a+b*arccsch(c*x))*ln(1+c*(1/c
/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^(3/2)/(-d)
^(1/2)+1/4*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*(a+b*arccsch(c*x))*ln(1
+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^(3/
2)/(-d)^(1/2)-1/4*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(-c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*b*polylog(2,c*(1/c/x+(1+1/c
^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/
4*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)
^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(
-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*b*arctanh((c^2
*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/e/d
^(1/2)/(c^2*d-e)^(1/2)-1/4*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)
/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/e/d^(1/2)/(c^2*d-e)^(1/2)+1/4*(a+b*ar
ccsch(c*x))/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(-a-b*arccsch(c*x))/e/(d/x+(-d)
^(1/2)*e^(1/2))
```

Rubi [A]
time = 0.84, antiderivative size = 719, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6439, 5793, 5828, 739, 212, 5827, 5680, 2221, 2317, 2438}

$$\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4\sqrt{-d}\,e^{3/2}}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{4\sqrt{-d}\,e^{3/2}}+\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4\sqrt{-d}\,e^{3/2}}-\frac{\left(a+b\operatorname{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{4\sqrt{-d}\,e^{3/2}}+\frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\operatorname{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}-\frac{b\,\mathrm{Li}_2\left(-\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4\sqrt{-d}\,e^{3/2}}+\frac{b\,\mathrm{Li}_2\left(\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4\sqrt{-d}\,e^{3/2}}-\frac{b\,\mathrm{Li}_2\left(-\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4\sqrt{-d}\,e^{3/2}}+\frac{b\,\mathrm{Li}_2\left(\frac{c\sqrt{-d}\,e^{\operatorname{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4\sqrt{-d}\,e^{3/2}}-\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{\frac{1}{c^2x^2}+1}\,\sqrt{c^2d-e}}\right)}{4\sqrt{d}\,e\sqrt{c^2d-e}}-\frac{b\tanh^{-1}\left(\frac{c^2d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{\frac{1}{c^2x^2}+1}\,\sqrt{c^2d-e}}\right)}{4\sqrt{d}\,e\sqrt{c^2d-e}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a + b*ArcCsch[c*x])/(4*e*(Sqrt[-d]*Sqrt[e] - d/x)) - (a + b*ArcCsch[c*x])/
(4*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/
(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*Sqrt[d]*Sqrt[c^2*d -
 e]*e) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d -
```

```
e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*Sqrt[d]*Sqrt[c^2*d - e]*e) + ((a + b*ArcCsch
[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])
/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch
[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*Sqrt[-d]*e^(3/2)) + ((a + b*ArcC
sch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]
)])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcC
sch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLo
g[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(4*Sqr
t[-d]*e^(3/2)) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[
-(c^2*d) + e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCs
ch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLo
g[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*Sqrt[-
d]*e^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\,\sqrt{e}-dx\right)^2} - \frac{d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\,\sqrt{e}+dx\right)^2} - \frac{d\left(a+b\sin\right.}{2e\left(-de\right.}\right.\right.$$

$$= \frac{d\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e} + \frac{d\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{4e}$$

$$= \frac{a+b\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} - \frac{b\text{Subst}\left(\int \frac{1}{\left(\sqrt{-d}\,\sqrt{e}-dx\right)\sqrt{1-}}\right.}{4ce}$$

$$= \frac{a+b\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\sqrt{e}-\sqrt{-d}\,x}\,dx, x, \frac{1}{x}\right)}{4e^{3/2}}$$

$$= \frac{a+b\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} - \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d-e}\,\sqrt{1+}}\right.}{4\sqrt{d}\,\sqrt{c^2d-e}\,e}$$

$$= \frac{a+b\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} - \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d-e}\,\sqrt{1+}}\right.}{4\sqrt{d}\,\sqrt{c^2d-e}\,e}$$

$$= \frac{a+b\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} - \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d-e}\,\sqrt{1+}}\right.}{4\sqrt{d}\,\sqrt{c^2d-e}\,e}$$

$$= \frac{a+b\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{a+b\text{csch}^{-1}(cx)}{4e\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} - \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d-e}\,\sqrt{1+}}\right.}{4\sqrt{d}\,\sqrt{c^2d-e}\,e}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.65, size = 1442, normalized size = 2.01

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] ((-4*a*Sqrt[e]*x)/(d + e*x^2) + (4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] +
 b*((2*ArcCsch[c*x])/(I*Sqrt[d] - Sqrt[e]*x) - (2*ArcCsch[c*x])/(I*Sqrt[d]
+ Sqrt[e]*x) + ((8*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[
((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]
])/Sqrt[d] + ((8*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((
c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]])
/Sqrt[d] - (Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])]])/Sqrt[d] + ((2*I)*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^
2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d] - (4*ArcSin[Sqrt[1 + Sqrt[
e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs
ch[c*x])/(c*Sqrt[d])]])/Sqrt[d] + (Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) +
 e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d] - ((2*I)*ArcCsch[c*x]*Log[1 + (I
*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d] + (4
*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[
-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d] + (Pi*Log[1 - (I*(Sqrt
[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d] - ((2*I)*Ar
cCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])]])/Sqrt[d] - (4*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 -
(I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d] - (
Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/
Sqrt[d] + ((2*I)*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^A
rcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d] + (4*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])
]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])]])/Sqrt[d] - (Pi*Log[Sqrt[e] - (I*Sqrt[d])/x]])/Sqrt[d] + (Pi*Log[Sqrt[
e] + (I*Sqrt[d])/x]])/Sqrt[d] + ((2*I)*Sqrt[e]*Log[(2*Sqrt[d]*Sqrt[e]*(I*Sqr
t[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt
[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]])/(Sqrt[d]*Sqrt[-(c^2*d) + e]) - ((
2*I)*Sqrt[e]*Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2
*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e
]*x))]])/(Sqrt[d]*Sqrt[-(c^2*d) + e]) - ((2*I)*PolyLog[2, ((-I)*(-Sqrt[e] +
Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d] + ((2*I)*PolyLog[
2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d]
 + ((2*I)*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])]])/Sqrt[d] - ((2*I)*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*
E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[d]))/(8*e^(3/2))
```

Maple [F]

time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\,\mathrm{arccsch}(cx))}{\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/sqrt(d) - x/(x^2*e^2 + d*e))*a + b*
integrate(x^2*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(x^4*e^2 + 2*d*x^2*e + d
^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\,\mathrm{acsch}\,(cx))}{\left(d+ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))/(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2,x)
```

```
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.109 $\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=713

$$-\frac{a+b\text{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\text{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}+\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d-e}\,\sqrt{1+\frac{1}{c^2x^2}}}\right)}{4d^{3/2}\sqrt{c^2d-e}}+\frac{b\tanh^{-1}\left(\frac{c^2 \cdots}{c\sqrt{d}\,\sqrt{c \cdots}}\right)}{4d^{3/2}\cdots}$$

```
[Out] 1/4*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c
^2/x^2)^(1/2))/d^(3/2)/(c^2*d-e)^(1/2)+1/4*b*arctanh((c^2*d+(-d)^(1/2)*e^(1
/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d-e)^(1/
2)-1/4*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arccsch(c*x))*ln(1+c*(
1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2
)/e^(1/2)-1/4*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1
/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arccsch(c*x))*l
n(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-
d)^(3/2)/e^(1/2)+1/4*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/
(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,c*(1/c/x+(1+
1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)
+1/4*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d
+e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2)
)*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(-a-b*arccs
ch(c*x))/d/(-d/x+(-d)^(1/2)*e^(1/2))+1/4*(a+b*arccsch(c*x))/d/(d/x+(-d)^(1/
2)*e^(1/2))
```

Rubi [A]
time = 1.51, antiderivative size = 713, normalized size of antiderivative = 1.00, number of steps used = 47, number of rules used = 11, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.611$, Rules used = {6429, 5823, 5793, 5828, 739, 212, 5827, 5680, 2221, 2317, 2438}

$$-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}+1\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(1-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)\log\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e-c^2d}+\sqrt{e}}+1\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{a+b\text{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\text{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}+\frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{\frac{1}{c^2x^2}+1}\,\sqrt{c^2d-e}}\right)}{4d^{3/2}\sqrt{c^2d-e}}+\frac{b\tanh^{-1}\left(\frac{c^2d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{\frac{1}{c^2x^2}+1}\,\sqrt{c^2d-e}}\right)}{4d^{3/2}\sqrt{c^2d-e}}+\frac{b\text{Li}_2\left(-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\text{Li}_2\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}-\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{b\text{Li}_2\left(-\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{b\text{Li}_2\left(\frac{c\sqrt{-d}\,e^{\text{csch}^{-1}(cx)}}{\sqrt{e}+\sqrt{e-c^2d}}\right)}{4(-d)^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x^2)^2,x]
```

```
[Out] -1/4*(a + b*ArcCsch[c*x])/(d*(Sqrt[-d]*Sqrt[e] - d/x)) + (a + b*ArcCsch[c*x
])/(4*d*(Sqrt[-d]*Sqrt[e] + d/x)) + (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/
x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*d^(3/2)*Sqrt[c^2*
d - e]) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d -
 e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*d^(3/2)*Sqrt[c^2*d - e]) - ((a + b*ArcCsch[
```

```
c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/
(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsc
h[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) - ((a + b*A
rcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) +
 e])])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E
^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b
*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])
/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e
] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((c*Sqrt[
-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(4*(-d)^(3/2)*Sqrt[e]
) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]
)])/(4*(-d)^(3/2)*Sqrt[e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
```

```
x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 6429

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x^(2*(p + 1)
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex^2)^2}\,dx = -\text{Subst}\left(\int \frac{x^2\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{(e+dx^2)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{e\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d\,(e+dx^2)^2} + \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{d\,(e+dx^2)}\right)\,dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{(e+dx^2)^2}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right)\,dx, x, \frac{1}{x}\right)}{d} + \frac{e\text{Subst}\left(\int\right.}{}$$

$$= -\left(\frac{1}{4}\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)\right) - \frac{1}{4}\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}+dx\right.}\right.$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} - \frac{1}{2}\text{Subst}\left(\int \left(-\frac{a+b\sinh^{-1}}{2d\sqrt{e}\left(\sqrt{e}-\right.}\right.\right.$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} - \frac{b\text{Subst}\left(\int \frac{1}{d^2-\frac{de}{c^2}-x^2}\,dx, x, \frac{-d-\frac{\sqrt{-}}{}}{\sqrt{1+}}\right.}{4cd}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d-e}\,\sqrt{1+}}\right.}{4d^{3/2}\sqrt{c^2d-e}}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d-e}\,\sqrt{1+}}\right.}{4d^{3/2}\sqrt{c^2d-e}}$$

$$= -\frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} + \frac{a+b\operatorname{csch}^{-1}(cx)}{4d\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \frac{b\tanh^{-1}\left(\frac{c^2d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2d-e}\,\sqrt{1+}}\right.}{4d^{3/2}\sqrt{c^2d-e}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.26, size = 1437, normalized size = 2.02

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2)^2,x]
```

```
[Out] ((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
+ (b*((2*Sqrt[d]*ArcCsch[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) + (2*Sqrt[d]*Ar
cCsch[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) + ((8*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*S
qrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x
])/4])/Sqrt[-(c^2*d) + e]])/Sqrt[e] + ((8*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqr
t[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])
/4])/Sqrt[-(c^2*d) + e]])/Sqrt[e] - (Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d
) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[e] + ((2*I)*ArcCsch[c*x]*Log[1 -
 (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] -
 (4*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sq
rt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] + (Pi*Log[1 + (I*(-
Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]])/Sqrt[e] - ((2*I
)*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])])/Sqrt[e] + (4*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log
[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[
e] + (Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[
d])]])/Sqrt[e] - ((2*I)*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e
])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] - (4*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sq
rt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/
(c*Sqrt[d])])/Sqrt[e] - (Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Arc
Csch[c*x])/(c*Sqrt[d])]])/Sqrt[e] + ((2*I)*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e]
+ Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] + (4*ArcSin[Sqr
t[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e
])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] - (Pi*Log[Sqrt[e] - (I*Sqrt[d])/x]
)/Sqrt[e] + (Pi*Log[Sqrt[e] + (I*Sqrt[d])/x])/Sqrt[e] - ((2*I)*Log[(2*Sqrt[
d]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2
*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) +
e] + ((2*I)*Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*
d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e]
*x))])/Sqrt[-(c^2*d) + e] - ((2*I)*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*
d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] + ((2*I)*PolyLog[2, (I*(-Sqr
t[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])])/Sqrt[e] + ((2*I)*P
olyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]
)/Sqrt[e] - ((2*I)*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c
*x])/(c*Sqrt[d])])/Sqrt[e]))/(4*d^(3/2)))/2
```

Maple [F]

time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arccsch}(cx)}{\left(e\,x^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(3/2) + x/(d*x^2*e + d^2)) + b*
integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(x^4*e^2 + 2*d*x^2*e + d^2),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acsch}\left(cx\right)}{\left(d + ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral((a + b*acsch(c*x))/(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x^2 + d)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^2,x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^2, x)
```

3.110 $\int \frac{a+b\,\text{csch}^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$

Optimal. Leaf size=758

$$\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2}-\frac{a}{d^2x}-\frac{b\text{csch}^{-1}(cx)}{d^2x}+\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}-\frac{be\tanh^{-1}\left(\frac{c^2d-}{c\sqrt{d}\sqrt{c^2d}}\right.}{4d^{5/2}\sqrt{c^2d}}$$

```
[Out] -a/d^2/x-b*arccsch(c*x)/d^2/x-1/4*b*e*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/
c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d-e)^(1/2)-1/4*
b*e*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2
/x^2)^(1/2))/d^(5/2)/(c^2*d-e)^(1/2)-3/4*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(
1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/
2)+3/4*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^
(1/2)-(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*(a+b*arccsch(c*x))*ln(1-c*(
1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*e^(1/2)/(
-d)^(5/2)+3/4*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1
/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*b*polylog(2,-c*(1/c/
x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^
(5/2)-3/4*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c
^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^
(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*b*poly
log(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))*
e^(1/2)/(-d)^(5/2)+1/4*e*(a+b*arccsch(c*x))/d^2/(-d/x+(-d)^(1/2)*e^(1/2))-1
/4*e*(a+b*arccsch(c*x))/d^2/(d/x+(-d)^(1/2)*e^(1/2))+b*c*(1+1/c^2/x^2)^(1/2
)/d^2
```

Rubi [A]
time = 1.56, antiderivative size = 758, normalized size of antiderivative = 1.00, number of steps used = 50, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {6439, 5823, 5772, 267, 5793, 5828, 739, 212, 5827, 5680, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)])/d^2 - a/(d^2*x) - (b*ArcCsch[c*x])/(d^2*x) + (e
*(a + b*ArcCsch[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] - d/x)) - (e*(a + b*ArcCsch
[c*x]))/(4*d^2*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*e*ArcTanh[(c^2*d - (Sqrt[-d]*
Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*d^(5/2)*
```

```
Sqrt[c^2*d - e]) - (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*S
qrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(4*d^(5/2)*Sqrt[c^2*d - e]) - (3*Sq
rt[e]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - S
qrt[-(c^2*d) + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1
 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/
2)) - (3*Sqrt[e]*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(
Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCsch[
c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/
(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqr
t[e] - Sqrt[-(c^2*d) + e]))])/(4*(-d)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (c*S
qrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/2)) + (
3*b*Sqrt[e]*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*
d) + e]))])/(4*(-d)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[
c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(4*(-d)^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5772

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcSinh[c*x])^n, x] - Dist[b*c*n, Int[x*((a + b*ArcSinh[c*x])^(n - 1)/Sqrt[1
 + c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = -\text{Subst}\left(\int \frac{x^4\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{d^2} + \frac{e^2\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2} - \frac{2e\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)}\right) d$$

$$= -\frac{\text{Subst}\left(\int \left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2}{}$$

$$= -\frac{a}{d^2x} - \frac{b\text{Subst}\left(\int \sinh^{-1}\left(\frac{x}{c}\right)\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \right.\right.}{d^2}$$

$$= -\frac{a}{d^2x} - \frac{b\text{csch}^{-1}(cx)}{d^2x} + \frac{b\text{Subst}\left(\int \frac{x}{\sqrt{1+\frac{x^2}{c^2}}}\,dx, x, \frac{1}{x}\right)}{cd^2} + \frac{\sqrt{e}\,\text{Subst}\left(\int \frac{a+b\sinh^{-1}}{\sqrt{e}-\sqrt{-}}\right.}{d^2}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{csch}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\text{csch}^{-1}(c\right.}{4d^2\left(\sqrt{-d}\sqrt{e}+\right.}$$

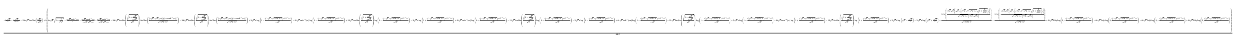
$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{csch}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\text{csch}^{-1}(c\right.}{4d^2\left(\sqrt{-d}\sqrt{e}+\right.}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{csch}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\text{csch}^{-1}(c\right.}{4d^2\left(\sqrt{-d}\sqrt{e}+\right.}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{csch}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\text{csch}^{-1}(c\right.}{4d^2\left(\sqrt{-d}\sqrt{e}+\right.}$$

$$= \frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{d^2} - \frac{a}{d^2x} - \frac{b\text{csch}^{-1}(cx)}{d^2x} + \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{e\left(a+b\text{csch}^{-1}(c\right.}{4d^2\left(\sqrt{-d}\sqrt{e}+\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.02, size = 1487, normalized size = 1.96



Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] ((-8*a*Sqrt[d])/x - (4*a*Sqrt[d]*e*x)/(d + e*x^2) - 12*a*Sqrt[e]*ArcTan[(Sq
rt[e]*x)/Sqrt[d]] + b*(8*c*Sqrt[d]*Sqrt[1 + 1/(c^2*x^2)] - (8*Sqrt[d]*ArcCs
ch[c*x])/x - (2*Sqrt[d]*e*ArcCsch[c*x])/((-I)*Sqrt[d]*Sqrt[e] + e*x) - (2*S
qrt[d]*e*ArcCsch[c*x])/(I*Sqrt[d]*Sqrt[e] + e*x) - (24*I)*Sqrt[e]*ArcSin[Sq
rt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi
+ (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - (24*I)*Sqrt[e]*ArcSin[Sqrt[
1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (
2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] + 3*Sqrt[e]*Pi*Log[1 - (I*(-Sqrt
[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (6*I)*Sqrt[e]*ArcC
sch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt
[d])] + 12*Sqrt[e]*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I
*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - 3*Sqrt[e]*P
i*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] +
 (6*I)*Sqrt[e]*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Ar
cCsch[c*x])/(c*Sqrt[d])] - 12*Sqrt[e]*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/
Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt
[d])] - 3*Sqrt[e]*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*
x])/(c*Sqrt[d])] + (6*I)*Sqrt[e]*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(
c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 12*Sqrt[e]*ArcSin[Sqrt[1 - Sqrt
[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs
ch[c*x])/(c*Sqrt[d])] + 3*Sqrt[e]*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) +
e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (6*I)*Sqrt[e]*ArcCsch[c*x]*Log[1 + (I*(S
qrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - 12*Sqrt[e]*ArcS
in[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*
d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 3*Sqrt[e]*Pi*Log[Sqrt[e] - (I*Sqrt[
d])/x] - 3*Sqrt[e]*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + ((2*I)*e*Log[(2*Sqrt[d
]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*
x^2)])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) + e
] - ((2*I)*e*Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2
*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[d] + I*Sqrt[e
]*x))])/Sqrt[-(c^2*d) + e] + (6*I)*Sqrt[e]*PolyLog[2, ((-I)*(-Sqrt[e] + Sqr
t[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (6*I)*Sqrt[e]*PolyLog[2, (I
*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (6*I)*Sqrt[
e]*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[
d])] + (6*I)*Sqrt[e]*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch
[c*x])/(c*Sqrt[d])]))/(8*d^(5/2))
```

Maple [F]
time = 8.60, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsch}(cx)}{x^2\,(e\,x^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x)
[Out] int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c^2*d-2.718281828459045>0)', see `a
ssume?`
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccsch(c*x) + a)/(x^6*e^2 + 2*d*x^4*e + d^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{acsch}\,(cx)}{x^2\,(d+ex^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x**2+d)**2,x)
[Out] Integral((a + b*acsch(c*x))/(x**2*(d + e*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^2*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x^2\,{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)^2),x)
[Out] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)^2), x)
```

3.111 $\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=694

$$\frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x}-\frac{a+b\text{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2}-\frac{a+b\text{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)}-\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{be^3}+\frac{b\left(c^2d-2e\right)\text{ArcTan}\left(\cdots\right)}{8\left(c^2d-e\right)^3\cdots}$$

```
[Out] 1/4*(-a-b*arccsch(c*x))/e/(e+d/x^2)^2+1/2*(-a-b*arccsch(c*x))/e^2/(e+d/x^2)
-(a+b*arccsch(c*x))^2/b/e^3+1/8*b*(c^2*d-2*e)*arctan((c^2*d-e)^(1/2)/c/x/e^
(1/2)/(1+1/c^2/x^2)^(1/2))/(c^2*d-e)^(3/2)/e^(5/2)-(a+b*arccsch(c*x))*ln(1-
1/(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e^3+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(
1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arc
csch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e
)^(1/2)))/e^3+1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d
)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3+1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/
x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^3+1/2*b*pol
ylog(2,1/(1/c/x+(1+1/c^2/x^2)^(1/2))^2)/e^3+1/2*b*polylog(2,-c*(1/c/x+(1+1/
c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3+1/2*b*polylog(2,
c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^3+1/
2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)
^(1/2)))/e^3+1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1
/2)+(-c^2*d+e)^(1/2)))/e^3+1/2*b*arctan((c^2*d-e)^(1/2)/c/x/e^(1/2)/(1+1/c^
2/x^2)^(1/2))/e^(5/2)/(c^2*d-e)^(1/2)+1/8*b*c*d*(1+1/c^2/x^2)^(1/2)/(c^2*d-
e)/e^2/(e+d/x^2)/x
```

Rubi [A]
time = 1.02, antiderivative size = 694, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {6439, 5823, 5775, 3797, 2221, 2317, 2438, 5821, 390, 385, 211, 5827, 5680}

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*d*Sqrt[1 + 1/(c^2*x^2)])/(8*(c^2*d - e)*e^2*(e + d/x^2)*x) - (a + b*Ar
cCsch[c*x])/(4*e*(e + d/x^2)^2) - (a + b*ArcCsch[c*x])/(2*e^2*(e + d/x^2))
- (a + b*ArcCsch[c*x])^2/(b*e^3) + (b*(c^2*d - 2*e)*ArcTan[Sqrt[c^2*d - e]/
(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])/(8*(c^2*d - e)^(3/2)*e^(5/2)) + (b*Ar
cTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])/(2*Sqrt[c^2*d -
```

```
e]*e^(5/2)) - ((a + b*ArcCsch[c*x])*Log[1 - E^(-2*ArcCsch[c*x])])/e^3 + ((a
 + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^
2*d) + e])])/(2*e^3) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[
c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*e^3) + ((a + b*ArcCsch[c*x])*Log[
1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^3) +
((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-
(c^2*d) + e])])/(2*e^3) + (b*PolyLog[2, E^(-2*ArcCsch[c*x])])/(2*e^3) + (b*
PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/
(2*e^3) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d
) + e])])/(2*e^3) + (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
Sqrt[-(c^2*d) + e]))])/(2*e^3) + (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/
(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*e^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3797

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist
[2*I, Int[((c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)
)/E^(2*I*k*Pi))))/E^(2*I*k*Pi), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Int
egerQ[4*k] && IGtQ[m, 0]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5775

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Dist[1/b,
Subst[Int[x^n*Coth[-a/b + x/b], x], x, a + b*ArcSinh[c*x]], x] /; FreeQ[{a,
 b, c}, x] && IGtQ[n, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 + c^2*x^2], x
], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{x^5\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e^3x} - \frac{dx\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e\left(e+dx^2\right)^3} - \frac{dx\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e^2\left(e+dx^2\right)^2}\right.\right.$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{x}\,dx, x, \frac{1}{x}\right)}{e^3} + \frac{d\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{e^3} + \frac{d\text{Sub}}{}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\text{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int(a+bx)\coth(x)\,dx, x, \text{csch}^{-}\right.}{e^3}$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\text{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\text{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{csch}^{-}\right.}{2be^3}$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\text{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\text{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{csch}^{-}\right.}{2be^3}$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\text{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\text{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{b(c^2d-2e)}{}$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\text{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\text{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{b(c^2d-2e)}{}$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\text{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\text{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{b(c^2d-2e)}{}$$

$$= \frac{bcd\sqrt{1+\frac{1}{c^2x^2}}}{8\left(c^2d-e\right)e^2\left(e+\frac{d}{x^2}\right)x} - \frac{a+b\text{csch}^{-1}(cx)}{4e\left(e+\frac{d}{x^2}\right)^2} - \frac{a+b\text{csch}^{-1}(cx)}{2e^2\left(e+\frac{d}{x^2}\right)} + \frac{b(c^2d-2e)}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.23, size = 2023, normalized size = 2.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/4*(a*d^2)/(e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*x
^2])/(2*e^3) + b*(-1/16*(d*((I*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*
(c^2*d - e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[c*x]/(Sqrt[e]*((-I)*Sqrt[
d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d - e)*Log[(4
*d*Sqrt[c^2*d - e]*Sqrt[e]*(Sqrt[e] + I*c*(c*Sqrt[d] - Sqrt[c^2*d - e]*Sqrt
[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d*(c^2*d
- e)^(3/2))))/e^(5/2) - (d*(((-I)*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[
d]*(c^2*d - e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[c*x]/(Sqrt[e]*(I*Sqrt[d]
+ Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]) + (I*(2*c^2*d - e)*Log[((4*I
)*d*Sqrt[c^2*d - e]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d - e]*Sqr
t[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d] - I*Sqrt[e]*x))])/(d*(c^2*d
 - e)^(3/2))))/(16*e^(5/2)) - (((7*I)/16)*Sqrt[d]*(-(ArcCsch[c*x]/(I*Sqrt[d
]*Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqrt[e]*(
I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/
(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d])
)/e^(5/2) + (((7*I)/16)*Sqrt[d]*(-(ArcCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x
)) + (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I
*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) +
 e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d]))/e^(5/2) + (Pi^
2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sqrt[e]/(
c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[
c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])]
 + (4*I)*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*S
qrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcC
sch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2
]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]
+ (4*I)*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch
[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*
Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4
*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8
*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]
 + 8*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])])/(16*e^3) + (Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 - 32*Ar
cSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*C
ot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1
 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e
```

```
])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqr
t[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 - Sqrt
[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcC
sch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])
*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-
(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 - Sqrt[e]
/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[
c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d])/x] + 4*PolyLog[2, E
^(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^
ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])
*E^ArcCsch[c*x])/(c*Sqrt[d])])/(16*e^3))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^5 (a + b \operatorname{arccsch}(cx))}{(e\,x^2 + d)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
[Out] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
[Out] 1/4*(2*e^(-3)*log(x^2*e + d) + (4*d*x^2*e + 3*d^2)/(x^4*e^5 + 2*d*x^2*e^4 +
 d^2*e^3))*a + b*integrate(x^5*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(x^6*e^
3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*x^5*arccsch(c*x) + a*x^5)/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e
+ d^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^5/(e*x^2 + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.112 $\int \frac{x^3\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=167

$$-\frac{bcx\sqrt{-1-c^2x^2}}{8\left(c^2d-e\right)e\sqrt{-c^2x^2}\left(d+ex^2\right)}+\frac{x^4\left(a+b\,\text{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{bc\left(c^2d-2e\right)x\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{\sqrt{c^2d-e}}\right)}{8d\left(c^2d-e\right)^{3/2}e^{3/2}\sqrt{-c^2x^2}}$$

```
[Out] 1/4*x^4*(a+b*arccsch(c*x))/d/(e*x^2+d)^2+1/8*b*c*(c^2*d-2*e)*x*arctanh(e^(1
/2)*(-c^2*x^2-1)^(1/2)/(c^2*d-e)^(1/2))/d/(c^2*d-e)^(3/2)/e^(3/2)/(-c^2*x^2
)^(1/2)-1/8*b*c*x*(-c^2*x^2-1)^(1/2)/(c^2*d-e)/e/(e*x^2+d)/(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {270, 6437, 12, 457, 79, 65, 214}

$$\frac{x^4\left(a+b\,\text{csch}^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}+\frac{bcx\left(c^2d-2e\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{8de^{3/2}\sqrt{-c^2x^2}\left(c^2d-e\right)^{3/2}}-\frac{bcx\sqrt{-c^2x^2-1}}{8e\sqrt{-c^2x^2}\left(c^2d-e\right)\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/8*(b*c*x*Sqrt[-1 - c^2*x^2])/((c^2*d - e)*e*Sqrt[-(c^2*x^2)]*(d + e*x^2)
) + (x^4*(a + b*ArcCsch[c*x]))/(4*d*(d + e*x^2)^2) + (b*c*(c^2*d - 2*e)*x*A
rcTanh[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/Sqrt[c^2*d - e]])/(8*d*(c^2*d - e)^(3/2
)*e^(3/2)*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
```

```
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx = \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\,(d+ex^2)^2} - \frac{(bcx)\int \frac{x^3}{4d\sqrt{-1-c^2x^2}\,(d+ex^2)^2}dx}{\sqrt{-c^2x^2}}$$

$$= \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\,(d+ex^2)^2} - \frac{(bcx)\int \frac{x^3}{\sqrt{-1-c^2x^2}\,(d+ex^2)^2}dx}{4d\sqrt{-c^2x^2}}$$

$$= \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\,(d+ex^2)^2} - \frac{(bcx)\text{Subst}\left(\int \frac{x}{\sqrt{-1-c^2x}\,(d+ex)^2}dx, x, x^2\right)}{8d\sqrt{-c^2x^2}}$$

$$= -\frac{bcx\sqrt{-1-c^2x^2}}{8\,(c^2d-e)\,e\sqrt{-c^2x^2}\,(d+ex^2)} + \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\,(d+ex^2)^2} - \frac{(bc(c^2d-2e)\,x)\,\text{S}\cdots}{16\cdots}$$

$$= -\frac{bcx\sqrt{-1-c^2x^2}}{8\,(c^2d-e)\,e\sqrt{-c^2x^2}\,(d+ex^2)} + \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\,(d+ex^2)^2} + \frac{(b(c^2d-2e)\,x)\,\text{Su}\cdots}{8\cdots}$$

$$= -\frac{bcx\sqrt{-1-c^2x^2}}{8\,(c^2d-e)\,e\sqrt{-c^2x^2}\,(d+ex^2)} + \frac{x^4\left(a+b\operatorname{csch}^{-1}(cx)\right)}{4d\,(d+ex^2)^2} + \frac{bc(c^2d-2e)\,x\tan\cdots}{8d\,(c^2d-\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.95, size = 375, normalized size = 2.25

$$-\frac{-\frac{4ad}{(d+ex^2)^2}+\frac{8a}{d+ex^2}-\frac{2bce\sqrt{1+\frac{1}{c^2x^2}}\,x}{(-c^2d+e)(d+ex^2)}+\frac{4b(d+2ex^2)\operatorname{csch}^{-1}(cx)}{(d+ex^2)^2}-\frac{4b\sinh^{-1}\left(\frac{1}{cx}\right)}{d}+\frac{b\sqrt{e}\,(-c^2d+2e)\log\left(\frac{16de^{3/2}\sqrt{-c^2d+e}\left(\sqrt{e}+c\left(-ic\sqrt{d}+\sqrt{-c^2d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{b(-c^2d+2e)\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right)}{d(-c^2d+e)^{3/2}}+\frac{b\sqrt{e}\,(-c^2d+2e)\log\left(-\frac{16ide^{3/2}\sqrt{-c^2d+e}\left(\sqrt{e}+c\left(ic\sqrt{d}+\sqrt{-c^2d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{b(c^2d-2e)\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right)}{d(-c^2d+e)^{3/2}}}{16e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/16*((-4*a*d)/(d + e*x^2)^2 + (8*a)/(d + e*x^2) - (2*b*c*e*Sqrt[1 + 1/(c^
2*x^2)]*x)/((-(c^2*d) + e)*(d + e*x^2)) + (4*b*(d + 2*e*x^2)*ArcCsch[c*x])/
(d + e*x^2)^2 - (4*b*ArcSinh[1/(c*x)])/d + (b*Sqrt[e]*(-(c^2*d) + 2*e)*Log[
(16*d*e^(3/2)*Sqrt[-(c^2*d) + e]*(Sqrt[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*
d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*(-(c^2*d) + 2*e)*(I*Sqrt[d] + Sqrt[e]
*x))])/(d*(-(c^2*d) + e)^(3/2)) + (b*Sqrt[e]*(-(c^2*d) + 2*e)*Log[((-16*I)*
d*e^(3/2)*Sqrt[-(c^2*d) + e]*(Sqrt[e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]
*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*(c^2*d - 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d*
(-(c^2*d) + e)^(3/2)))/e^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1880 vs. 2(145) = 290.
time = 6.34, size = 1881, normalized size = 11.26

method	result	size
derivativedivides	Expression too large to display	1881
default	Expression too large to display	1881

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^4*(a*c^6*(-1/2/e^2/(c^2*e*x^2+c^2*d)+1/4*d*c^2/e^2/(c^2*e*x^2+c^2*d)^2)
-1/2*b*c^6*arccsch(c*x)/e^2/(c^2*e*x^2+c^2*d)+1/4*b*c^8*arccsch(c*x)*d/e^2/
(c^2*e*x^2+c^2*d)^2-1/4*b*c^7*(c^2*x^2+1)^(1/2)/e/((c^2*x^2+1)/c^2/x^2)^(1/
2)/x*d/(c^2*d-e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c^2*d*e)^(1/2))*arctanh
(1/(c^2*x^2+1)^(1/2))-1/4*b*c^7*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/
2)*x/(c^2*d-e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c^2*d*e)^(1/2))*arctanh(1
/(c^2*x^2+1)^(1/2))+1/16*b*c^7*(c^2*x^2+1)^(1/2)/e/((c^2*x^2+1)/c^2/x^2)^(1
/2)/x*d/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c
^2*d*e)^(1/2))*ln(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(
1/2)*c*x+e)/(-e*c*x+(-c^2*d*e)^(1/2)))+1/16*b*c^7*(c^2*x^2+1)^(1/2)/((c^2*x
^2+1)/c^2/x^2)^(1/2)*x/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(-e*c*x+(-c^2*d*e)^(1
/2))/(e*c*x+(-c^2*d*e)^(1/2))*ln(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)
*e+(-c^2*d*e)^(1/2)*c*x+e)/(-e*c*x+(-c^2*d*e)^(1/2)))+1/16*b*c^7*(c^2*x^2+1
)^(1/2)/e/((c^2*x^2+1)/c^2/x^2)^(1/2)/x*d/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(-
e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c^2*d*e)^(1/2))*ln(-2*(-(c^2*x^2+1)^(1/2)*
(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x+(-c^2*d*e)^(1/2)))+1/
16*b*c^7*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)*x/(-(c^2*d-e)/e)^(1/
2)/(c^2*d-e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c^2*d*e)^(1/2))*ln(-2*(-(c^
2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e*c*x+(-c^2*
d*e)^(1/2)))+1/8*b*c^5*(c^2*x^2+1)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/(c^2*d-e)/
(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c^2*d*e)^(1/2))+1/4*b*c^5*(c^2*x^2+1)^(1
/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/(c^2*d-e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*
x+(-c^2*d*e)^(1/2))*arctanh(1/(c^2*x^2+1)^(1/2))+1/4*b*c^5*(c^2*x^2+1)^(1/2
)*e/((c^2*x^2+1)/c^2/x^2)^(1/2)*x/d/(c^2*d-e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*
c*x+(-c^2*d*e)^(1/2))*arctanh(1/(c^2*x^2+1)^(1/2))-1/8*b*c^5*(c^2*x^2+1)^(1
/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(-e*c*x+(-
c^2*d*e)^(1/2))/(e*c*x+(-c^2*d*e)^(1/2))*ln(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-
e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-e*c*x+(-c^2*d*e)^(1/2)))-1/8*b*c^5*
(c^2*x^2+1)^(1/2)*e/((c^2*x^2+1)/c^2/x^2)^(1/2)*x/d/(-(c^2*d-e)/e)^(1/2)/(c
^2*d-e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c^2*d*e)^(1/2))*ln(-2*((c^2*x^2+
1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-e*c*x+(-c^2*d*e)^
(1/2)))-1/8*b*c^5*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/(-(c^2*d-
e)/e)^(1/2)/(c^2*d-e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c^2*d*e)^(1/2))*ln
(-2*(-(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(e*c
*x+(-c^2*d*e)^(1/2)))-1/8*b*c^5*(c^2*x^2+1)^(1/2)*e/((c^2*x^2+1)/c^2/x^2)^(
1/2)*x/d/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-
```

```
c^2*d*e)^(1/2))*ln(-2*(-(c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)
^(1/2)*c*x-e)/(e*c*x+(-c^2*d*e)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*b*((2*c^4*d^4*log(c) - 2*(c^4*d^2*e^2 - 2*c^2*d*e^3 + e^4)*x^4*log(x) -
 (4*d^3*log(c) + d^3)*c^2*e + (4*c^4*d^3*e*log(c) - (8*d^2*log(c) + d^2)*c^
2*e^2 + (4*d*log(c) + d)*e^3)*x^2 + (2*d^2*log(c) + d^2)*e^2 + (c^4*d^4 - 2
*c^2*d^3*e + (c^4*d^2*e^2 - 2*c^2*d*e^3)*x^4 + 2*(c^4*d^3*e - 2*c^2*d^2*e^2
)*x^2)*log(c^2*x^2 + 1) - 2*(c^4*d^4 - 2*c^2*d^3*e + 2*(c^4*d^3*e - 2*c^2*d
^2*e^2 + d*e^3)*x^2 + d^2*e^2)*log(sqrt(c^2*x^2 + 1) + 1))/(c^4*d^5*e^2 - 2
*c^2*d^4*e^3 + (c^4*d^3*e^4 - 2*c^2*d^2*e^5 + d*e^6)*x^4 + d^3*e^4 + 2*(c^4
*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5)*x^2) + log(x^2*e + d)/(c^4*d^3 - 2*c^2*
d^2*e + d*e^2) - 8*integrate(1/4*(2*c^2*x^3*e + c^2*d*x)/(c^2*x^6*e^4 + (2*
c^2*d*e^3 + e^4)*x^4 + (c^2*d^2*e^2 + 2*d*e^3)*x^2 + d^2*e^2 + (c^2*x^6*e^4
 + (2*c^2*d*e^3 + e^4)*x^4 + (c^2*d^2*e^2 + 2*d*e^3)*x^2 + d^2*e^2)*sqrt(c^
2*x^2 + 1)), x)) - 1/4*(2*x^2*e + d)*a/(x^4*e^4 + 2*d*x^2*e^3 + d^2*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1953 vs. $2(148) = 296$.
time = 0.66, size = 3944, normalized size = 23.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 + 8*a*d*x^2*cosh(1)^3 + 8*a*d*x^2*sinh(1)^3 - 4*(4*a*c^
2*d^2*x^2 - a*d^2)*cosh(1)^2 - 4*(4*a*c^2*d^2*x^2 - 6*a*d*x^2*cosh(1) - a*d
^2)*sinh(1)^2 - (2*b*x^4*cosh(1)^3 + 2*b*x^4*sinh(1)^3 - b*c^2*d^3 - (b*c^2
*d*x^4 - 4*b*d*x^2)*cosh(1)^2 - (b*c^2*d*x^4 - 6*b*x^4*cosh(1) - 4*b*d*x^2)
*sinh(1)^2 - 2*(b*c^2*d^2*x^2 - b*d^2)*cosh(1) - 2*(b*c^2*d^2*x^2 - 3*b*x^4
*cosh(1)^2 - b*d^2 + (b*c^2*d*x^4 - 4*b*d*x^2)*cosh(1))*sinh(1))*sqrt(-(c^2
*d - cosh(1) - sinh(1))/(cosh(1) - sinh(1)))*log(-(c^2*d + 2*c*x*sqrt(-(c^2
*d - cosh(1) - sinh(1))/(cosh(1) - sinh(1)))*sqrt((c^2*x^2 + 1)/(c^2*x^2))
- (c^2*x^2 + 2)*cosh(1) - (c^2*x^2 + 2)*sinh(1))/(x^2*cosh(1) + x^2*sinh(1)
 + d)) + 8*(a*c^4*d^3*x^2 - a*c^2*d^3)*cosh(1) - 4*(b*c^4*d^4 + b*x^4*cosh(
1)^4 + b*x^4*sinh(1)^4 - 2*(b*c^2*d*x^4 - b*d*x^2)*cosh(1)^3 - 2*(b*c^2*d*x
^4 - 2*b*x^4*cosh(1) - b*d*x^2)*sinh(1)^3 + (b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^
2 + b*d^2)*cosh(1)^2 + (b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + 6*b*x^4*cosh(1)^2
```

+ b*d^2 - 6*(b*c^2*d*x^4 - b*d*x^2)*cosh(1))*sinh(1)^2 + 2*(b*c^4*d^3*x^2 - b*c^2*d^3)*cosh(1) + 2*(b*c^4*d^3*x^2 + 2*b*x^4*cosh(1)^3 - b*c^2*d^3 - 3*(b*c^2*d*x^4 - b*d*x^2)*cosh(1)^2 + (b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*cosh(1))*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 4*(b*c^4*d^4 + b*x^4*cosh(1)^4 + b*x^4*sinh(1)^4 - 2*(b*c^2*d*x^4 - b*d*x^2)*cosh(1)^3 - 2*(b*c^2*d*x^4 - 2*b*x^4*cosh(1) - b*d*x^2)*sinh(1)^3 + (b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*cosh(1)^2 + (b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + 6*b*x^4*cosh(1)^2 + b*d^2 - 6*(b*c^2*d*x^4 - b*d*x^2)*cosh(1))*sinh(1)^2 + 2*(b*c^4*d^3*x^2 - b*c^2*d^3)*cosh(1) + 2*(b*c^4*d^3*x^2 + 2*b*x^4*cosh(1)^3 - b*c^2*d^3 - 3*(b*c^2*d*x^4 - b*d*x^2)*cosh(1)^2 + (b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*cosh(1))*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 4*(b*c^4*d^4 + 2*b*d*x^2*cosh(1)^3 + 2*b*d*x^2*sinh(1)^3 - (4*b*c^2*d^2*x^2 - b*d^2)*cosh(1)^2 - (4*b*c^2*d^2*x^2 - 6*b*d*x^2*cosh(1) - b*d^2)*sinh(1)^2 + 2*(b*c^4*d^3*x^2 - b*c^2*d^3)*cosh(1) + 2*(b*c^4*d^3*x^2 - b*c^2*d^3 + 3*b*d*x^2*cosh(1)^2 - (4*b*c^2*d^2*x^2 - b*d^2)*cosh(1))*sinh(1))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 8*(a*c^4*d^3*x^2 - a*c^2*d^3 + 3*a*d*x^2*cosh(1)^2 - (4*a*c^2*d^2*x^2 - a*d^2)*cosh(1))*sinh(1) + 2*(b*c^3*d^3*x*cosh(1) - b*c*d*x^3*cosh(1)^3 - b*c*d*x^3*sinh(1)^3 + (b*c^3*d^2*x^3 - b*c*d^2*x)*cosh(1)^2 + (b*c^3*d^2*x^3 - 3*b*c*d*x^3*cosh(1) - b*c*d^2*x)*sinh(1)^2 + (b*c^3*d^3*x - 3*b*c*d*x^3*cosh(1)^2 + 2*(b*c^3*d^2*x^3 - b*c*d^2*x)*cosh(1))*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(c^4*d^5*cosh(1)^2 + d*x^4*cosh(1)^6 + d*x^4*sinh(1)^6 - 2*(c^2*d^2*x^4 - d^2*x^2)*cosh(1)^5 - 2*(c^2*d^2*x^4 - 3*d*x^4*cosh(1) - d^2*x^2)*sinh(1)^5 + (c^4*d^3*x^4 - 4*c^2*d^3*x^2 + d^3)*cosh(1)^4 + (c^4*d^3*x^4 - 4*c^2*d^3*x^2 + 15*d*x^4*cosh(1)^2 + d^3 - 10*(c^2*d^2*x^4 - d^2*x^2)*cosh(1))*sinh(1)^4 + 2*(c^4*d^4*x^2 - c^2*d^4)*cosh(1)^3 + 2*(c^4*d^4*x^2 + 10*d*x^4*cosh(1)^3 - c^2*d^4 - 10*(c^2*d^2*x^4 - d^2*x^2)*cosh(1)^2 + 2*(c^4*d^3*x^4 - 4*c^2*d^3*x^2 + d^3)*cosh(1))*sinh(1)^3 + (c^4*d^5 + 15*d*x^4*cosh(1)^4 - 20*(c^2*d^2*x^4 - d^2*x^2)*cosh(1)^3 + 6*(c^4*d^3*x^4 - 4*c^2*d^3*x^2 + d^3)*cosh(1)^2 + 6*(c^4*d^4*x^2 - c^2*d^4)*cosh(1))*sinh(1)^2 + 2*(c^4*d^5*cosh(1) + 3*d*x^4*cosh(1)^5 - 5*(c^2*d^2*x^4 - d^2*x^2)*cosh(1)^4 + 2*(c^4*d^3*x^4 - 4*c^2*d^3*x^2 + d^3)*cosh(1)^3 + 3*(c^4*d^4*x^2 - c^2*d^4)*cosh(1)^2)*sinh(1)), -1/8*(2*a*c^4*d^4 + 4*a*d*x^2*cosh(1)^3 + 4*a*d*x^2*sinh(1)^3 - 2*(4*a*c^2*d^2*x^2 - a*d^2)*cosh(1)^2 - 2*(4*a*c^2*d^2*x^2 - 6*a*d*x^2*cosh(1) - a*d^2)*sinh(1)^2 - (2*b*x^4*cosh(1)^3 + 2*b*x^4*sinh(1)^3 - b*c^2*d^3 - (b*c^2*d*x^4 - 4*b*d*x^2)*cosh(1)^2 - (b*c^2*d*x^4 - 6*b*x^4*cosh(1) - 4*b*d*x^2)*sinh(1)^2 - 2*(b*c^2*d^2*x^2 - b*d^2)*cosh(1) - 2*(b*c^2*d^2*x^2 - 3*b*x^4*cosh(1)^2 - b*d^2 + (b*c^2*d*x^4 - 4*b*d*x^2)*cosh(1))*sinh(1))*sqrt((c^2*d - cosh(1) - sinh(1))/(cosh(1) - sinh(1)))*arctan(-c*x*sqrt((c^2*d - cosh(1) - sinh(1))/(cosh(1) - sinh(1)))*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d - cosh(1) - sinh(1))) + 4*(a*c^4*d^3*x^2 - a*c^2*d^3)*cosh(1) - 2*(b*c^4*d^4 + b*x^4*cosh(1)^4 + b*x^4*sinh(1)^4 - 2*(b*c^2*d*x^4 - b*d*x^2)*cosh(1)^3 - 2*(b*c^2*d*x^4 - 2*b*x^4*cosh(1) - b*d*x^2)*sinh(1)^3 + (b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*cosh(1)^2 + (b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + 6*b*x^4*cosh(1)^2 + b*d^2 - 6*(b*c^2*d*x^4 - b*d*x^2)*cosh(1))*sinh(1)

```
^2 + 2*(b*c^4*d^3*x^2 - b*c^2*d^3)*cosh(1) + 2*(b*c^4*d^3*x^2 + 2*b*x^4*cos
h(1)^3 - b*c^2*d^3 - 3*(b*c^2*d*x^4 - b*d*x^2)*cosh(1)^2 + (b*c^4*d^2*x^4 -
 4*b*c^2*d^2*x^2 + b*d^2)*cosh(1))*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2
*x^2)) - c*x + 1) + 2*(b*c^4*d^4 + b*x^4*cosh(1)^4 + b*x^4*sinh(1)^4 - 2*(b
*c^2*d*x^4 - b*d*x^2)*cosh(1)^3 - 2*(b*c^2*d*x^4 - 2*b*x^4*cosh(1) - b*d*x^
2)*sinh(1)^3 + (b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*cosh(1)^2 + (b*c^4
*d^2*x^4 - 4*b*c^2*d^2*x^2 + 6*b*x^4*cosh(1)^2 + b*d^2 - 6*(b*c^2*d*x^4 - b
*d*x^2)*cosh(1))*sinh(1)^2 + 2*(b*c^4*d^3*x^2 - b*c^2*d^3)*cosh(1) + 2*(b*c
^4*d^3*x^2 + 2*b*x^4*cosh(1)^3 - b*c^2*d^3 - 3*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x^2 + d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.113 $\int \frac{x\left(a+b\,\text{csch}^{-1}(cx)\right)}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=205

$$\frac{bcx\sqrt{-1-c^2x^2}}{8d\left(c^2d-e\right)\sqrt{-c^2x^2}\,\left(d+ex^2\right)}-\frac{a+b\,\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{bcx\text{ArcTan}\left(\sqrt{-1-c^2x^2}\right)}{4d^2e\sqrt{-c^2x^2}}+\frac{bc\left(3c^2d-2e\right)x\tanh^{-1}\left(\sqrt{\cdots}\right.}{8d^2\left(c^2d-e\right)^{3/2}\sqrt{e}\cdots}$$

```
[Out] 1/4*(-a-b*arccsch(c*x))/e/(e*x^2+d)^2+1/4*b*c*x*arctan((-c^2*x^2-1)^(1/2))/
d^2/e/(-c^2*x^2)^(1/2)+1/8*b*c*(3*c^2*d-2*e)*x*arctanh(e^(1/2)*(-c^2*x^2-1)
^(1/2)/(c^2*d-e)^(1/2))/d^2/(c^2*d-e)^(3/2)/e^(1/2)/(-c^2*x^2)^(1/2)+1/8*b*
c*x*(-c^2*x^2-1)^(1/2)/d/(c^2*d-e)/(e*x^2+d)/(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {6435, 457, 105, 162, 65, 211, 214}

$$-\frac{a+b\,\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{bcx\text{ArcTan}\left(\sqrt{-c^2x^2-1}\right)}{4d^2e\sqrt{-c^2x^2}}+\frac{bcx\left(3c^2d-2e\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{\sqrt{c^2d-e}}\right)}{8d^2\sqrt{e}\sqrt{-c^2x^2}\left(c^2d-e\right)^{3/2}}+\frac{bcx\sqrt{-c^2x^2-1}}{8d\sqrt{-c^2x^2}\left(c^2d-e\right)\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*x*Sqrt[-1 - c^2*x^2])/(8*d*(c^2*d - e)*Sqrt[-(c^2*x^2)]*(d + e*x^2)) -
 (a + b*ArcCsch[c*x])/(4*e*(d + e*x^2)^2) + (b*c*x*ArcTan[Sqrt[-1 - c^2*x^2
]])/(4*d^2*e*Sqrt[-(c^2*x^2)]) + (b*c*(3*c^2*d - 2*e)*x*ArcTanh[(Sqrt[e]*Sq
rt[-1 - c^2*x^2])/Sqrt[c^2*d - e]])/(8*d^2*(c^2*d - e)^(3/2)*Sqrt[e]*Sqrt[-
(c^2*x^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 105

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
```

```
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (Integer
Q[n] || IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

Rule 162

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6435

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2])), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rubi steps

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\frac{a+b\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{(bcx)\int \frac{1}{x\sqrt{-1-c^2x^2}\,(d+ex^2)^2}\,dx}{4e\sqrt{-c^2x^2}}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{(bcx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\,(d+ex)^2}\,dx, x, x^2\right)}{8e\sqrt{-c^2x^2}}$$

$$= \frac{bcx\sqrt{-1-c^2x^2}}{8d\left(c^2d-e\right)\sqrt{-c^2x^2}\left(d+ex^2\right)} - \frac{a+b\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{(bcx)\text{Subst}\left(\int \frac{c^2d-e-\frac{1}{2}}{x\sqrt{-1-c^2}}\right.}{8d\left(c^2d-e\right)e\sqrt{}}$$

$$= \frac{bcx\sqrt{-1-c^2x^2}}{8d\left(c^2d-e\right)\sqrt{-c^2x^2}\left(d+ex^2\right)} - \frac{a+b\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{(bcx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2}}\right.}{8d^2e\sqrt{-c^2x^2}}$$

$$= \frac{bcx\sqrt{-1-c^2x^2}}{8d\left(c^2d-e\right)\sqrt{-c^2x^2}\left(d+ex^2\right)} - \frac{a+b\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2} - \frac{(bx)\text{Subst}\left(\int \frac{1}{-\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx, x\right.}{4cd^2e\sqrt{-c^2}}$$

$$= \frac{bcx\sqrt{-1-c^2x^2}}{8d\left(c^2d-e\right)\sqrt{-c^2x^2}\left(d+ex^2\right)} - \frac{a+b\text{csch}^{-1}(cx)}{4e\left(d+ex^2\right)^2} + \frac{bcx\tan^{-1}\left(\sqrt{-1-c^2x^2}\right)}{4d^2e\sqrt{-c^2x^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.62, size = 368, normalized size = 1.80

$$\frac{1}{16}\left(-\frac{4a}{e\left(d+ex^2\right)^2} + \frac{2bc\sqrt{1+\frac{1}{c^2x^2}}\,x}{d\left(c^2d-e\right)\left(d+ex^2\right)} - \frac{4b\text{csch}^{-1}(cx)}{e\left(d+ex^2\right)^2} + \frac{4b\sinh^{-1}\left(\frac{1}{cx}\right)}{d^2e} + \frac{b\left(3c^2d-2e\right)\log\left(\frac{16d^2\sqrt{e}\sqrt{-c^2d+e}\left(\sqrt{e}+c\left(-ic\sqrt{d}+\sqrt{-c^2d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{b\left(-3c^2d+2e\right)\left(i\sqrt{d}+\sqrt{e}x\right)}\right)}{d^2\sqrt{e}\left(-c^2d+e\right)^{3/2}} + \frac{b\left(3c^2d-2e\right)\log\left(-\frac{16id^2\sqrt{e}\sqrt{-c^2d+e}\left(\sqrt{e}+c\left(ic\sqrt{d}+\sqrt{-c^2d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{b\left(3c^2d-2e\right)\left(\sqrt{d}+i\sqrt{e}x\right)}\right)}{d^2\sqrt{e}\left(-c^2d+e\right)^{3/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] ((-4*a)/(e*(d + e*x^2)^2) + (2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x)/(d*(c^2*d - e)*
(d + e*x^2)) - (4*b*ArcCsch[c*x])/(e*(d + e*x^2)^2) + (4*b*ArcSinh[1/(c*x)]
)/(d^2*e) + (b*(3*c^2*d - 2*e)*Log[(16*d^2*Sqrt[e]*Sqrt[-(c^2*d) + e]*(Sqrt
[e] + c*((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*
(-3*c^2*d + 2*e)*(I*Sqrt[d] + Sqrt[e]*x))])/(d^2*Sqrt[e]*(-(c^2*d) + e)^(3/
2)) + (b*(3*c^2*d - 2*e)*Log[((-16*I)*d^2*Sqrt[e]*Sqrt[-(c^2*d) + e]*(Sqrt[
e] + c*(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(b*(3*c
^2*d - 2*e)*(Sqrt[d] + I*Sqrt[e]*x))])/(d^2*Sqrt[e]*(-(c^2*d) + e)^(3/2)))/
16
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1837 vs. $2(180) = 360$.
time = 6.44, size = 1838, normalized size = 8.97

method	result	size
derivativedivides	Expression too large to display	1838
default	Expression too large to display	1838

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(-1/4*a*c^6/e/(c^2*e*x^2+c^2*d)^2-1/4*b*c^6/e/(c^2*e*x^2+c^2*d)^2*arc
csch(c*x)-1/4*b*c^5*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/(c^2*d-
e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c^2*d*e)^(1/2))*arctanh(1/(c^2*x^2+1)
^(1/2))-1/4*b*c^5*(c^2*x^2+1)^(1/2)*e/((c^2*x^2+1)/c^2/x^2)^(1/2)*x/d/(c^2*
d-e)/(-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c^2*d*e)^(1/2))*arctanh(1/(c^2*x^2+
1)^(1/2))+3/16*b*c^5*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/(-(c^2
*d-e)/e)^(1/2)/(c^2*d-e)/(e*c*x+(-c^2*d*e)^(1/2))/(-e*c*x+(-c^2*d*e)^(1/2))
*ln(2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(e*
c*x+(-c^2*d*e)^(1/2)))+3/16*b*c^5*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(
1/2)*x/d/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(e*c*x+(-c^2*d*e)^(1/2))/(-e*c*x+(-
c^2*d*e)^(1/2))*ln(2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e-(-c^2*d*e)^(
1/2)*c*x+e)/(e*c*x+(-c^2*d*e)^(1/2)))*e+3/16*b*c^5*(c^2*x^2+1)^(1/2)/((c^2*
x^2+1)/c^2/x^2)^(1/2)/x/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(-e*c*x+(-c^2*d*e)^(
1/2))/(e*c*x+(-c^2*d*e)^(1/2))*ln(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2
)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-e*c*x+(-c^2*d*e)^(1/2)))+3/16*b*c^5*(c^2*x^2+
1)^(1/2)*e/((c^2*x^2+1)/c^2/x^2)^(1/2)*x/d/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(
-e*c*x+(-c^2*d*e)^(1/2))/(e*c*x+(-c^2*d*e)^(1/2))*ln(-2*((c^2*x^2+1)^(1/2)*
(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(-e*c*x+(-c^2*d*e)^(1/2)))-1
/8*b*c^3*(c^2*x^2+1)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/(c^2*d-e)/(e*c*x+(-c^2
*d*e)^(1/2))/(-e*c*x+(-c^2*d*e)^(1/2))*e+1/4*b*c^3*(c^2*x^2+1)^(1/2)/((c^2*
x^2+1)/c^2/x^2)^(1/2)/x/d/(c^2*d-e)/(e*c*x+(-c^2*d*e)^(1/2))/(-e*c*x+(-c^2*
d*e)^(1/2))*arctanh(1/(c^2*x^2+1)^(1/2))*e+1/4*b*c^3*(c^2*x^2+1)^(1/2)/((c^
2*x^2+1)/c^2/x^2)^(1/2)*x/d^2/(c^2*d-e)/(e*c*x+(-c^2*d*e)^(1/2))/(-e*c*x+(-
c^2*d*e)^(1/2))*arctanh(1/(c^2*x^2+1)^(1/2))*e^2-1/8*b*c^3*(c^2*x^2+1)^(1/2
)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(e*c*x+(-c
^2*d*e)^(1/2))/(-e*c*x+(-c^2*d*e)^(1/2))*ln(2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e
)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(e*c*x+(-c^2*d*e)^(1/2)))*e-1/8*b*c^3*
(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)*x/d^2/(-(c^2*d-e)/e)^(1/2)/(c
^2*d-e)/(e*c*x+(-c^2*d*e)^(1/2))/(-e*c*x+(-c^2*d*e)^(1/2))*ln(2*((c^2*x^2+1
)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e-(-c^2*d*e)^(1/2)*c*x+e)/(e*c*x+(-c^2*d*e)^(1
/2)))*e^2-1/8*b*c^3*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^2)^(1/2)/x/d/(-(c^
2*d-e)/e)^(1/2)/(c^2*d-e)/(e*c*x+(-c^2*d*e)^(1/2))/(-e*c*x+(-c^2*d*e)^(1/2)
)*ln(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(
```

```
-e*c*x+(-c^2*d*e)^(1/2)))*e-1/8*b*c^3*(c^2*x^2+1)^(1/2)/((c^2*x^2+1)/c^2/x^
2)^(1/2)*x/d^2/(-(c^2*d-e)/e)^(1/2)/(c^2*d-e)/(e*c*x+(-c^2*d*e)^(1/2))/(-e*
c*x+(-c^2*d*e)^(1/2))*ln(-2*((c^2*x^2+1)^(1/2)*(-(c^2*d-e)/e)^(1/2)*e+(-c^2
*d*e)^(1/2)*c*x+e)/(-e*c*x+(-c^2*d*e)^(1/2)))*e^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(8*c^2*integrate(1/4*x/(c^2*x^6*e^3 + (2*c^2*d*e^2 + e^3)*x^4 + (c^2*d
^2*e + 2*d*e^2)*x^2 + d^2*e + (c^2*x^6*e^3 + (2*c^2*d*e^2 + e^3)*x^4 + (c^2
*d^2*e + 2*d*e^2)*x^2 + d^2*e)*sqrt(c^2*x^2 + 1)), x) + (2*c^2*d - e)*log(x
^2*e + d)/(c^4*d^4 - 2*c^2*d^3*e + d^2*e^2) - (2*c^4*d^4*log(c) - (4*d^3*lo
g(c) - d^3)*c^2*e + (c^2*d^2*e^2 - d*e^3)*x^2 + (2*d^2*log(c) - d^2)*e^2 +
(c^4*d^2*x^4*e^2 + 2*c^4*d^3*x^2*e + c^4*d^4)*log(c^2*x^2 + 1) - 2*((c^4*d^
2*e^2 - 2*c^2*d*e^3 + e^4)*x^4 + 2*(c^4*d^3*e - 2*c^2*d^2*e^2 + d*e^3)*x^2)
*log(x) - 2*(c^4*d^4 - 2*c^2*d^3*e + d^2*e^2)*log(sqrt(c^2*x^2 + 1) + 1))/(
c^4*d^6*e - 2*c^2*d^5*e^2 + (c^4*d^4*e^3 - 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + d
^4*e^3 + 2*(c^4*d^5*e^2 - 2*c^2*d^4*e^3 + d^3*e^4)*x^2))*b - 1/4*a/(x^4*e^3
 + 2*d*x^2*e^2 + d^2*e)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1635 vs. $2(179) = 358$.
time = 0.58, size = 3307, normalized size = 16.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c^4*d^4 - 8*a*c^2*d^3*cosh(1) + 4*a*d^2*cosh(1)^2 + 4*a*d^2*sin
h(1)^2 - (2*b*x^4*cosh(1)^3 + 2*b*x^4*sinh(1)^3 - 3*b*c^2*d^3 - (3*b*c^2*d*
x^4 - 4*b*d*x^2)*cosh(1)^2 - (3*b*c^2*d*x^4 - 6*b*x^4*cosh(1) - 4*b*d*x^2)*
sinh(1)^2 - 2*(3*b*c^2*d^2*x^2 - b*d^2)*cosh(1) - 2*(3*b*c^2*d^2*x^2 - 3*b*
x^4*cosh(1)^2 - b*d^2 + (3*b*c^2*d*x^4 - 4*b*d*x^2)*cosh(1))*sinh(1))*sqrt(
-(c^2*d - cosh(1) - sinh(1))/(cosh(1) - sinh(1)))*log(-(c^2*d + 2*c*x*sqrt(
-(c^2*d - cosh(1) - sinh(1))/(cosh(1) - sinh(1)))*sqrt((c^2*x^2 + 1)/(c^2*x
^2)) - (c^2*x^2 + 2)*cosh(1) - (c^2*x^2 + 2)*sinh(1))/(x^2*cosh(1) + x^2*si
nh(1) + d)) - 4*(b*c^4*d^4 + b*x^4*cosh(1)^4 + b*x^4*sinh(1)^4 - 2*(b*c^2*d
*x^4 - b*d*x^2)*cosh(1)^3 - 2*(b*c^2*d*x^4 - 2*b*x^4*cosh(1) - b*d*x^2)*sin
h(1)^3 + (b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*cosh(1)^2 + (b*c^4*d^2*x
^4 - 4*b*c^2*d^2*x^2 + 6*b*x^4*cosh(1)^2 + b*d^2 - 6*(b*c^2*d*x^4 - b*d*x^2
```

```
)*cosh(1))*sinh(1)^2 + 2*(b*c^4*d^3*x^2 - b*c^2*d^3)*cosh(1) + 2*(b*c^4*d^3
*x^2 + 2*b*x^4*cosh(1)^3 - b*c^2*d^3 - 3*(b*c^2*d*x^4 - b*d*x^2)*cosh(1)^2
+ (b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*cosh(1))*sinh(1))*log(c*x*sqrt(
(c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 4*(b*c^4*d^4 + b*x^4*cosh(1)^4 + b*x^
4*sinh(1)^4 - 2*(b*c^2*d*x^4 - b*d*x^2)*cosh(1)^3 - 2*(b*c^2*d*x^4 - 2*b*x^
4*cosh(1) - b*d*x^2)*sinh(1)^3 + (b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*
cosh(1)^2 + (b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + 6*b*x^4*cosh(1)^2 + b*d^2 -
6*(b*c^2*d*x^4 - b*d*x^2)*cosh(1))*sinh(1)^2 + 2*(b*c^4*d^3*x^2 - b*c^2*d^3
)*cosh(1) + 2*(b*c^4*d^3*x^2 + 2*b*x^4*cosh(1)^3 - b*c^2*d^3 - 3*(b*c^2*d*x
^4 - b*d*x^2)*cosh(1)^2 + (b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*cosh(1)
)*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x - 1) + 4*(b*c^4*d^4
- 2*b*c^2*d^3*cosh(1) + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - 2*(b*c^2*d^3 -
b*d^2*cosh(1))*sinh(1))*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))
- 8*(a*c^2*d^3 - a*d^2*cosh(1))*sinh(1) - 2*(b*c^3*d^3*x*cosh(1) - b*c*d*x^
3*cosh(1)^3 - b*c*d*x^3*sinh(1)^3 + (b*c^3*d^2*x^3 - b*c*d^2*x)*cosh(1)^2 +
 (b*c^3*d^2*x^3 - 3*b*c*d*x^3*cosh(1) - b*c*d^2*x)*sinh(1)^2 + (b*c^3*d^3*x
 - 3*b*c*d*x^3*cosh(1)^2 + 2*(b*c^3*d^2*x^3 - b*c*d^2*x)*cosh(1))*sinh(1))*
sqrt((c^2*x^2 + 1)/(c^2*x^2)))/(c^4*d^6*cosh(1) + d^2*x^4*cosh(1)^5 + d^2*x
^4*sinh(1)^5 - 2*(c^2*d^3*x^4 - d^3*x^2)*cosh(1)^4 - (2*c^2*d^3*x^4 - 5*d^2
*x^4*cosh(1) - 2*d^3*x^2)*sinh(1)^4 + (c^4*d^4*x^4 - 4*c^2*d^4*x^2 + d^4)*c
osh(1)^3 + (c^4*d^4*x^4 - 4*c^2*d^4*x^2 + 10*d^2*x^4*cosh(1)^2 + d^4 - 8*(c
^2*d^3*x^4 - d^3*x^2)*cosh(1))*sinh(1)^3 + 2*(c^4*d^5*x^2 - c^2*d^5)*cosh(1
)^2 + (2*c^4*d^5*x^2 + 10*d^2*x^4*cosh(1)^3 - 2*c^2*d^5 - 12*(c^2*d^3*x^4 -
 d^3*x^2)*cosh(1)^2 + 3*(c^4*d^4*x^4 - 4*c^2*d^4*x^2 + d^4)*cosh(1))*sinh(1
)^2 + (c^4*d^6 + 5*d^2*x^4*cosh(1)^4 - 8*(c^2*d^3*x^4 - d^3*x^2)*cosh(1)^3
+ 3*(c^4*d^4*x^4 - 4*c^2*d^4*x^2 + d^4)*cosh(1)^2 + 4*(c^4*d^5*x^2 - c^2*d^
5)*cosh(1))*sinh(1)), -1/8*(2*a*c^4*d^4 - 4*a*c^2*d^3*cosh(1) + 2*a*d^2*cos
h(1)^2 + 2*a*d^2*sinh(1)^2 - (2*b*x^4*cosh(1)^3 + 2*b*x^4*sinh(1)^3 - 3*b*c
^2*d^3 - (3*b*c^2*d*x^4 - 4*b*d*x^2)*cosh(1)^2 - (3*b*c^2*d*x^4 - 6*b*x^4*c
osh(1) - 4*b*d*x^2)*sinh(1)^2 - 2*(3*b*c^2*d^2*x^2 - b*d^2)*cosh(1) - 2*(3*
b*c^2*d^2*x^2 - 3*b*x^4*cosh(1)^2 - b*d^2 + (3*b*c^2*d*x^4 - 4*b*d*x^2)*cos
h(1))*sinh(1))*sqrt((c^2*d - cosh(1) - sinh(1))/(cosh(1) - sinh(1)))*arctan
(-c*x*sqrt((c^2*d - cosh(1) - sinh(1))/(cosh(1) - sinh(1)))*sqrt((c^2*x^2 +
 1)/(c^2*x^2))/(c^2*d - cosh(1) - sinh(1))) - 2*(b*c^4*d^4 + b*x^4*cosh(1)^
4 + b*x^4*sinh(1)^4 - 2*(b*c^2*d*x^4 - b*d*x^2)*cosh(1)^3 - 2*(b*c^2*d*x^4
- 2*b*x^4*cosh(1) - b*d*x^2)*sinh(1)^3 + (b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 +
 b*d^2)*cosh(1)^2 + (b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + 6*b*x^4*cosh(1)^2 +
b*d^2 - 6*(b*c^2*d*x^4 - b*d*x^2)*cosh(1))*sinh(1)^2 + 2*(b*c^4*d^3*x^2 - b
*c^2*d^3)*cosh(1) + 2*(b*c^4*d^3*x^2 + 2*b*x^4*cosh(1)^3 - b*c^2*d^3 - 3*(b
*c^2*d*x^4 - b*d*x^2)*cosh(1)^2 + (b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)
*cosh(1))*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - c*x + 1) + 2*(b*
c^4*d^4 + b*x^4*cosh(1)^4 + b*x^4*sinh(1)^4 - 2*(b*c^2*d*x^4 - b*d*x^2)*cos
h(1)^3 - 2*(b*c^2*d*x^4 - 2*b*x^4*cosh(1) - b*d*x^2)*sinh(1)^3 + (b*c^4*d^2
*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*cosh(1)^2 + (b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^
2 + 6*b*x^4*cosh(1)^2 + b*d^2 - 6*(b*c^2*d*x^4 - b*d*x^2)*cosh(1))*sinh(1)^
```

```
2 + 2*(b*c^4*d^3*x^2 - b*c^2*d^3)*cosh(1) + 2*(b*c^4*d^3*x^2 + 2*b*x^4*cosh
(1)^3 - b*c^2*d^3 - 3*(b*c^2*d*x^4 - b*d*x^2)*cosh(1)^2 + (b*c^4*d^2*x^4 -
4*b*c^2*d^2*x^2 + b*d^2)*cosh(1))*sinh(1))*log(c*x*sqrt((c^2*x^2 + 1)/(c^2*
x^2)) - c*x - 1) + 2*(b*c^4*d^4 - 2*b*c^2*d^3*cosh(1) + b*d^2*cosh(1)^2 + b
*d^2*sinh(1)^2 - 2*(b*c^2*d^3 - b*d^2*cosh(1))*sinh(1))*log((c*x*sqrt((c^2*
x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 4*(a*c^2*d^3 - a*d^2*cosh(1))*sinh(1) - (
b*c^3*d^3*x*cosh(1) - b*c*d*x^3*cosh(1)^3 - b*c*d*x^3*sinh(1)^3 + (b*c^3*d^
2*x^3 - b*c*d^2*x)*cosh(1)^2 + (b*c^3*d^2*x^3 - 3*b*c*d*x^3*cosh(1) - b*c*d
^2*x)*sinh(1)^2 + (b*c^3*d^3*x - 3*b*c*d*x^3*cosh(1)^2 + 2*(b*c^3*d^2*x^3 -
 b*c*d^2*x)*cosh(1))*sinh(1))*sqrt((c^2*x^2 + 1...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.114 $\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex^2)^3}\,dx$

Optimal. Leaf size=657

$$-\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x}+\frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2}-\frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)}+\frac{\left(a+b\text{csch}^{-1}(cx)\right)^2}{2bd^3}-\frac{b(c^2d-2e)\sqrt{\vphantom{x}}}{}$$

[Out] 1/4*e^2*(a+b*arccsch(c*x))/d^3/(e+d/x^2)^2-e*(a+b*arccsch(c*x))/d^3/(e+d/x^2)+1/2*(a+b*arccsch(c*x))^2/b/d^3-1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d^3-1/2*(a+b*arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d^3-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d^3-1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/d^3-1/2*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d^3-1/2*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/d^3-1/8*b*(c^2*d-2*e)*arctan((c^2*d-e)^(1/2)/c/x/e^(1/2)/(1+1/c^2/x^2)^(1/2))*e^(1/2)/d^3/(c^2*d-e)^(3/2)+b*arctan((c^2*d-e)^(1/2)/c/x/e^(1/2)/(1+1/c^2/x^2)^(1/2))*e^(1/2)/d^3/(c^2*d-e)^(1/2)-1/8*b*c*e*(1+1/c^2/x^2)^(1/2)/d^2/(c^2*d-e)/(e+d/x^2)/x

Rubi [A]
time = 0.86, antiderivative size = 657, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {6439, 5823, 5821, 390, 385, 211, 5827, 5680, 2221, 2317, 2438}

Antiderivative was successfully verified.

[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^3),x]

[Out] -1/8*(b*c*e*Sqrt[1 + 1/(c^2*x^2)])/(d^2*(c^2*d - e)*(e + d/x^2)*x) + (e^2*(a + b*ArcCsch[c*x]))/(4*d^3*(e + d/x^2)^2) - (e*(a + b*ArcCsch[c*x]))/(d^3*(e + d/x^2)) + (a + b*ArcCsch[c*x])^2/(2*b*d^3) - (b*(c^2*d - 2*e)*Sqrt[e]*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])/(8*d^3*(c^2*d - e)^(3/2)) + (b*Sqrt[e]*ArcTan[Sqrt[c^2*d - e]/(c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)])/(d^3*Sqrt[c^2*d - e]) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(2*d^3) - ((a + b*ArcCsc

```
h[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])]
)/(2*d^3) - ((a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt
[e] + Sqrt[-(c^2*d) + e])])/(2*d^3) - ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt
[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(2*d^3) - (b*PolyLog[
2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(2*d^3)
- (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])]
)/(2*d^3) - (b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c
^2*d) + e]))])/(2*d^3) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
 + Sqrt[-(c^2*d) + e])])/(2*d^3)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(-b)*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*n*(p + 1)*(b*c -
 a*d))), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)),
 Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q},
 x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !L
tQ[q, -1]) && NeQ[p, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5821

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcSinh[c*x])/(2*e*(p + 1))),
x] - Dist[b*(c/(2*e*(p + 1))), Int[(d + e*x^2)^(p + 1)/Sqrt[1 + c^2*x^2], x
], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[e, c^2*d] && NeQ[p, -1]
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = -\text{Subst}\left(\int \frac{x^5\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{e^2x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^3} - \frac{2ex\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d^2\left(e+dx^2\right)^2} + \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right.}{d^2\left(e+dx^2\right)}\right.\right.$$

$$= -\frac{\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\text{Subst}\left(\int \frac{x\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\text{Sul}}{}$$

$$= \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int\left(-\frac{\sqrt{-d}\,\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{2d\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \right.\right.}{d^2}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{b\sqrt{e}\,\tan}{}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{cs}\right.}{2}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{cs}\right.}{2}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{cs}\right.}{2}$$

$$= -\frac{bce\sqrt{1+\frac{1}{c^2x^2}}}{8d^2\left(c^2d-e\right)\left(e+\frac{d}{x^2}\right)x} + \frac{e^2\left(a+b\text{csch}^{-1}(cx)\right)}{4d^3\left(e+\frac{d}{x^2}\right)^2} - \frac{e\left(a+b\text{csch}^{-1}(cx)\right)}{d^3\left(e+\frac{d}{x^2}\right)} + \frac{\left(a+b\text{cs}\right.}{2}$$

Mathematica [F]
time = 61.31, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex^2)^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^3), x]
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsch}(cx)}{x\,(e\,x^2+d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((2*x^2*e + 3*d)/(d^2*x^4*e^2 + 2*d^3*x^2*e + d^4) - 2*log(x^2*e + d)
/d^3 + 4*log(x)/d^3) + b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(x^
7*e^3 + 3*d*x^5*e^2 + 3*d^2*x^3*e + d^3*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/(x^7*e^3 + 3*d*x^5*e^2 + 3*d^2*x^3*e + d^3*x)
, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^3*x), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x\,{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^3),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^3), x)
```

3.115 $\int \frac{x^4\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=1106

$$-\frac{bc\sqrt{-d}\,\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\,\sqrt{e}\,-\frac{d}{x}\right)}-\frac{bc\sqrt{-d}\,\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\,\sqrt{e}\,+\frac{d}{x}\right)}+\frac{\sqrt{-d}\,\left(a+b\mathrm{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\,\sqrt{e}\,-\frac{d}{x}\right)^2}+\frac{3(a}{16e^2\,(}$$

```
[Out] 3/16*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1
/2)-(-c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*(a+b*arccsch(c*x))*ln(1+c*(1
/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^(5/2)/(-
d)^(1/2)+3/16*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1
/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*(a+b*arccsch(c*x))*
ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e
^(5/2)/(-d)^(1/2)-3/16*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*b*polylog(2,c*(1/c/x+
(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1
/2)-3/16*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c
^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+3/16*b*polylog(2,c*(1/c/x+(1+1/c^2/x^2)^
(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/e^(5/2)/(-d)^(1/2)+1/16*b*arc
tanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(
1/2))/(c^2*d-e)^(3/2)/e/d^(1/2)+1/16*b*arctanh((c^2*d+(-d)^(1/2)*e^(1/2)/x)
/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/(c^2*d-e)^(3/2)/e/d^(1/2)-3
/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c
^2/x^2)^(1/2))/e^2/d^(1/2)/(c^2*d-e)^(1/2)-3/16*b*arctanh((c^2*d+(-d)^(1/2)
*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/e^2/d^(1/2)/(c^2
*d-e)^(1/2)+1/16*(a+b*arccsch(c*x))*(-d)^(1/2)/e^(3/2)/(-d/x+(-d)^(1/2)*e^(
1/2))^2+3/16*(a+b*arccsch(c*x))/e^2/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*(a+b*arc
csch(c*x))*(-d)^(1/2)/e^(3/2)/(d/x+(-d)^(1/2)*e^(1/2))^2-3/16*(a+b*arccsch(
c*x))/e^2/(d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*(-d)^(1/2)*(1+1/c^2/x^2)^(1/2)/
(c^2*d-e)/e^(3/2)/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*(-d)^(1/2)*(1+1/c^2/x^
2)^(1/2)/(c^2*d-e)/e^(3/2)/(d/x+(-d)^(1/2)*e^(1/2))
```

Rubi [A]

time = 1.06, antiderivative size = 1106, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {6439, 5793, 5828, 745, 739, 212, 5827, 5680, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/16*(b*c*Sqrt[-d]*Sqrt[1 + 1/(c^2*x^2)])/((c^2*d - e)*e^(3/2)*(Sqrt[-d]*S
qrt[e] - d/x)) - (b*c*Sqrt[-d]*Sqrt[1 + 1/(c^2*x^2)])/(16*(c^2*d - e)*e^(3/
2)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[-d]*(a + b*ArcCsch[c*x]))/(16*e^(3/2)*
(Sqrt[-d]*Sqrt[e] - d/x)^2) + (3*(a + b*ArcCsch[c*x]))/(16*e^2*(Sqrt[-d]*Sq
rt[e] - d/x)) - (Sqrt[-d]*(a + b*ArcCsch[c*x]))/(16*e^(3/2)*(Sqrt[-d]*Sqrt[
e] + d/x)^2) - (3*(a + b*ArcCsch[c*x]))/(16*e^2*(Sqrt[-d]*Sqrt[e] + d/x)) -
 (3*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqr
t[1 + 1/(c^2*x^2)])])/(16*Sqrt[d]*Sqrt[c^2*d - e]*e^2) + (b*ArcTanh[(c^2*d
- (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])
/(16*Sqrt[d]*(c^2*d - e)^(3/2)*e) - (3*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e]
)/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*Sqrt[d]*Sqrt[c
^2*d - e]*e^2) + (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[
c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*Sqrt[d]*(c^2*d - e)^(3/2)*e) + (3*(
a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c
^2*d) + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcCsch[c*x])*Log[1 + (c*Sq
rt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e^(5/2
)) + (3*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] +
 Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*(a + b*ArcCsch[c*x])*Log[
1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*Sqrt[-
d]*e^(5/2)) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt
[-(c^2*d) + e]))])/(16*Sqrt[-d]*e^(5/2)) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^Ar
cCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e^(5/2)) - (3*b*P
olyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(
16*Sqrt[-d]*e^(5/2)) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e]
 + Sqrt[-(c^2*d) + e])])/(16*Sqrt[-d]*e^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 739

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 745

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
```

```
0] && NeQ[m, -1]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int\left(-\frac{d^3\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{8(-d)^{3/2}e^{3/2}\left(\sqrt{-d}\,\sqrt{e}-dx\right)^3} - \frac{3d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{16e^2\left(\sqrt{-d}\,\sqrt{e}-dx\right)^2}\right.\right.$$

$$= \frac{(3d)\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e^2} + \frac{(3d)\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\,\sqrt{e}+dx\right)^2}\,d\right.}{16e^2}$$

$$= \frac{\sqrt{-d}\left(a+b\text{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)^2} + \frac{3\left(a+b\text{csch}^{-1}(cx)\right)}{16e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{-d}\left(a+b\text{csch}^{-1}(cx)\right)}{16e^{3/2}\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}$$

$$= -\frac{bc\sqrt{-d}\,\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\,\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \cdots$$

$$= -\frac{bc\sqrt{-d}\,\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\,\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \cdots$$

$$= -\frac{bc\sqrt{-d}\,\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\,\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \cdots$$

$$= -\frac{bc\sqrt{-d}\,\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\,\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \cdots$$

$$= -\frac{bc\sqrt{-d}\,\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{-d}\,\sqrt{1+\frac{1}{c^2x^2}}}{16\left(c^2d-e\right)e^{3/2}\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)} + \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.05, size = 2045, normalized size = 1.85

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*(((I/16)*Sqrt[d]*((I*c*Sqrt[e]
*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*((-I)*Sqrt[d] + Sqrt[e]*x))
- ArcCsch[c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d
*Sqrt[e]) + (I*(2*c^2*d - e)*Log[(4*d*Sqrt[c^2*d - e]*Sqrt[e]*(Sqrt[e] + I*
c*(c*Sqrt[d] - Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(S
qrt[d] + I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/e^2 - ((I/16)*Sqrt[d]*(((-
I)*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*(I*Sqrt[d] + Sqr
t[e]*x)) - ArcCsch[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*
x)]/(d*Sqrt[e]) + (I*(2*c^2*d - e)*Log[((4*I)*d*Sqrt[c^2*d - e]*Sqrt[e]*(I*
Sqrt[e] + c*(c*Sqrt[d] + Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2
*d - e)*(Sqrt[d] - I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/e^2 + (5*(-(ArcC
sch[c*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2
*Sqrt[d]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 +
1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c^2*
d) + e]))/Sqrt[d]))/(16*e^2) + (5*(-(ArcCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e
*x)) + (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*
(I*c*Sqrt[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d)
 + e]*(Sqrt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d]))/(16*e^2) + (
((3*I)/128)*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sq
rt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi
+ (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-
2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Ar
cCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2
*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*
Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x
])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcC
sch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d)
 + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqr
t[d])]/Sqrt[2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(
c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*Ar
cCsch[c*x])] + 8*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*
x])/(c*Sqrt[d])] + 8*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcC
sch[c*x])/(c*Sqrt[d])]))/(Sqrt[d]*e^(5/2)) - (((3*I)/128)*(Pi^2 - (4*I)*Pi*
ArcCsch[c*x] - 8*ArcCsch[c*x]^2 - 32*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/S
qrt[2]]*ArcTan[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqr
t[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*L
```

```
og[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*
ArcCsch[c*x]*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*
Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I
*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Lo
g[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*Ar
cCsch[c*x]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqr
t[d])] - (16*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(S
qrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sq
rt[e] - (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, (
(-I)*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyL
og[2, (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]))/(Sqr
t[d]*e^(5/2)))
```

Maple [**F**]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + b\,\mathrm{arccsch}(cx))}{(e\,x^2 + d)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

```
[Out] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/sqrt(d) - (5*x^3*e + 3*d*x)/(x^4*
e^4 + 2*d*x^2*e^3 + d^2*e^2))*a + b*integrate(x^4*log(sqrt(1/(c^2*x^2) + 1)
 + 1/(c*x))/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccsch(c*x) + a*x^4)/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e
+ d^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^4/(e*x^2 + d)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3,x)
[Out] int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.116 $\int \frac{x^2\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=1106

$$-\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\,\left(c^2d-e\right)\sqrt{e}\,\left(\sqrt{-d}\,\sqrt{e}\,-\frac{d}{x}\right)}-\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\,\left(c^2d-e\right)\sqrt{e}\,\left(\sqrt{-d}\,\sqrt{e}\,+\frac{d}{x}\right)}+\frac{a+b\operatorname{csch}^{-1}(}{16\sqrt{-d}\,\sqrt{e}\,\left(\sqrt{-d}\right.}$$

```
[Out] -1/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1
/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d-e)^(3/2)-1/16*b*arctanh((c^2*d+(-d)^(1/2)*e
^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(3/2)/(c^2*d-e)^
(3/2)-1/16*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)
/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arccsch(c*x))*ln(
1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)
^(3/2)/e^(3/2)-1/16*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(
-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arccsch(
c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/
2)))/(-d)^(3/2)/e^(3/2)+1/16*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d
)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*b*polylog(2,c*(
1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(3/2
)/e^(3/2)+1/16*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/
2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16*b*polylog(2,c*(1/c/x+(1+1/c^2
/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(3/2)/e^(3/2)-1/16
*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/
x^2)^(1/2))/d^(3/2)/e/(c^2*d-e)^(1/2)-1/16*b*arctanh((c^2*d+(-d)^(1/2)*e^(1
/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(3/2)/e/(c^2*d-e)^(
1/2)+1/16*(a+b*arccsch(c*x))/(-d)^(1/2)/e^(1/2)/(-d/x+(-d)^(1/2)*e^(1/2))^2
+1/16*(a+b*arccsch(c*x))/d/e/(-d/x+(-d)^(1/2)*e^(1/2))+1/16*(-a-b*arccsch(c
*x))/(-d)^(1/2)/e^(1/2)/(d/x+(-d)^(1/2)*e^(1/2))^2+1/16*(-a-b*arccsch(c*x))
/d/e/(d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*(1+1/c^2/x^2)^(1/2)/(c^2*d-e)/(-d)^(
1/2)/e^(1/2)/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*(1+1/c^2/x^2)^(1/2)/(c^2*d-
e)/(-d)^(1/2)/e^(1/2)/(d/x+(-d)^(1/2)*e^(1/2))
```

Rubi [A]

time = 2.04, antiderivative size = 1106, normalized size of antiderivative = 1.00, number of steps used = 63, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6439, 5823, 5793, 5828, 745, 739, 212, 5827, 5680, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/16*(b*c*Sqrt[1 + 1/(c^2*x^2)])/(Sqrt[-d]*(c^2*d - e)*Sqrt[e]*(Sqrt[-d]*S
qrt[e] - d/x)) - (b*c*Sqrt[1 + 1/(c^2*x^2)])/(16*Sqrt[-d]*(c^2*d - e)*Sqrt[
e]*(Sqrt[-d]*Sqrt[e] + d/x)) + (a + b*ArcCsch[c*x])/(16*Sqrt[-d]*Sqrt[e]*(S
qrt[-d]*Sqrt[e] - d/x)^2) + (a + b*ArcCsch[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e]
- d/x)) - (a + b*ArcCsch[c*x])/(16*Sqrt[-d]*Sqrt[e]*(Sqrt[-d]*Sqrt[e] + d/x
)^2) - (a + b*ArcCsch[c*x])/(16*d*e*(Sqrt[-d]*Sqrt[e] + d/x)) - (b*ArcTanh[
(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x
^2)])])/(16*d^(3/2)*(c^2*d - e)^(3/2)) - (b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt
[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(3/2)*Sqr
t[c^2*d - e]*e) - (b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt
[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(3/2)*(c^2*d - e)^(3/2)) - (b*Ar
cTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/
(c^2*x^2)])])/(16*d^(3/2)*Sqrt[c^2*d - e]*e) - ((a + b*ArcCsch[c*x])*Log[1
- (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(3/
2)*e^(3/2)) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sq
rt[e] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(3/2)*e^(3/2)) - ((a + b*ArcCsch[c*x
])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16
*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcCsch[c*x])*Log[1 + (c*Sqrt[-d]*E^ArcCsch[
c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog
[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e]))])/(16*(-d
)^(3/2)*e^(3/2)) - (b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqr
t[-(c^2*d) + e])])/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog[2, -((c*Sqrt[-d]*E^
ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e]))])/(16*(-d)^(3/2)*e^(3/2)) - (
b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(
16*(-d)^(3/2)*e^(3/2))
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 739</u>

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

<u>Rule 745</u>

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 6439

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_.) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x
^(m + 2*(p + 1))), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0
] && IntegersQ[m, p]
```

Rubi steps

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\text{Subst}\left(\int \frac{x^2\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{e\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d\left(e+dx^2\right)^3}+\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{d\left(e+dx^2\right)^2}\right)dx, x, \frac{1}{x}\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^2}\,dx, x, \frac{1}{x}\right)}{d}+\frac{e\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(e+dx^2\right)^3}\,dx, x, \frac{1}{x}\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}-dx\right)^2}-\frac{d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{4e\left(\sqrt{-d}\sqrt{e}+dx\right)^2}-\frac{d\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{2e\left(-de-d^2x^2\right)}\right) dx}{d}$$

$$= -\frac{3\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16e}-\frac{3\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x,}{16e}$$

$$= \frac{a+b\text{csch}^{-1}(cx)}{16\sqrt{-d}\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2}+\frac{a+b\text{csch}^{-1}(cx)}{16de\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\text{csch}}{16\sqrt{-d}\sqrt{e}\left(\sqrt{\right.}}$$

$$= -\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\right.}$$

$$= -\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\right.}$$

$$= -\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\right.}$$

$$= -\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{bc\sqrt{1+\frac{1}{c^2x^2}}}{16\sqrt{-d}\left(c^2d-e\right)\sqrt{e}\left(\sqrt{-d}\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.05, size = 2053, normalized size = 1.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*(((-1/16*I)*((I*c*Sqrt[e]*Sqrt[1 +
1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[
c*x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e])
+ (I*(2*c^2*d - e)*Log[(4*d*Sqrt[c^2*d - e]*Sqrt[e]*(Sqrt[e] + I*c*(c*Sqrt[
d] - Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d] + I
*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/(Sqrt[d]*e) + ((I/16)*(((-I)*c*Sqrt[
e]*Sqrt[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*(I*Sqrt[d] + Sqrt[e]*x)) -
 ArcCsch[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqr
t[e]) + (I*(2*c^2*d - e)*Log[((4*I)*d*Sqrt[c^2*d - e]*Sqrt[e]*(I*Sqrt[e] +
c*(c*Sqrt[d] + Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(S
qrt[d] - I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/(Sqrt[d]*e) - (-(ArcCsch[c
*x]/(I*Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt
[d]*Sqrt[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^
2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c^2*d) +
e]))/Sqrt[d])/(16*d*e) - (-(ArcCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) + (I
*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt
[d] + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sq
rt[d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d])/(16*d*e) + ((I/128)*(P
i^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sqrt[e]
/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcCsc
h[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x]
)] + (4*I)*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Ar
cCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt
[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])
] + (4*I)*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*S
qrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCs
ch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]
]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] -
(4*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] +
 8*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d]
)] + 8*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*S
qrt[d])]))/(d^(3/2)*e^(3/2)) - ((I/128)*(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*A
rcCsch[c*x]^2 - 32*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c
*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) + e]] -
 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 + (I*(-Sqrt[e
```

```
] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1
 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)
*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] + Sqrt[
-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 - (I*(Sqrt[e]
+ Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 -
 (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16*I)*Ar
cSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqrt[-(c^
2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] - (I*Sqrt[d]
)/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((-I)*(-Sqrt[e] + S
qrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (I*(Sqrt[e]
+ Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]))/(d^(3/2)*e^(3/2)))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^{2} (a +b \,\mathrm{arccsch}\left(c x \right))}{\left(e \,x^{2}+d \right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*(arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/d^(3/2) + (x^3*e - d*x)/(d*x^4*e^
3 + 2*d^2*x^2*e^2 + d^3*e)) + b*integrate(x^2*log(sqrt(1/(c^2*x^2) + 1) + 1
/(c*x))/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e
+ d^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3,x)
```

```
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^3, x)
```

3.117 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=1096

$$-\frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d-e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)}-\frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d-e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)}+\frac{\sqrt{e}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right.}$$

```
[Out] 1/16*b*e*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+
1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d-e)^(3/2)+1/16*b*e*arctanh((c^2*d+(-d)^(1/2
)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d-
e)^(3/2)+5/16*b*arctanh((c^2*d-(-d)^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1
/2)/(1+1/c^2/x^2)^(1/2))/d^(5/2)/(c^2*d-e)^(1/2)+5/16*b*arctanh((c^2*d+(-d)
^(1/2)*e^(1/2)/x)/c/d^(1/2)/(c^2*d-e)^(1/2)/(1+1/c^2/x^2)^(1/2))/d^(5/2)/(c
^2*d-e)^(1/2)+3/16*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-
d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*arccsch(c
*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2
)))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arccsch(c*x))*ln(1-c*(1/c/x+(1+1/c^2/x^2)^
(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*
arccsch(c*x))*ln(1+c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*
d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1
/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*b*polyl
og(2,c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)-(-c^2*d+e)^(1/2)))/(
-d)^(5/2)/e^(1/2)-3/16*b*polylog(2,-c*(1/c/x+(1+1/c^2/x^2)^(1/2))*(-d)^(1/2
)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*b*polylog(2,c*(1/c/x+
(1+1/c^2/x^2)^(1/2))*(-d)^(1/2)/(e^(1/2)+(-c^2*d+e)^(1/2)))/(-d)^(5/2)/e^(1
/2)+1/16*(a+b*arccsch(c*x))*e^(1/2)/(-d)^(3/2)/(-d/x+(-d)^(1/2)*e^(1/2))^2-
5/16*(a+b*arccsch(c*x))/d^2/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*(a+b*arccsch(c*x
))*e^(1/2)/(-d)^(3/2)/(d/x+(-d)^(1/2)*e^(1/2))^2+5/16*(a+b*arccsch(c*x))/d^
2/(d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*e^(1/2)*(1+1/c^2/x^2)^(1/2)/(-d)^(3/2)/
(c^2*d-e)/(-d/x+(-d)^(1/2)*e^(1/2))-1/16*b*c*e^(1/2)*(1+1/c^2/x^2)^(1/2)/(-
d)^(3/2)/(c^2*d-e)/(d/x+(-d)^(1/2)*e^(1/2))
```

Rubi [A]
time = 2.50, antiderivative size = 1096, normalized size of antiderivative = 1.00, number of steps used = 81, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6429, 5823, 5793, 5828, 745, 739, 212, 5827, 5680, 2221, 2317, 2438}

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x^2)^3,x]
```

```
[Out] -1/16*(b*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)])/((-d)^(3/2)*(c^2*d - e)*(Sqrt[-d]
*Sqrt[e] - d/x)) - (b*c*Sqrt[e]*Sqrt[1 + 1/(c^2*x^2)])/(16*(-d)^(3/2)*(c^2*
d - e)*(Sqrt[-d]*Sqrt[e] + d/x)) + (Sqrt[e]*(a + b*ArcCsch[c*x]))/(16*(-d)^
(3/2)*(Sqrt[-d]*Sqrt[e] - d/x)^2) - (5*(a + b*ArcCsch[c*x]))/(16*d^2*(Sqrt[
-d]*Sqrt[e] - d/x)) - (Sqrt[e]*(a + b*ArcCsch[c*x]))/(16*(-d)^(3/2)*(Sqrt[-
d]*Sqrt[e] + d/x)^2) + (5*(a + b*ArcCsch[c*x]))/(16*d^2*(Sqrt[-d]*Sqrt[e] +
 d/x)) + (5*b*ArcTanh[(c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d
- e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(5/2)*Sqrt[c^2*d - e]) + (b*e*ArcTanh[(
c^2*d - (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^
2)])])/(16*d^(5/2)*(c^2*d - e)^(3/2)) + (5*b*ArcTanh[(c^2*d + (Sqrt[-d]*Sqr
t[e])/x)/(c*Sqrt[d]*Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(5/2)*Sq
rt[c^2*d - e]) + (b*e*ArcTanh[(c^2*d + (Sqrt[-d]*Sqrt[e])/x)/(c*Sqrt[d]*Sqr
t[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])])/(16*d^(5/2)*(c^2*d - e)^(3/2)) + (3*(
a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c
^2*d) + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCsch[c*x])*Log[1 + (c*
Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(5/2)*Sq
rt[e]) + (3*(a + b*ArcCsch[c*x])*Log[1 - (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[
e] + Sqrt[-(c^2*d) + e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCsch[c*x]
)*Log[1 + (c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*
(-d)^(5/2)*Sqrt[e]) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e
] - Sqrt[-(c^2*d) + e]))])/(16*(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (c*Sqr
t[-d]*E^ArcCsch[c*x])/(Sqrt[e] - Sqrt[-(c^2*d) + e])])/(16*(-d)^(5/2)*Sqrt[
e]) - (3*b*PolyLog[2, -((c*Sqrt[-d]*E^ArcCsch[c*x])/(Sqrt[e] + Sqrt[-(c^2*d
) + e]))])/(16*(-d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (c*Sqrt[-d]*E^ArcCsch[
c*x])/(Sqrt[e] + Sqrt[-(c^2*d) + e])])/(16*(-d)^(5/2)*Sqrt[e])
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

<u>Rule 739</u>

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

<u>Rule 745</u>

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
e*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/((m + 1)*(c*d^2 + a*e^2))), x] + D
ist[c*(d/(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5680

```
Int[(Cosh[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))^(m_.))/((a_) + (b_.)*Sin
h[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 + b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 + b^2, 0]
```

Rule 5793

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[e, c^2*d] && IntegerQ[p] && (p >
 0 || IGtQ[n, 0])
```

Rule 5823

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcSinh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[e, c^
2*d] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5827

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[(a + b*x)^n*(Cosh[x]/(c*d + e*Sinh[x])), x], x, ArcSinh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5828

```
Int[((a_.) + ArcSinh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^n/(e*(m + 1))), x]
 - Dist[b*c*(n/(e*(m + 1))), Int[(d + e*x)^(m + 1)*((a + b*ArcSinh[c*x])^(n
 - 1)/Sqrt[1 + c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[n,
 0] && NeQ[m, -1]
```

Rule 6429

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_)^2)^(p_.),
 x_Symbol] :> -Subst[Int[(e + d*x^2)^p*((a + b*ArcSinh[x/c])^n/x^(2*(p + 1)
)), x], x, 1/x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[n, 0] && IntegerQ[p
]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^3}\,dx = -\text{Subst}\left(\int \frac{x^4\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{(e+dx^2)^3}\,dx, x, \frac{1}{x}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{e^2\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d^2(e+dx^2)^3} - \frac{2e\left(a+b\sinh^{-1}\left(\frac{x}{c}\right)\right)}{d^2(e+dx^2)^2} + \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{d^2(e+dx^2)}\right) d$$

$$= -\frac{\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{e+dx^2}\,dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{(e+dx^2)^2}\,dx, x, \frac{1}{x}\right)}{d^2} - \frac{e^2\text{Subst}\left(\right.}{}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}-\sqrt{-d}\,x\right)} + \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{2\sqrt{e}\left(\sqrt{e}+\sqrt{-d}\,x\right)}\right) dx, x, \frac{1}{x}\right)}{d^2} + \frac{(2e)\text{Subst}}{}$$

$$= \frac{3\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}-dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16d} + \frac{3\text{Subst}\left(\int \frac{a+b\sinh^{-1}\left(\frac{x}{c}\right)}{\left(\sqrt{-d}\sqrt{e}+dx\right)^2}\,dx, x, \frac{1}{x}\right)}{16d} +$$

$$= \frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx)\right)}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)^2} - \frac{5\left(a+b\text{csch}^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{e}\left(a+b\text{csch}^{-1}(cx\right.}{16(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+\right.}$$

$$= -\frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d-e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d-e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} +$$

$$= -\frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d-e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d-e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} +$$

$$= -\frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d-e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d-e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} +$$

$$= -\frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d-e)\left(\sqrt{-d}\sqrt{e}-\frac{d}{x}\right)} - \frac{bc\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}}{16(-d)^{3/2}(c^2d-e)\left(\sqrt{-d}\sqrt{e}+\frac{d}{x}\right)} +$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.03, size = 2038, normalized size = 1.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2)^3,x]
```

```
[Out] (a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + b*(((I/16)*((I*c*Sqrt[e]*Sqrt[1 + 1/
(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCsch[c*
x]/(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e]) +
(I*(2*c^2*d - e)*Log[(4*d*Sqrt[c^2*d - e]*Sqrt[e]*(Sqrt[e] + I*c*(c*Sqrt[d]
 - Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d] + I*S
qrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/d^(3/2) - ((I/16)*(((-I)*c*Sqrt[e]*Sqr
t[1 + 1/(c^2*x^2)]*x)/(Sqrt[d]*(c^2*d - e)*(I*Sqrt[d] + Sqrt[e]*x)) - ArcCs
ch[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - ArcSinh[1/(c*x)]/(d*Sqrt[e])
 + (I*(2*c^2*d - e)*Log[((4*I)*d*Sqrt[c^2*d - e]*Sqrt[e]*(I*Sqrt[e] + c*(c*S
qrt[d] + Sqrt[c^2*d - e]*Sqrt[1 + 1/(c^2*x^2)])*x))/((2*c^2*d - e)*(Sqrt[d]
 - I*Sqrt[e]*x))])/(d*(c^2*d - e)^(3/2))))/d^(3/2) - (3*(-(ArcCsch[c*x]/(I*
Sqrt[d]*Sqrt[e] + e*x)) - (I*(ArcSinh[1/(c*x)]/Sqrt[e] - Log[(2*Sqrt[d]*Sqr
t[e]*(I*Sqrt[e] + c*(c*Sqrt[d] + I*Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)]
)*x))/(Sqrt[-(c^2*d) + e]*(I*Sqrt[d] + Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sq
rt[d]))/(16*d^2) - (3*(-(ArcCsch[c*x]/((-I)*Sqrt[d]*Sqrt[e] + e*x)) + (I*(A
rcSinh[1/(c*x)]/Sqrt[e] - Log[(-2*Sqrt[d]*Sqrt[e]*(Sqrt[e] + c*(I*c*Sqrt[d]
 + Sqrt[-(c^2*d) + e]*Sqrt[1 + 1/(c^2*x^2)])*x))/(Sqrt[-(c^2*d) + e]*(Sqrt[
d] + I*Sqrt[e]*x))]/Sqrt[-(c^2*d) + e]))/Sqrt[d]))/(16*d^2) + (((3*I)/128)*
(Pi^2 - (4*I)*Pi*ArcCsch[c*x] - 8*ArcCsch[c*x]^2 + 32*ArcSin[Sqrt[1 + Sqrt[
e]/(c*Sqrt[d])]/Sqrt[2]]*ArcTan[((c*Sqrt[d] - Sqrt[e])*Cot[(Pi + (2*I)*ArcC
sch[c*x])/4])/Sqrt[-(c^2*d) + e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*
x])] + (4*I)*Pi*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/
(c*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^
ArcCsch[c*x])/(c*Sqrt[d])] + (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sq
rt[2]]*Log[1 - (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d
])] + (4*I)*Pi*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])] + 8*ArcCsch[c*x]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^Arc
Csch[c*x])/(c*Sqrt[d])] - (16*I)*ArcSin[Sqrt[1 + Sqrt[e]/(c*Sqrt[d])]/Sqrt[
2]]*Log[1 + (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]
 - (4*I)*Pi*Log[Sqrt[e] + (I*Sqrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])]
 + 8*PolyLog[2, (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[
d])] + 8*PolyLog[2, ((-I)*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c
*Sqrt[d])]))/(d^(5/2)*Sqrt[e]) - (((3*I)/128)*(Pi^2 - (4*I)*Pi*ArcCsch[c*x]
 - 8*ArcCsch[c*x]^2 - 32*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*ArcT
an[((c*Sqrt[d] + Sqrt[e])*Cot[(Pi + (2*I)*ArcCsch[c*x])/4])/Sqrt[-(c^2*d) +
 e]] - 8*ArcCsch[c*x]*Log[1 - E^(-2*ArcCsch[c*x])] + (4*I)*Pi*Log[1 + (I*(-
```

```
Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]
*Log[1 + (I*(-Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] +
(16*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 + (I*(-Sqrt[e] +
 Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + (4*I)*Pi*Log[1 - (I*(Sq
rt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*ArcCsch[c*x]*L
og[1 - (I*(Sqrt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (16
*I)*ArcSin[Sqrt[1 - Sqrt[e]/(c*Sqrt[d])]/Sqrt[2]]*Log[1 - (I*(Sqrt[e] + Sqr
t[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] - (4*I)*Pi*Log[Sqrt[e] - (I*S
qrt[d])/x] + 4*PolyLog[2, E^(-2*ArcCsch[c*x])] + 8*PolyLog[2, ((-I)*(-Sqrt[
e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])] + 8*PolyLog[2, (I*(Sq
rt[e] + Sqrt[-(c^2*d) + e])*E^ArcCsch[c*x])/(c*Sqrt[d])]))/(d^(5/2)*Sqrt[e]
))
```

Maple [**F**]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arccsch}(cx)}{\left(e\,x^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

```
[Out] int((a+b*arccsch(c*x))/(e*x^2+d)^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*((3*x^3*e + 5*d*x)/(d^2*x^4*e^2 + 2*d^3*x^2*e + d^4) + 3*arctan(x*e^(
1/2)/sqrt(d))*e^(-1/2)/d^(5/2)) + b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1
/(c*x))/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/(x^6*e^3 + 3*d*x^4*e^2 + 3*d^2*x^2*e + d^3),
x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d)**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^3,x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/(e*x^2 + d)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{\left(e\,x^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^3,x)
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^3, x)
```

3.118 $\int x^5 \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=413

$$-\frac{b(23c^4d^2-12c^2de-75e^2)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}}-\frac{b(29c^2d+25e)\,x\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{840c^3e^2\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1}}{4}$$

```
[Out] 1/3*d^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/e^3-2/5*d*(e*x^2+d)^(5/2)*(a+b*a
rccsch(c*x))/e^3+1/7*(e*x^2+d)^(7/2)*(a+b*arccsch(c*x))/e^3+1/1680*b*(105*c
^6*d^3+35*c^4*d^2*e+63*c^2*d*e^2-75*e^3)*x*arctan(e^(1/2)*(-c^2*x^2-1)^(1/2
)/c/(e*x^2+d)^(1/2))/c^6/e^(5/2)/(-c^2*x^2)^(1/2)+8/105*b*c*d^(7/2)*x*arcta
n((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^3/(-c^2*x^2)^(1/2)-1/840*b*
(29*c^2*d+25*e)*x*(e*x^2+d)^(3/2)*(-c^2*x^2-1)^(1/2)/c^3/e^2/(-c^2*x^2)^(1/
2)+1/42*b*x*(e*x^2+d)^(5/2)*(-c^2*x^2-1)^(1/2)/c/e^2/(-c^2*x^2)^(1/2)-1/168
0*b*(23*c^4*d^2-12*c^2*d*e-75*e^2)*x*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^5
/e^2/(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.92, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {272, 45, 6437, 12, 1629, 159, 163, 65, 223, 209, 95, 210}

$$\frac{d^2(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3}-\frac{2d(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}+\frac{(d+ex^2)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^3}+\frac{8bcd^{7/2}x\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{105e^3\sqrt{-c^2x^2}}+\frac{bx(105c^6d^3+35c^4d^2e+63c^2de^2-75e^3)\text{ArcTan}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{1680c^6e^{5/2}\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}\,(d+ex^2)^{5/2}}{42ce^2\sqrt{-c^2x^2}}-\frac{bx\sqrt{-c^2x^2-1}\,(29c^2d+25e)\,(d+ex^2)^{3/2}}{840c^3e^2\sqrt{-c^2x^2}}-\frac{bx\sqrt{-c^2x^2-1}\,(23c^4d^2-12c^2de-75e^2)\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -1/1680*(b*(23*c^4*d^2 - 12*c^2*d*e - 75*e^2)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d +
 e*x^2])/(c^5*e^2*Sqrt[-(c^2*x^2)]) - (b*(29*c^2*d + 25*e)*x*Sqrt[-1 - c^2*
x^2]*(d + e*x^2)^(3/2))/(840*c^3*e^2*Sqrt[-(c^2*x^2)]) + (b*x*Sqrt[-1 - c^2
*x^2]*(d + e*x^2)^(5/2))/(42*c*e^2*Sqrt[-(c^2*x^2)]) + (d^2*(d + e*x^2)^(3/
2)*(a + b*ArcCsch[c*x]))/(3*e^3) - (2*d*(d + e*x^2)^(5/2)*(a + b*ArcCsch[c*
x]))/(5*e^3) + ((d + e*x^2)^(7/2)*(a + b*ArcCsch[c*x]))/(7*e^3) + (b*(105*c
^6*d^3 + 35*c^4*d^2*e + 63*c^2*d*e^2 - 75*e^3)*x*ArcTan[(Sqrt[e]*Sqrt[-1 -
c^2*x^2])/(c*Sqrt[d + e*x^2])])/(1680*c^6*e^(5/2)*Sqrt[-(c^2*x^2)]) + (8*b*
c*d^(7/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(105*e^3*
Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^5\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{d^2(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$= \frac{d^2(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$= \frac{d^2(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3} - \frac{2d(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^3}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\,(d+ex^2)^{5/2}}{42ce^2\sqrt{-c^2x^2}} + \frac{d^2(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3} - 2$$

$$= -\frac{b(29c^2d+25e)\,x\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{840c^3e^2\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\,(d+e}{42ce^2\sqrt{-c^2x^2}}$$

$$= -\frac{b(23c^4d^2-12c^2de-75e^2)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}} - \frac{b(29c^2d+2}{}$$

$$= -\frac{b(23c^4d^2-12c^2de-75e^2)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}} - \frac{b(29c^2d+2}{}$$

$$= -\frac{b(23c^4d^2-12c^2de-75e^2)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}} - \frac{b(29c^2d+2}{}$$

$$= -\frac{b(23c^4d^2-12c^2de-75e^2)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}} - \frac{b(29c^2d+2}{}$$

$$= -\frac{b(23c^4d^2-12c^2de-75e^2)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{1680c^5e^2\sqrt{-c^2x^2}} - \frac{b(29c^2d+2}{}$$

Mathematica [A]
time = 3.86, size = 319, normalized size = 0.77

$$\frac{\sqrt{d+ex^2}\left(16ac^5(8d^3-4d^2ex^2+3de^2x^4+15e^3x^6)+be\sqrt{1+\frac{1}{c^2x^2}}\,x(75e^2-2c^2e(19d+25ex^2)+c^4(-41d^2+22dex^2+40e^2x^4))+16bc^5(8d^3-4d^2ex^2+3de^2x^4+15e^3x^6)\,\text{csch}^{-1}(cx)\right)}{1680c^5e^3} - \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(128c^7d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}\,(-105c^6d^3-35c^4d^2e-63c^2de^2+75e^3)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{1680c^6e^3\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(16*a*c^5*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^2*x^4 + 15*e^3*x^6)
 + b*e*Sqrt[1 + 1/(c^2*x^2)]*x*(75*e^2 - 2*c^2*e*(19*d + 25*e*x^2) + c^4*(-
41*d^2 + 22*d*e*x^2 + 40*e^2*x^4)) + 16*b*c^5*(8*d^3 - 4*d^2*e*x^2 + 3*d*e^
2*x^4 + 15*e^3*x^6)*ArcCsch[c*x]))/(1680*c^5*e^3) - (b*Sqrt[1 + 1/(c^2*x^2)
]*x*(128*c^7*d^(7/2)*ArcTanh[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2]] +
 Sqrt[e]*(-105*c^6*d^3 - 35*c^4*d^2*e - 63*c^2*d*e^2 + 75*e^3)*ArcTanh[(Sqr
t[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]))/(1680*c^6*e^3*Sqrt[1 + c^2*x
^2])
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int x^{5}(a+b\,\mathrm{arccsch}(cx))\,\sqrt{e\,x^{2}+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*(x^2*e + d)^(3/2)*x^4*e^(-1) - 12*(x^2*e + d)^(3/2)*d*x^2*e^(-2)
+ 8*(x^2*e + d)^(3/2)*d^2*e^(-3))*a + 1/105*((15*x^6*e^3 + 3*d*x^4*e^2 - 4*
d^2*x^2*e + 8*d^3)*sqrt(x^2*e + d)*e^(-3)*log(sqrt(c^2*x^2 + 1) + 1) + 105*
integrate(1/105*(15*c^2*x^7*e^3 + 3*c^2*d*x^5*e^2 - 4*c^2*d^2*x^3*e + 8*c^2
*d^3*x)*sqrt(x^2*e + d)/(c^2*x^2*e^3 + (c^2*x^2*e^3 + e^3)*sqrt(c^2*x^2 + 1
) + e^3), x) - 105*integrate(1/105*(15*c^2*x^7*(7*log(c) + 1)*e^3 - 4*c^2*d
^2*x^3*e + 8*c^2*d^3*x + 3*(c^2*d*e^2 + 35*e^3*log(c))*x^5 + 105*(c^2*x^7*e
^3 + x^5*e^3)*log(x))*sqrt(x^2*e + d)/(c^2*x^2*e^3 + e^3), x))*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1059 vs. 2(352) = 704.
time = 1.53, size = 2155, normalized size = 5.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6720*(128*b*c^7*d^(7/2)*log((c^4*d^2*x^4 + 8*c^2*d^2*x^2 + x^4*cosh(1)^2
 + x^4*sinh(1)^2 - 4*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*
sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 8*d^2 + 2*(3*c^2*d*x^4 + 4*d*x^2)*cosh(1) + 2*(3*c^2*d*x^4 + x^4*cosh(1) +
 4*d*x^2)*sinh(1))/x^4) - (105*b*c^6*d^3 + 35*b*c^4*d^2*cosh(1) + 63*b*c^2*
d*cosh(1)^2 - 75*b*cosh(1)^3 - 75*b*sinh(1)^3 + 9*(7*b*c^2*d - 25*b*cosh(1)
)*sinh(1)^2 + (35*b*c^4*d^2 + 126*b*c^2*d*cosh(1) - 225*b*cosh(1)^2)*sinh(1
))*sqrt(cosh(1) + sinh(1))*log(c^4*d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1
)^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*sinh(1)^2 - 4*(c^4*d*x + (2*c^4*x^3 + c^2
*x)*cosh(1) + (2*c^4*x^3 + c^2*x)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) +
 d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))*sqrt(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2
+ 3*c^2*d)*cosh(1) + 2*(4*c^4*d*x^2 + 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2 + 1)
*cosh(1))*sinh(1)) + 64*(15*b*c^7*x^6*cosh(1)^3 + 15*b*c^7*x^6*sinh(1)^3 +
3*b*c^7*d*x^4*cosh(1)^2 - 4*b*c^7*d^2*x^2*cosh(1) + 8*b*c^7*d^3 + 3*(15*b*c
^7*x^6*cosh(1) + b*c^7*d*x^4)*sinh(1)^2 + (45*b*c^7*x^6*cosh(1)^2 + 6*b*c^7
*d*x^4*cosh(1) - 4*b*c^7*d^2*x^2)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) +
 d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(240*a*c^7*x^6*c
osh(1)^3 + 240*a*c^7*x^6*sinh(1)^3 + 48*a*c^7*d*x^4*cosh(1)^2 - 64*a*c^7*d^
2*x^2*cosh(1) + 128*a*c^7*d^3 + 48*(15*a*c^7*x^6*cosh(1) + a*c^7*d*x^4)*sin
h(1)^2 + 16*(45*a*c^7*x^6*cosh(1)^2 + 6*a*c^7*d*x^4*cosh(1) - 4*a*c^7*d^2*x
^2)*sinh(1) - (41*b*c^6*d^2*x*cosh(1) - 5*(8*b*c^6*x^5 - 10*b*c^4*x^3 + 15*
b*c^2*x)*cosh(1)^3 - 5*(8*b*c^6*x^5 - 10*b*c^4*x^3 + 15*b*c^2*x)*sinh(1)^3
- 2*(11*b*c^6*d*x^3 - 19*b*c^4*d*x)*cosh(1)^2 - (22*b*c^6*d*x^3 - 38*b*c^4*
d*x + 15*(8*b*c^6*x^5 - 10*b*c^4*x^3 + 15*b*c^2*x)*cosh(1))*sinh(1)^2 + (41
*b*c^6*d^2*x - 15*(8*b*c^6*x^5 - 10*b*c^4*x^3 + 15*b*c^2*x)*cosh(1)^2 - 4*(
11*b*c^6*d*x^3 - 19*b*c^4*d*x)*cosh(1))*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^
2)))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(c^7*cosh(1)^3 + 3*c^7*cosh(1)^2*
sinh(1) + 3*c^7*cosh(1)*sinh(1)^2 + c^7*sinh(1)^3), 1/6720*(256*b*c^7*sqrt(
-d)*d^3*arctan(1/2*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*sq
rt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c
^2*d^2*x^2 + d^2 + (c^2*d*x^4 + d*x^2)*cosh(1) + (c^2*d*x^4 + d*x^2)*sinh(1
))) - (105*b*c^6*d^3 + 35*b*c^4*d^2*cosh(1) + 63*b*c^2*d*cosh(1)^2 - 75*b*c
osh(1)^3 - 75*b*sinh(1)^3 + 9*(7*b*c^2*d - 25*b*cosh(1))*sinh(1)^2 + (35*b*
c^4*d^2 + 126*b*c^2*d*cosh(1) - 225*b*cosh(1)^2)*sinh(1))*sqrt(cosh(1) + si
nh(1))*log(c^4*d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 + 8
*c^2*x^2 + 1)*sinh(1)^2 - 4*(c^4*d*x + (2*c^4*x^3 + c^2*x)*cosh(1) + (2*c^4
*x^3 + c^2*x)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt((c^2*x^2 +
1)/(c^2*x^2))*sqrt(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1) +
 2*(4*c^4*d*x^2 + 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) +
 64*(15*b*c^7*x^6*cosh(1)^3 + 15*b*c^7*x^6*sinh(1)^3 + 3*b*c^7*d*x^4*cosh(1
)^2 - 4*b*c^7*d^2*x^2*cosh(1) + 8*b*c^7*d^3 + 3*(15*b*c^7*x^6*cosh(1) + b*c
^7*d*x^4)*sinh(1)^2 + (45*b*c^7*x^6*cosh(1)^2 + 6*b*c^7*d*x^4*cosh(1) - 4*b
*c^7*d^2*x^2)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*log((c*x*sqrt((c
^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(240*a*c^7*x^6*cosh(1)^3 + 240*a*c^7
*x^6*sinh(1)^3 + 48*a*c^7*d*x^4*cosh(1)^2 - 64*a*c^7*d^2*x^2*cosh(1) + 128*
```

```
a*c^7*d^3 + 48*(15*a*c^7*x^6*cosh(1) + a*c^7*d*x^4)*sinh(1)^2 + 16*(45*a*c^
7*x^6*cosh(1)^2 + 6*a*c^7*d*x^4*cosh(1) - 4*a*c^7*d^2*x^2)*sinh(1) - (41*b*
c^6*d^2*x*cosh(1) - 5*(8*b*c^6*x^5 - 10*b*c^4*x^3 + 15*b*c^2*x)*cosh(1)^3 -
 5*(8*b*c^6*x^5 - 10*b*c^4*x^3 + 15*b*c^2*x)*sinh(1)^3 - 2*(11*b*c^6*d*x^3
- 19*b*c^4*d*x)*cosh(1)^2 - (22*b*c^6*d*x^3 - 38*b*c^4*d*x + 15*(8*b*c^6*x^
5 - 10*b*c^4*x^3 + 15*b*c^2*x)*cosh(1))*sinh(1)^2 + (41*b*c^6*d^2*x - 15*(8
*b*c^6*x^5 - 10*b*c^4*x^3 + 15*b*c^2*x)*cosh(1)^2 - 4*(11*b*c^6*d*x^3 - 19*
b*c^4*d*x)*cosh(1))*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(x^2*cosh(1
) + x^2*sinh(1) + d))/(c^7*cosh(1)^3 + 3*c^7*cosh(1)^2*sinh(1) + 3*c^7*cosh
(1)*sinh(1)^2 + c^7*sinh(1)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*x^5, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x^5*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)
```

3.119 $\int x^3\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=302

$$\frac{b(c^2d-9e)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{20ce\sqrt{-c^2x^2}}-\frac{d(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{(d+e}{}$$

```
[Out] -1/3*d*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/e^2+1/5*(e*x^2+d)^(5/2)*(a+b*arcc
sch(c*x))/e^2-1/120*b*(15*c^4*d^2+10*c^2*d*e-9*e^2)*x*arctan(e^(1/2)*(-c^2*
x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(3/2)/(-c^2*x^2)^(1/2)-2/15*b*c*d^(5/
2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^2/(-c^2*x^2)^(1/2
)+1/20*b*x*(e*x^2+d)^(3/2)*(-c^2*x^2-1)^(1/2)/c/e/(-c^2*x^2)^(1/2)+1/120*b*
(c^2*d-9*e)*x*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/e/(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {272, 45, 6437, 12, 587, 159, 163, 65, 223, 209, 95, 210}

$$-\frac{d(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}+\frac{(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}-\frac{2bcd^{5/2}x\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{15e^2\sqrt{-c^2x^2}}-\frac{bx(15c^4d^2+10c^2de-9e^2)\text{ArcTan}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{3/2}\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}\,(d+ex^2)^{3/2}}{20ce\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}\,(c^2d-9e)\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(c^2*d - 9*e)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(120*c^3*e*Sqrt[-(c^
2*x^2)]) + (b*x*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e*Sqrt[-(c^2*x^
2)]) - (d*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^2) + ((d + e*x^2)^(5
/2)*(a + b*ArcCsch[c*x]))/(5*e^2) - (b*(15*c^4*d^2 + 10*c^2*d*e - 9*e^2)*x*
ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^4*e^(3/2)*
Sqrt[-(c^2*x^2)]) - (2*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-
1 - c^2*x^2])])/(15*e^2*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^3\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)\,dx = -\frac{d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2}$$

$$= -\frac{d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2}$$

$$= -\frac{d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+\frac{(d+ex^2)^{5/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5e^2}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{20ce\sqrt{-c^2x^2}}-\frac{d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^2}+$$

$$= \frac{b(c^2d-9e)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{20ce\sqrt{-c^2x^2}}$$

$$= \frac{b(c^2d-9e)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{20ce\sqrt{-c^2x^2}}$$

$$= \frac{b(c^2d-9e)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{20ce\sqrt{-c^2x^2}}$$

$$= \frac{b(c^2d-9e)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{20ce\sqrt{-c^2x^2}}$$

$$= \frac{b(c^2d-9e)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{120c^3e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{20ce\sqrt{-c^2x^2}}$$

Mathematica [A]
time = 2.36, size = 255, normalized size = 0.84

$$\frac{\sqrt{d+ex^2}\left(8ac^3(-2d^2+dex^2+3e^2x^4)+be\sqrt{1+\frac{1}{c^2x^2}}\,x(-9e+c^2(7d+6ex^2))+8bc^3(-2d^2+dex^2+3e^2x^4)\operatorname{csch}^{-1}(cx)\right)}{120c^3e^2}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(16c^5d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}\,(-15c^4d^2-10c^2de+9e^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{120c^4e^2\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^3*(-2*d^2 + d*e*x^2 + 3*e^2*x^4) + b*e*Sqrt[1 + 1/(
c^2*x^2)]*x*(-9*e + c^2*(7*d + 6*e*x^2)) + 8*b*c^3*(-2*d^2 + d*e*x^2 + 3*e^
```

```
2*x^4)*ArcCsch[c*x]))/(120*c^3*e^2) + (b*Sqrt[1 + 1/(c^2*x^2)]*x*(16*c^5*d^
(5/2)*ArcTanh[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2]] + Sqrt[e]*(-15*c
^4*d^2 - 10*c^2*d*e + 9*e^2)*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d
+ e*x^2])]))/(120*c^4*e^2*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int x^3(a + b\,\mathrm{arccsch}(cx))\sqrt{e\,x^2 + d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*(x^2*e + d)^(3/2)*x^2*e^(-1) - 2*(x^2*e + d)^(3/2)*d*e^(-2))*a + 1/
15*((3*x^4*e^2 + d*x^2*e - 2*d^2)*sqrt(x^2*e + d)*e^(-2)*log(sqrt(c^2*x^2 +
 1) + 1) + 15*integrate(1/15*(3*c^2*x^5*e^2 + c^2*d*x^3*e - 2*c^2*d^2*x)*sq
rt(x^2*e + d)/(c^2*x^2*e^2 + (c^2*x^2*e^2 + e^2)*sqrt(c^2*x^2 + 1) + e^2),
x) - 15*integrate(1/15*(3*c^2*x^5*(5*log(c) + 1)*e^2 - 2*c^2*d^2*x + (c^2*d
*e + 15*e^2*log(c))*x^3 + 15*(c^2*x^5*e^2 + x^3*e^2)*log(x))*sqrt(x^2*e + d
)/(c^2*x^2*e^2 + e^2), x))*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(254) = 508$.
time = 0.85, size = 1524, normalized size = 5.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(16*b*c^5*d^(5/2)*log((c^4*d^2*x^4 + 8*c^2*d^2*x^2 + x^4*cosh(1)^2 +
 x^4*sinh(1)^2 + 4*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*sq
rt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8
*d^2 + 2*(3*c^2*d*x^4 + 4*d*x^2)*cosh(1) + 2*(3*c^2*d*x^4 + x^4*cosh(1) + 4
*d*x^2)*sinh(1))/x^4) - (15*b*c^4*d^2 + 10*b*c^2*d*cosh(1) - 9*b*cosh(1)^2
```

```
- 9*b*sinh(1)^2 + 2*(5*b*c^2*d - 9*b*cosh(1))*sinh(1))*sqrt(cosh(1) + sinh(
1))*log(c^4*d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 + 8*c^
2*x^2 + 1)*sinh(1)^2 + 4*(c^4*d*x + (2*c^4*x^3 + c^2*x)*cosh(1) + (2*c^4*x^
3 + c^2*x)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt((c^2*x^2 + 1)/
(c^2*x^2))*sqrt(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1) + 2*
(4*c^4*d*x^2 + 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) + 32
*(3*b*c^5*x^4*cosh(1)^2 + 3*b*c^5*x^4*sinh(1)^2 + b*c^5*d*x^2*cosh(1) - 2*b
*c^5*d^2 + (6*b*c^5*x^4*cosh(1) + b*c^5*d*x^2)*sinh(1))*sqrt(x^2*cosh(1) +
x^2*sinh(1) + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(24
*a*c^5*x^4*cosh(1)^2 + 24*a*c^5*x^4*sinh(1)^2 + 8*a*c^5*d*x^2*cosh(1) - 16*
a*c^5*d^2 + 8*(6*a*c^5*x^4*cosh(1) + a*c^5*d*x^2)*sinh(1) + (7*b*c^4*d*x*co
sh(1) + 3*(2*b*c^4*x^3 - 3*b*c^2*x)*cosh(1)^2 + 3*(2*b*c^4*x^3 - 3*b*c^2*x)
*sinh(1)^2 + (7*b*c^4*d*x + 6*(2*b*c^4*x^3 - 3*b*c^2*x)*cosh(1))*sinh(1))*s
qrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(c^5*cos
h(1)^2 + 2*c^5*cosh(1)*sinh(1) + c^5*sinh(1)^2), -1/480*(32*b*c^5*sqrt(-d)*
d^2*arctan(1/2*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*sqrt(x
^2*cosh(1) + x^2*sinh(1) + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d
^2*x^2 + d^2 + (c^2*d*x^4 + d*x^2)*cosh(1) + (c^2*d*x^4 + d*x^2)*sinh(1)))
+ (15*b*c^4*d^2 + 10*b*c^2*d*cosh(1) - 9*b*cosh(1)^2 - 9*b*sinh(1)^2 + 2*(5
*b*c^2*d - 9*b*cosh(1))*sinh(1))*sqrt(cosh(1) + sinh(1))*log(c^4*d^2 + (8*c
^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*sinh(1)^2 +
 4*(c^4*d*x + (2*c^4*x^3 + c^2*x)*cosh(1) + (2*c^4*x^3 + c^2*x)*sinh(1))*sq
rt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))*sqrt(cosh(1
) + sinh(1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1) + 2*(4*c^4*d*x^2 + 3*c^2*d
 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) - 32*(3*b*c^5*x^4*cosh(1)^
2 + 3*b*c^5*x^4*sinh(1)^2 + b*c^5*d*x^2*cosh(1) - 2*b*c^5*d^2 + (6*b*c^5*x^
4*cosh(1) + b*c^5*d*x^2)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*log((
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 4*(24*a*c^5*x^4*cosh(1)^2 +
 24*a*c^5*x^4*sinh(1)^2 + 8*a*c^5*d*x^2*cosh(1) - 16*a*c^5*d^2 + 8*(6*a*c^5
*x^4*cosh(1) + a*c^5*d*x^2)*sinh(1) + (7*b*c^4*d*x*cosh(1) + 3*(2*b*c^4*x^3
 - 3*b*c^2*x)*cosh(1)^2 + 3*(2*b*c^4*x^3 - 3*b*c^2*x)*sinh(1)^2 + (7*b*c^4*
d*x + 6*(2*b*c^4*x^3 - 3*b*c^2*x)*cosh(1))*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2
*x^2)))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(c^5*cosh(1)^2 + 2*c^5*cosh(1)
*sinh(1) + c^5*sinh(1)^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3(a + b\,\mathrm{acsch}\,(cx))\,\sqrt{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))*sqrt(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x^3*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)
```

3.120 $\int x\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=203

$$\frac{bx\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}+\frac{b(3c^2d-e)\,x\text{ArcTan}\left(\frac{\sqrt{e}\,\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\,\sqrt{-c^2x^2}}+\frac{bc\ldots}{\ldots}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/e+1/3*b*c*d^(3/2)*x*arctan((e*x^2+d)
^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e/(-c^2*x^2)^(1/2)+1/6*b*(3*c^2*d-e)*x*a
rctan(e^(1/2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(1/2)/(-c^2*x^2)^
(1/2)+1/6*b*x*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6435, 457, 104, 163, 65, 223, 209, 95, 210}

$$\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}+\frac{bcd^{3/2}x\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{-c^2x^2-1}}\right)}{3e\sqrt{-c^2x^2}}+\frac{bx(3c^2d-e)\,\text{ArcTan}\left(\frac{\sqrt{e}\,\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2\sqrt{e}\,\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}\,\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(6*c*Sqrt[-(c^2*x^2)]) + ((d + e*x
^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e) + (b*(3*c^2*d - e)*x*ArcTan[(Sqrt[e]*
Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6*c^2*Sqrt[e]*Sqrt[-(c^2*x^2)])
+ (b*c*d^(3/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(3*e
*Sqrt[-(c^2*x^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6435

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2])), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
```

]

Rubi steps

$$\int x\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{(bcx)\int\frac{\left(d+ex^2\right)^{3/2}}{x\sqrt{-1-c^2x^2}}\,dx}{3e\sqrt{-c^2x^2}}$$

$$=\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{(bcx)\text{Subst}\left(\int\frac{(d+ex)^{3/2}}{x\sqrt{-1-c^2x}}\,dx,x$$

$$=\frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}+\frac{(bx)}{}$$

$$=\frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}-\frac{(b(3c}{}$$

$$=\frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}+\frac{(b(3c}{}$$

$$=\frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}+\frac{bcd^{3/}}{}$$

$$=\frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6c\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e}+\frac{b(3c^}{}$$

Mathematica [A]
time = 1.49, size = 194, normalized size = 0.96

$$\frac{\sqrt{d+ex^2}\left(be\sqrt{1+\frac{1}{c^2x^2}}\,x+2ac(d+ex^2)+2bc(d+ex^2)\,\text{csch}^{-1}(cx)\right)}{6ce}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(2c^3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}\,(-3c^2d+e)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{6c^2e\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]

[Out] (Sqrt[d + e*x^2]*(b*e*Sqrt[1 + 1/(c^2*x^2)]*x + 2*a*c*(d + e*x^2) + 2*b*c*(
d + e*x^2)*ArcCsch[c*x]))/(6*c*e) - (b*Sqrt[1 + 1/(c^2*x^2)]*x*(2*c^3*d^(3/
2)*ArcTanh[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2]] + Sqrt[e]*(-3*c^2*d
```

```
 + e)*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]))/(6*c^2*e*S
qrt[1 + c^2*x^2])
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int x(a + b\,\mathrm{arccsch}(cx))\sqrt{e\,x^2 + d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^2*e + d)^(3/2)*a*e^(-1) + 1/3*((x^2*e + d)^(3/2)*e^(-1)*log(sqrt(c^2
*x^2 + 1) + 1) + 3*integrate(1/3*(c^2*x^3*e + c^2*d*x)*sqrt(x^2*e + d)/(c^2
*x^2*e + (c^2*x^2*e + e)*sqrt(c^2*x^2 + 1) + e), x) - 3*integrate(1/3*(c^2*
x^3*(3*log(c) + 1)*e + (c^2*d + 3*e*log(c))*x + 3*(c^2*x^3*e + x*e)*log(x))
*sqrt(x^2*e + d)/(c^2*x^2*e + e), x))*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(168) = 336$.
time = 0.62, size = 1113, normalized size = 5.48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(2*b*c^3*d^(3/2)*log((c^4*d^2*x^4 + 8*c^2*d^2*x^2 + x^4*cosh(1)^2 + x
^4*sinh(1)^2 - 4*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*sqrt
(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d
^2 + 2*(3*c^2*d*x^4 + 4*d*x^2)*cosh(1) + 2*(3*c^2*d*x^4 + x^4*cosh(1) + 4*d
*x^2)*sinh(1))/x^4) - (3*b*c^2*d - b*cosh(1) - b*sinh(1))*sqrt(cosh(1) + si
nh(1))*log(c^4*d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 + 8
*c^2*x^2 + 1)*sinh(1)^2 - 4*(c^4*d*x + (2*c^4*x^3 + c^2*x)*cosh(1) + (2*c^4
*x^3 + c^2*x)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt((c^2*x^2 +
1)/(c^2*x^2))*sqrt(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1) +
```

```
 2*(4*c^4*d*x^2 + 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) +
 8*(b*c^3*x^2*cosh(1) + b*c^3*x^2*sinh(1) + b*c^3*d)*sqrt(x^2*cosh(1) + x^2
*sinh(1) + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c
^3*x^2*cosh(1) + 2*a*c^3*x^2*sinh(1) + 2*a*c^3*d + (b*c^2*x*cosh(1) + b*c^2
*x*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(x^2*cosh(1) + x^2*sinh(1) +
 d))/(c^3*cosh(1) + c^3*sinh(1)), 1/24*(4*b*c^3*sqrt(-d)*d*arctan(1/2*(c^3*
d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*sqrt(x^2*cosh(1) + x^2*sin
h(1) + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d^2*x^2 + d^2 + (c^2*
d*x^4 + d*x^2)*cosh(1) + (c^2*d*x^4 + d*x^2)*sinh(1))) - (3*b*c^2*d - b*cos
h(1) - b*sinh(1))*sqrt(cosh(1) + sinh(1))*log(c^4*d^2 + (8*c^4*x^4 + 8*c^2*
x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*sinh(1)^2 - 4*(c^4*d*x + (
2*c^4*x^3 + c^2*x)*cosh(1) + (2*c^4*x^3 + c^2*x)*sinh(1))*sqrt(x^2*cosh(1)
+ x^2*sinh(1) + d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))*sqrt(cosh(1) + sinh(1)) +
2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1) + 2*(4*c^4*d*x^2 + 3*c^2*d + (8*c^4*x^4 +
 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) + 8*(b*c^3*x^2*cosh(1) + b*c^3*x^2*sinh(1
) + b*c^3*d)*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*log((c*x*sqrt((c^2*x^2 + 1
)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a*c^3*x^2*cosh(1) + 2*a*c^3*x^2*sinh(1) + 2
*a*c^3*d + (b*c^2*x*cosh(1) + b*c^2*x*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)
))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(c^3*cosh(1) + c^3*sinh(1))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b\,\mathrm{acsch}\,(cx))\,\sqrt{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))*sqrt(d + e*x**2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*x, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)
[Out] int(x*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)
```

3.121 $\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\sqrt{d+ex^2}\,(a+b\text{csch}^{-1}(cx))}{x},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\,(a+b\text{csch}^{-1}(cx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x,x]
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\,(a+b\text{csch}^{-1}(cx))}{x}\,dx = \int \frac{\sqrt{d+ex^2}\,(a+b\text{csch}^{-1}(cx))}{x}\,dx$$

Mathematica [A]
time = 3.38, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}\,(a+b\text{csch}^{-1}(cx))}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x, x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{arccsch}(cx))\,\sqrt{e\,x^2+d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x)
```

```
[Out] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -(sqrt(d)*arcsinh(sqrt(d)*e^(-1/2)/abs(x)) - sqrt(x^2*e + d))*a + b*integra
te(sqrt(x^2*e + d)*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsch(c*x) + a)/x, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsch}(cx))\sqrt{d + ex^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x**2)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x,x)
[Out] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x, x)
```

3.122 $\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\sqrt{d+ex^2}\,\left(a+b\text{csch}^{-1}(cx)\right)}{x^3},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\,\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^3,x]
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^3, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\,\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx=\int \frac{\sqrt{d+ex^2}\,\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx$$

Mathematica [A]
time = 3.40, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}\,\left(a+b\text{csch}^{-1}(cx)\right)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^3,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^3, x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\text{arccsch}(cx)\right)\sqrt{e\,x^2+d}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

```
[Out] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(arcsinh(sqrt(d)*e^(-1/2)/abs(x))*e/sqrt(d) - sqrt(x^2*e + d)*e/d + (x
^2*e + d)^(3/2)/(d*x^2))*a + b*integrate(sqrt(x^2*e + d)*log(sqrt(1/(c^2*x^
2) + 1) + 1/(c*x))/x^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsch(c*x) + a)/x^3, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsch}(cx))\sqrt{d + ex^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x**2)/x**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^3,x)
[Out] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^3, x)
```

3.123 $\int x^2 \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

Optimal. Leaf size=26

$$\text{Int}\left(x^2 \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2 \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
[Out] Defer[Int][x^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\int x^2 \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int x^2 \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

Mathematica [A]
time = 5.96, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
[Out] Integrate[x^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Maple [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int x^2 (a+b\,\text{arccsch}(cx)) \sqrt{e\,x^2+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(d^2*arcsinh(x*e^(1/2)/sqrt(d))*e^(-3/2) - 2*(x^2*e + d)^(3/2)*x*e^(-1
) + sqrt(x^2*e + d)*d*x*e^(-1))*a + b*integrate(sqrt(x^2*e + d)*x^2*log(sqr
t(1/(c^2*x^2) + 1) + 1/(c*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)*sqrt(x^2*e + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a + b\,\mathrm{acsch}\,(cx))\,\sqrt{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))*sqrt(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^2\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)
[Out] int(x^2*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)
```

3.124 $\int \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

Optimal. Leaf size=23

$$\text{Int}\left(\sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\int \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx = \int \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

Mathematica [A]
time = 1.58, size = 0, normalized size = 0.00

$$\int \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Maple [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{arccsch}(cx)\right) \sqrt{e\,x^2+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
[Out] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] 1/2*(d*arcsinh(x*e^(1/2)/sqrt(d))*e^(-1/2) + sqrt(x^2*e + d)*x)*a + b*integ
rate(sqrt(x^2*e + d)*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^2*e + d)*(b*arccsch(c*x) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acsch}{(cx)}) \sqrt{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)
[Out] int((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)
```

3.125 $\int \frac{\sqrt{d+ex^2}\left(a+b\mathbf{csch}^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^2,x]
[Out] Defer[Int][(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^2, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx=\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Mathematica [A]
time = 1.10, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^2,x]
[Out] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^2, x]
```

Maple [A]
time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\text{arccsch}(cx)\right)\sqrt{e\,x^2+d}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

```
[Out] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] (arcsinh(x*e^(1/2)/sqrt(d))*e^(1/2) - sqrt(x^2*e + d)/x)*a + b*integrate(sq
rt(x^2*e + d)*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/x^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsch(c*x) + a)/x^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsch}(cx))\sqrt{d + ex^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x**2)/x**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^2,x)
[Out] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^2, x)
```

3.126 $\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=389

$$-\frac{2bc^3(c^2d-2e)\,x^2\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}\,\sqrt{-1-c^2x^2}}-\frac{2bc(c^2d-2e)\,\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}}-\frac{(d+ex^2}{}$$

```
[Out] -1/3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/d/x^3-2/9*b*c^3*(c^2*d-2*e)*x^2*(e*
x^2+d)^(1/2)/d/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)-2/9*b*c*(c^2*d-2*e)*(-c^
2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/(-c^2*x^2)^(1/2)+1/9*b*c*(-c^2*x^2-1)^(1/2
)*(e*x^2+d)^(1/2)/x^2/(-c^2*x^2)^(1/2)+2/9*b*c^2*(c^2*d-2*e)*x*(1/(c^2*x^2+
1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/
2))*(e*x^2+d)^(1/2)/d/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2
*x^2+1))^(1/2)-1/9*b*(c^2*d-3*e)*e*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2
)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^2/(-
c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {270, 6437, 12, 485, 597, 545, 429, 506, 422}

$$-\frac{(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3dx^3}-\frac{bex(c^2d-3e)\sqrt{d+ex^2}\,F\left(\operatorname{ArcTan}(cx)\left|1-\frac{e}{c^2d}\right.\right)}{9d^2\sqrt{-c^2x^2}\,\sqrt{-c^2x^2-1}\,\sqrt{\frac{d+ex^2}{d\,(c^2x^2+1)}}}+\frac{2bc^2x(c^2d-2e)\sqrt{d+ex^2}\,E\left(\operatorname{ArcTan}(cx)\left|1-\frac{e}{c^2d}\right.\right)}{9d\sqrt{-c^2x^2}\,\sqrt{-c^2x^2-1}\,\sqrt{\frac{d+ex^2}{d\,(c^2x^2+1)}}}-\frac{2bc\sqrt{-c^2x^2-1}\,(c^2d-2e)\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}}+\frac{bc\sqrt{-c^2x^2-1}\,\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}}-\frac{2bc^3x^2(c^2d-2e)\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}\,\sqrt{-c^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] (-2*b*c^3*(c^2*d - 2*e)*x^2*Sqrt[d + e*x^2])/(9*d*Sqrt[-(c^2*x^2)]*Sqrt[-1
- c^2*x^2]) - (2*b*c*(c^2*d - 2*e)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(9*d
*Sqrt[-(c^2*x^2)]) + (b*c*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(9*x^2*Sqrt[-
(c^2*x^2)]) - ((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*d*x^3) + (2*b*c^2
*(c^2*d - 2*e)*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(9*
d*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))])
- (b*(c^2*d - 3*e)*e*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)
])/(9*d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*
x^2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^4}\,dx = -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3} - \frac{(bcx)\int -\frac{\left(d+ex^2\right)^{3/2}}{3dx^4\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3} + \frac{(bcx)\int \frac{\left(d+ex^2\right)^{3/2}}{x^4\sqrt{-1-c^2x^2}}\,dx}{3d\sqrt{-c^2x^2}}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3} - \frac{(bcx)\,\cdots}{\cdots}$$

$$= -\frac{2bc(c^2d-2e)\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}}$$

$$= -\frac{2bc(c^2d-2e)\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{9x^2\sqrt{-c^2x^2}}$$

$$= -\frac{2bc^3(c^2d-2e)\,x^2\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}\,\sqrt{-1-c^2x^2}} - \frac{2bc(c^2d-2e)\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}}$$

$$= -\frac{2bc^3(c^2d-2e)\,x^2\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}\,\sqrt{-1-c^2x^2}} - \frac{2bc(c^2d-2e)\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{9d\sqrt{-c^2x^2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.46, size = 237, normalized size = 0.61

$$-\frac{\sqrt{d+ex^2}\left(bc\sqrt{1+\frac{1}{c^2x^2}}\,x(-d+2c^2dx^2-4ex^2)+3a(d+ex^2)+3b(d+ex^2)\,\text{csch}^{-1}(cx)\right)}{9dx^3} - \frac{ibc\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\left(2c^2d(c^2d-2e)\,E\left(i\sinh^{-1}\left(\sqrt{c^2}\,x\right)|\frac{e}{c^2d}\right)+(-2c^4d^2+5c^2de-3e^2)\,F\left(i\sinh^{-1}\left(\sqrt{c^2}\,x\right)|\frac{e}{c^2d}\right)\right)}{9\sqrt{c^2}\,d\sqrt{1+c^2x^2}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] -1/9*(Sqrt[d + e*x^2]*(b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-d + 2*c^2*d*x^2 - 4*e*
x^2) + 3*a*(d + e*x^2) + 3*b*(d + e*x^2)*ArcCsch[c*x]))/(d*x^3) - ((I/9)*b*
c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(2*c^2*d*(c^2*d - 2*e)*Ellipt
icE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] + (-2*c^4*d^2 + 5*c^2*d*e - 3*e^2)*E
llipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d*Sqrt[1 + c^2*x^2
]*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{arccsch}(cx))\,\sqrt{e\,x^2+d}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

```
[Out] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*b*((x^2*e + d)^(3/2)*log(sqrt(c^2*x^2 + 1) + 1)/(d*x^3) + 3*integrate(
-1/3*(c^2*x^4*e - (3*d*log(c) - d)*c^2*x^2 - 3*d*log(c) - 3*(c^2*d*x^2 + d)
*log(x))*sqrt(x^2*e + d)/(c^2*d*x^6 + d*x^4), x) + 3*integrate(1/3*(c^2*x^2
*e + c^2*d)*sqrt(x^2*e + d)/(c^2*d*x^4 + d*x^2 + (c^2*d*x^4 + d*x^2)*sqrt(c
^2*x^2 + 1)), x)) - 1/3*(x^2*e + d)^(3/2)*a/(d*x^3)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsch}(cx))\sqrt{d + ex^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x**4,x)
```

```
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x**2)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^4,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^4, x)
```

3.127 $\int \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=527

$$\frac{bc^3(24c^4d^2-19c^2de-31e^2)\,x^2\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}\,\sqrt{-1-c^2x^2}}+\frac{bc(24c^4d^2-19c^2de-31e^2)\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}}-\frac{bc(12c^2d+$$

```
[Out] -1/5*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/d/x^5+2/15*e*(e*x^2+d)^(3/2)*(a+b*a
rccsch(c*x))/d^2/x^3-1/45*b*c^3*(2*c^2*d-e)*e*x^2*(e*x^2+d)^(1/2)/d^2/(-c^2
*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)-2/15*b*c^3*e^2*x^2*(e*x^2+d)^(1/2)/d^2/(-c^2
*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)+1/75*b*c^3*(8*c^4*d^2-3*c^2*d*e-2*e^2)*x^2*(
e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)-1/45*b*c*(2*c^2*d-e)
*e*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)-2/15*b*c*e^2*(-c
^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)+1/75*b*c*(8*c^4*d^2-3*
c^2*d*e-2*e^2)*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)+1/25
*b*c*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/x^4/(-c^2*x^2)^(1/2)-1/75*b*c*(4*c^
2*d-e)*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(-c^2*x^2)^(1/2)+1/45*b*c*e
*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(-c^2*x^2)^(1/2)+1/45*b*c^2*(2*c^
2*d-e)*e*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1
)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1
)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)+2/15*b*c^2*e^2*x*(1/(c^2*x^2+1))^(1
/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e
*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2
+1))^(1/2)-1/75*b*c^2*(8*c^4*d^2-3*c^2*d*e-2*e^2)*x*(1/(c^2*x^2+1))^(1/2)*(
c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+
d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^
(1/2)+1/75*b*c^2*(4*c^2*d-e)*e*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*El
lipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^2/(-c^2*
x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)-1/45*b*c^2*e^
2*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticF(c*x/(c^2*x^2+1)^(1/2)
,(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)
/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)-2/15*b*e^3*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^
2+1)^(1/2)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/
2)/d^3/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

Rubi [A]

time = 0.43, antiderivative size = 527, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {277, 270, 6437, 12, 594, 597, 545, 429, 506, 422}

$$\frac{2e(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{15d^2x^3}-\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5dx^5}-\frac{bc^2x(24c^4d^2-19c^2de-31e^2)\sqrt{d+ex^2}\,E\left(\text{ArcTan}(cx)\middle|1-\frac{e}{c^2d}\right)}{225d^2\sqrt{-c^2x^2}\,\sqrt{-c^2x^2-1}\,\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}+\frac{2bex(6c^4d^2-4c^2de-15e^2)\sqrt{d+ex^2}\,F\left(\text{ArcTan}(cx)\middle|1-\frac{e}{c^2d}\right)}{225d^3\sqrt{-c^2x^2}\,\sqrt{-c^2x^2-1}\,\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}-\frac{bc\sqrt{-c^2x^2-1}\,(12c^2d+e)\sqrt{d+ex^2}}{225dx^2\sqrt{-c^2x^2}}+\frac{bc\sqrt{-c^2x^2-1}\,(d+ex^2)^{3/2}}{25dx^4\sqrt{-c^2x^2}}+\frac{bc\sqrt{-c^2x^2-1}\,(24c^4d^2-19c^2de-31e^2)\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}}+\frac{bc^3x^2(24c^4d^2-19c^2de-31e^2)\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}\,\sqrt{-c^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (b*c^3*(24*c^4*d^2 - 19*c^2*d*e - 31*e^2)*x^2*Sqrt[d + e*x^2])/(225*d^2*Sqr
t[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]) + (b*c*(24*c^4*d^2 - 19*c^2*d*e - 31*e^2)
*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(225*d^2*Sqrt[-(c^2*x^2)]) - (b*c*(12*
c^2*d + e)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(225*d*x^2*Sqrt[-(c^2*x^2)])
 + (b*c*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(25*d*x^4*Sqrt[-(c^2*x^2)]) -
 ((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(5*d*x^5) + (2*e*(d + e*x^2)^(3/2
)*(a + b*ArcCsch[c*x]))/(15*d^2*x^3) - (b*c^2*(24*c^4*d^2 - 19*c^2*d*e - 31
*e^2)*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(225*d^2*Sqr
t[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]) + (2*
b*e*(6*c^4*d^2 - 4*c^2*d*e - 15*e^2)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x
], 1 - e/(c^2*d)])/(225*d^3*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e
*x^2)/(d*(1 + c^2*x^2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{x^6}\,dx = -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{15d^2x^3}$$

$$= -\frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} + \frac{2e\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{15d^2x^3}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{25dx^4\sqrt{-c^2x^2}} - \frac{\left(d+ex^2\right)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{5dx^5} + \frac{2e(d}{}$$

$$= -\frac{bc\left(12c^2d+e\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225dx^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/}}{25dx^4\sqrt{-c^2x^2}}$$

$$= \frac{bc\left(24c^4d^2-19c^2de-31e^2\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}} - \frac{bc\left(12c^2d+e\right)}{225}$$

$$= \frac{bc\left(24c^4d^2-19c^2de-31e^2\right)\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}} - \frac{bc\left(12c^2d+e\right)}{225}$$

$$= \frac{bc^3\left(24c^4d^2-19c^2de-31e^2\right)x^2\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc\left(24c^4d^2-19c^2de-31\right.}{225d^2}$$

$$= \frac{bc^3\left(24c^4d^2-19c^2de-31e^2\right)x^2\sqrt{d+ex^2}}{225d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}} + \frac{bc\left(24c^4d^2-19c^2de-31\right.}{225d^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.07, size = 314, normalized size = 0.60

$$\frac{\sqrt{d+ex^2}\left(-15a\left(3d^2+dex^2-2e^2x^4\right)+bc\sqrt{1+\frac{1}{c^2x^2}}\,x\left(-31e^2x^4+dex^2\left(8-19c^2x^2\right)+3d^2\left(3-4c^2x^2+8c^4x^4\right)\right)-15b\left(3d^2+dex^2-2e^2x^4\right)\operatorname{csch}^{-1}(cx)\right)}{225d^2x^5}+\frac{ibc\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\left(c^2d\left(24c^4d^2-19c^2de-31e^2\right)E\left(i\sinh^{-1}\left(\sqrt{c^2}\,x\right)\middle|\frac{e}{c^2d}\right)+\left(-24c^6d^3+31c^4d^2e+23c^2de^2-30e^3\right)F\left(i\sinh^{-1}\left(\sqrt{c^2}\,x\right)\middle|\frac{e}{c^2d}\right)\right)}{225\sqrt{c^2}\,d^2\sqrt{1+c^2x^2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (Sqrt[d + e*x^2]*(-15*a*(3*d^2 + d*e*x^2 - 2*e^2*x^4) + b*c*Sqrt[1 + 1/(c^2
*x^2)]*x*(-31*e^2*x^4 + d*e*x^2*(8 - 19*c^2*x^2) + 3*d^2*(3 - 4*c^2*x^2 + 8
*c^4*x^4)) - 15*b*(3*d^2 + d*e*x^2 - 2*e^2*x^4)*ArcCsch[c*x]))/(225*d^2*x^5
) + ((I/225)*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(24*c^4
*d^2 - 19*c^2*d*e - 31*e^2)*EllipticE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] +
```

```
(-24*c^6*d^3 + 31*c^4*d^2*e + 23*c^2*d*e^2 - 30*e^3)*EllipticF[I*ArcSinh[Sq
rt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccsch}(cx))\sqrt{e\,x^2+d}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

```
[Out] int((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="maxima")
```

```
[Out] 1/15*a*(2*(x^2*e + d)^(3/2)*e/(d^2*x^3) - 3*(x^2*e + d)^(3/2)/(d*x^5)) + 1/
15*b*((2*x^4*e^2 - d*x^2*e - 3*d^2)*sqrt(x^2*e + d)*log(sqrt(c^2*x^2 + 1) +
 1)/(d^2*x^5) - 15*integrate(1/15*(2*c^2*x^6*e^2 - c^2*d*x^4*e + 3*(5*d^2*l
og(c) - d^2)*c^2*x^2 + 15*d^2*log(c) + 15*(c^2*d^2*x^2 + d^2)*log(x))*sqrt(
x^2*e + d)/(c^2*d^2*x^8 + d^2*x^6), x) + 15*integrate(1/15*(2*c^2*x^4*e^2 -
 c^2*d*x^2*e - 3*c^2*d^2)*sqrt(x^2*e + d)/(c^2*d^2*x^6 + d^2*x^4 + (c^2*d^2
*x^6 + d^2*x^4)*sqrt(c^2*x^2 + 1)), x))
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acsch}\,(cx))\sqrt{d+ex^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))*(e*x**2+d)**(1/2)/x**6,x)
```

```
[Out] Integral((a + b*acsch(c*x))*sqrt(d + e*x**2)/x**6, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)/x^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))))/x^6, x)
```

3.128 $\int x^3(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=384

$$-\frac{b(3c^4d^2+38c^2de-25e^2)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}}+\frac{b(13c^2d-25e)\,x\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{840c^3e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-}}{42}$$

```
[Out] -1/5*d*(e*x^2+d)^(5/2)*(a+b*arccsch(c*x))/e^2+1/7*(e*x^2+d)^(7/2)*(a+b*arcc
sch(c*x))/e^2-1/560*b*(35*c^6*d^3+35*c^4*d^2*e-63*c^2*d*e^2+25*e^3)*x*arcta
n(e^(1/2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^6/e^(3/2)/(-c^2*x^2)^(1/2
)-2/35*b*c*d^(7/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^2
/(-c^2*x^2)^(1/2)+1/840*b*(13*c^2*d-25*e)*x*(e*x^2+d)^(3/2)*(-c^2*x^2-1)^(1
/2)/c^3/e/(-c^2*x^2)^(1/2)+1/42*b*x*(e*x^2+d)^(5/2)*(-c^2*x^2-1)^(1/2)/c/e/
(-c^2*x^2)^(1/2)-1/560*b*(3*c^4*d^2+38*c^2*d*e-25*e^2)*x*(-c^2*x^2-1)^(1/2)
*(e*x^2+d)^(1/2)/c^5/e/(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {272, 45, 6437, 12, 587, 159, 163, 65, 223, 209, 95, 210}

$$-\frac{d(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}+\frac{(d+ex^2)^{7/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7e^2}-\frac{2bcd^{7/2}x\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{35e^2\sqrt{-c^2x^2}}-\frac{bx(35c^6d^3+35c^4d^2e-63c^2de^2+25e^3)\text{ArcTan}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{560c^6e^{3/2}\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}\,(d+ex^2)^{5/2}}{42ce\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}\,(13c^2d-25e)(d+ex^2)^{3/2}}{840c^3e\sqrt{-c^2x^2}}-\frac{bx\sqrt{-c^2x^2-1}\,(3c^4d^2+38c^2de-25e^2)\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -1/560*(b*(3*c^4*d^2 + 38*c^2*d*e - 25*e^2)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e
*x^2])/(c^5*e*Sqrt[-(c^2*x^2)]) + (b*(13*c^2*d - 25*e)*x*Sqrt[-1 - c^2*x^2]
*(d + e*x^2)^(3/2))/(840*c^3*e*Sqrt[-(c^2*x^2)]) + (b*x*Sqrt[-1 - c^2*x^2]*
(d + e*x^2)^(5/2))/(42*c*e*Sqrt[-(c^2*x^2)]) - (d*(d + e*x^2)^(5/2)*(a + b*
ArcCsch[c*x]))/(5*e^2) + ((d + e*x^2)^(7/2)*(a + b*ArcCsch[c*x]))/(7*e^2) -
 (b*(35*c^6*d^3 + 35*c^4*d^2*e - 63*c^2*d*e^2 + 25*e^3)*x*ArcTan[(Sqrt[e]*S
qrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(560*c^6*e^(3/2)*Sqrt[-(c^2*x^2)])
 - (2*b*c*d^(7/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(
35*e^2*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int x^3\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx = -\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(c\right.}{7e^2}$$

$$= -\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(c\right.}{7e^2}$$

$$= -\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}+\frac{\left(d+ex^2\right)^{7/2}\left(a+b\text{csch}^{-1}(c\right.}{7e^2}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{5/2}}{42ce\sqrt{-c^2x^2}}-\frac{d\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e^2}+$$

$$= \frac{b(13c^2d-25e)\,x\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{840c^3e\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\left(d+\right.}{42ce\sqrt{-c^2x^2}}$$

$$= -\frac{b(3c^4d^2+38c^2de-25e^2)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}}+\frac{b(13c^2d-}{}$$

$$= -\frac{b(3c^4d^2+38c^2de-25e^2)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}}+\frac{b(13c^2d-}{}$$

$$= -\frac{b(3c^4d^2+38c^2de-25e^2)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}}+\frac{b(13c^2d-}{}$$

$$= -\frac{b(3c^4d^2+38c^2de-25e^2)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}}+\frac{b(13c^2d-}{}$$

$$= -\frac{b(3c^4d^2+38c^2de-25e^2)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{560c^5e\sqrt{-c^2x^2}}+\frac{b(13c^2d-}{}$$

<u>Mathematica</u> [A]
time = 3.28, size = 293, normalized size = 0.76

$$\frac{\sqrt{d+ex^2}\left(-48ac^5(2d-5ex^2)\left(d+ex^2\right)^2+be\sqrt{1+\frac{1}{c^2x^2}}\,x(75e^2-2c^2e(82d+25ex^2)+c^4(57d^2+106dex^2+40e^2x^4))-48bc^5(2d-5ex^2)\left(d+ex^2\right)^2\text{csch}^{-1}(cx)\right)}{1680c^5e^2}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(-32c^7d^{7/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}\,(35c^6d^3+35c^4d^2e-63c^2de^2+25e^3)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{560c^6e^2\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-48*a*c^5*(2*d - 5*e*x^2)*(d + e*x^2)^2 + b*e*Sqrt[1 + 1/
(c^2*x^2)]*x*(75*e^2 - 2*c^2*e*(82*d + 25*e*x^2) + c^4*(57*d^2 + 106*d*e*x^
2 + 40*e^2*x^4)) - 48*b*c^5*(2*d - 5*e*x^2)*(d + e*x^2)^2*ArcCsch[c*x]))/(1
680*c^5*e^2) - (b*Sqrt[1 + 1/(c^2*x^2)]*x*(-32*c^7*d^(7/2)*ArcTanh[(Sqrt[d]
*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2]] + Sqrt[e]*(35*c^6*d^3 + 35*c^4*d^2*e -
 63*c^2*d*e^2 + 25*e^3)*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x
^2])]))/(560*c^6*e^2*Sqrt[1 + c^2*x^2])
```

Maple [**F**]
time = 0.14, size = 0, normalized size = 0.00

$$\int x^{3}\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccsch}(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

```
[Out] int(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/35*(5*(x^2*e + d)^(5/2)*x^2*e^(-1) - 2*(x^2*e + d)^(5/2)*d*e^(-2))*a + 1/
35*((5*x^6*e^3 + 8*d*x^4*e^2 + d^2*x^2*e - 2*d^3)*sqrt(x^2*e + d)*e^(-2)*lo
g(sqrt(c^2*x^2 + 1) + 1) + 35*integrate(1/35*(5*c^2*x^7*e^3 + 8*c^2*d*x^5*e
^2 + c^2*d^2*x^3*e - 2*c^2*d^3*x)*sqrt(x^2*e + d)/(c^2*x^2*e^2 + (c^2*x^2*e
^2 + e^2)*sqrt(c^2*x^2 + 1) + e^2), x) - 35*integrate(1/35*(5*c^2*x^7*(7*lo
g(c) + 1)*e^3 - 2*c^2*d^3*x + ((35*d*log(c) + 8*d)*c^2*e^2 + 35*e^3*log(c))
*x^5 + (c^2*d^2*e + 35*d*e^2*log(c))*x^3 + 35*(c^2*x^7*e^3 + (c^2*d*e^2 + e
^3)*x^5 + d*x^3*e^2)*log(x))*sqrt(x^2*e + d)/(c^2*x^2*e^2 + e^2), x))*b
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. $2(327) = 654$.
time = 1.64, size = 2121, normalized size = 5.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] [1/6720*(96*b*c^7*d^(7/2)*log((c^4*d^2*x^4 + 8*c^2*d^2*x^2 + x^4*cosh(1)^2
+ x^4*sinh(1)^2 + 4*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*s
qrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
8*d^2 + 2*(3*c^2*d*x^4 + 4*d*x^2)*cosh(1) + 2*(3*c^2*d*x^4 + x^4*cosh(1) +
4*d*x^2)*sinh(1))/x^4) + 3*(35*b*c^6*d^3 + 35*b*c^4*d^2*cosh(1) - 63*b*c^2*
d*cosh(1)^2 + 25*b*cosh(1)^3 + 25*b*sinh(1)^3 - 3*(21*b*c^2*d - 25*b*cosh(1
))*sinh(1)^2 + (35*b*c^4*d^2 - 126*b*c^2*d*cosh(1) + 75*b*cosh(1)^2)*sinh(1
))*sqrt(cosh(1) + sinh(1))*log(c^4*d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1
)^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*sinh(1)^2 - 4*(c^4*d*x + (2*c^4*x^3 + c^2
*x)*cosh(1) + (2*c^4*x^3 + c^2*x)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) +
 d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))*sqrt(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2
+ 3*c^2*d)*cosh(1) + 2*(4*c^4*d*x^2 + 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2 + 1)
*cosh(1))*sinh(1)) + 192*(5*b*c^7*x^6*cosh(1)^3 + 5*b*c^7*x^6*sinh(1)^3 + 8
*b*c^7*d*x^4*cosh(1)^2 + b*c^7*d^2*x^2*cosh(1) - 2*b*c^7*d^3 + (15*b*c^7*x^
6*cosh(1) + 8*b*c^7*d*x^4)*sinh(1)^2 + (15*b*c^7*x^6*cosh(1)^2 + 16*b*c^7*d
*x^4*cosh(1) + b*c^7*d^2*x^2)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*
log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(240*a*c^7*x^6*cosh(
1)^3 + 240*a*c^7*x^6*sinh(1)^3 + 384*a*c^7*d*x^4*cosh(1)^2 + 48*a*c^7*d^2*x
^2*cosh(1) - 96*a*c^7*d^3 + 48*(15*a*c^7*x^6*cosh(1) + 8*a*c^7*d*x^4)*sinh(
1)^2 + 48*(15*a*c^7*x^6*cosh(1)^2 + 16*a*c^7*d*x^4*cosh(1) + a*c^7*d^2*x^2)
*sinh(1) + (57*b*c^6*d^2*x*cosh(1) + 5*(8*b*c^6*x^5 - 10*b*c^4*x^3 + 15*b*c
^2*x)*cosh(1)^3 + 5*(8*b*c^6*x^5 - 10*b*c^4*x^3 + 15*b*c^2*x)*sinh(1)^3 + 2
*(53*b*c^6*d*x^3 - 82*b*c^4*d*x)*cosh(1)^2 + (106*b*c^6*d*x^3 - 164*b*c^4*d
*x + 15*(8*b*c^6*x^5 - 10*b*c^4*x^3 + 15*b*c^2*x)*cosh(1))*sinh(1)^2 + (57*
b*c^6*d^2*x + 15*(8*b*c^6*x^5 - 10*b*c^4*x^3 + 15*b*c^2*x)*cosh(1)^2 + 4*(5
3*b*c^6*d*x^3 - 82*b*c^4*d*x)*cosh(1))*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2
)))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(c^7*cosh(1)^2 + 2*c^7*cosh(1)*sin
h(1) + c^7*sinh(1)^2), -1/6720*(192*b*c^7*sqrt(-d)*d^3*arctan(1/2*(c^3*d*x^
3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*sqrt(x^2*cosh(1) + x^2*sinh(1)
 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d^2*x^2 + d^2 + (c^2*d*x^
4 + d*x^2)*cosh(1) + (c^2*d*x^4 + d*x^2)*sinh(1))) - 3*(35*b*c^6*d^3 + 35*b
*c^4*d^2*cosh(1) - 63*b*c^2*d*cosh(1)^2 + 25*b*cosh(1)^3 + 25*b*sinh(1)^3 -
 3*(21*b*c^2*d - 25*b*cosh(1))*sinh(1)^2 + (35*b*c^4*d^2 - 126*b*c^2*d*cosh
(1) + 75*b*cosh(1)^2)*sinh(1))*sqrt(cosh(1) + sinh(1))*log(c^4*d^2 + (8*c^4
*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*sinh(1)^2 - 4
*(c^4*d*x + (2*c^4*x^3 + c^2*x)*cosh(1) + (2*c^4*x^3 + c^2*x)*sinh(1))*sqrt
(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))*sqrt(cosh(1)
+ sinh(1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1) + 2*(4*c^4*d*x^2 + 3*c^2*d +
 (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) - 192*(5*b*c^7*x^6*cosh(1)^3
 + 5*b*c^7*x^6*sinh(1)^3 + 8*b*c^7*d*x^4*cosh(1)^2 + b*c^7*d^2*x^2*cosh(1)
- 2*b*c^7*d^3 + (15*b*c^7*x^6*cosh(1) + 8*b*c^7*d*x^4)*sinh(1)^2 + (15*b*c^
7*x^6*cosh(1)^2 + 16*b*c^7*d*x^4*cosh(1) + b*c^7*d^2*x^2)*sinh(1))*sqrt(x^2
*cosh(1) + x^2*sinh(1) + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*
x)) - 4*(240*a*c^7*x^6*cosh(1)^3 + 240*a*c^7*x^6*sinh(1)^3 + 384*a*c^7*d*x^
4*cosh(1)^2 + 48*a*c^7*d^2*x^2*cosh(1) - 96*a*c^7*d^3 + 48*(15*a*c^7*x^6*co
```

```
sh(1) + 8*a*c^7*d*x^4)*sinh(1)^2 + 48*(15*a*c^7*x^6*cosh(1)^2 + 16*a*c^7*d*
x^4*cosh(1) + a*c^7*d^2*x^2)*sinh(1) + (57*b*c^6*d^2*x*cosh(1) + 5*(8*b*c^6
*x^5 - 10*b*c^4*x^3 + 15*b*c^2*x)*cosh(1)^3 + 5*(8*b*c^6*x^5 - 10*b*c^4*x^3
 + 15*b*c^2*x)*sinh(1)^3 + 2*(53*b*c^6*d*x^3 - 82*b*c^4*d*x)*cosh(1)^2 + (1
06*b*c^6*d*x^3 - 164*b*c^4*d*x + 15*(8*b*c^6*x^5 - 10*b*c^4*x^3 + 15*b*c^2*
x)*cosh(1))*sinh(1)^2 + (57*b*c^6*d^2*x + 15*(8*b*c^6*x^5 - 10*b*c^4*x^3 +
15*b*c^2*x)*cosh(1)^2 + 4*(53*b*c^6*d*x^3 - 82*b*c^4*d*x)*cosh(1))*sinh(1))
*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(c^7*c
osh(1)^2 + 2*c^7*cosh(1)*sinh(1) + c^7*sinh(1)^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*x^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x^3*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))), x)
```

3.129 $\int x(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=270

$$\frac{b(7c^2d-3e)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{20c\sqrt{-c^2x^2}}+\frac{(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}+\frac{b(15}{}$$

```
[Out] 1/5*(e*x^2+d)^(5/2)*(a+b*arccsch(c*x))/e+1/5*b*c*d^(5/2)*x*arctan((e*x^2+d)
^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e/(-c^2*x^2)^(1/2)+1/40*b*(15*c^4*d^2-10
*c^2*d*e+3*e^2)*x*arctan(e^(1/2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/
e^(1/2)/(-c^2*x^2)^(1/2)+1/20*b*x*(e*x^2+d)^(3/2)*(-c^2*x^2-1)^(1/2)/c/(-c^
2*x^2)^(1/2)+1/40*b*(7*c^2*d-3*e)*x*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c^3/
(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6435, 457, 104, 159, 163, 65, 223, 209, 95, 210}

$$\frac{(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}+\frac{bcd^{5/2}x\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{-c^2x^2-1}}\right)}{5e\sqrt{-c^2x^2}}+\frac{bx(15c^4d^2-10c^2de+3e^2)\,\text{ArcTan}\left(\frac{\sqrt{e}\,\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{40c^4\sqrt{e}\,\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}\,(d+ex^2)^{3/2}}{20c\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}\,(7c^2d-3e)\,\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*(7*c^2*d - 3*e)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(40*c^3*Sqrt[-(c^2
*x^2)]) + (b*x*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*Sqrt[-(c^2*x^2)]
) + ((d + e*x^2)^(5/2)*(a + b*ArcCsch[c*x]))/(5*e) + (b*(15*c^4*d^2 - 10*c^
2*d*e + 3*e^2)*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/
(40*c^4*Sqrt[e]*Sqrt[-(c^2*x^2)]) + (b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(
Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(5*e*Sqrt[-(c^2*x^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
```

```
&& LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/(d*f*(m + n + p + 1))), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6435

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2])), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rubi steps

$$\int x\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}-\frac{(bcx)\int\frac{\left(d+ex^2\right)^{5/2}}{x\sqrt{-1-c^2x^2}}\,dx}{5e\sqrt{-c^2x^2}}$$

$$=\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}-\frac{(bcx)\text{Subst}\left(\int\frac{(d+ex)^{5/2}}{x\sqrt{-1-c^2x}}\,dx,x\right.}{10e\sqrt{-c^2x^2}}$$

$$=\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}{20c\sqrt{-c^2x^2}}+\frac{\left(d+ex^2\right)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5e}+\frac{(b}{}$$

$$=\frac{b(7c^2d-3e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^3}{20c\sqrt{-c^2x^2}}$$

$$=\frac{b(7c^2d-3e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^3}{20c\sqrt{-c^2x^2}}$$

$$=\frac{b(7c^2d-3e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^3}{20c\sqrt{-c^2x^2}}$$

$$=\frac{b(7c^2d-3e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^3}{20c\sqrt{-c^2x^2}}$$

$$=\frac{b(7c^2d-3e)\,x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{40c^3\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\left(d+ex^2\right)^3}{20c\sqrt{-c^2x^2}}$$

<u>Mathematica</u> [A]
time = 2.58, size = 233, normalized size = 0.86

$$\frac{\sqrt{d+ex^2}\left(8ac^3\left(d+ex^2\right)^2+be\sqrt{1+\frac{1}{c^2x^2}}\,x\left(-3e+c^2\left(9d+2ex^2\right)\right)+8bc^3\left(d+ex^2\right)^2\text{csch}^{-1}(cx)\right)}{40c^3e}+\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(-8c^5d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}\left(15c^4d^2-10c^2de+3e^2\right)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{40c^4e\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^3*(d + e*x^2)^2 + b*e*Sqrt[1 + 1/(c^2*x^2)]*x*(-3*e
 + c^2*(9*d + 2*e*x^2)) + 8*b*c^3*(d + e*x^2)^2*ArcCsch[c*x]))/(40*c^3*e) +
 (b*Sqrt[1 + 1/(c^2*x^2)]*x*(-8*c^5*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[1 + c^2*x
^2])/Sqrt[d + e*x^2]] + Sqrt[e]*(15*c^4*d^2 - 10*c^2*d*e + 3*e^2)*ArcTanh[(
```

```
Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]))/(40*c^4*e*Sqrt[1 + c^2*x^
2])
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int x\left(e\,x^{2}+d\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccsch}\left(c x\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

```
[Out] int(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*(x^2*e + d)^(5/2)*a*e^(-1) + 1/5*((x^4*e^2 + 2*d*x^2*e + d^2)*sqrt(x^2*
e + d)*e^(-1)*log(sqrt(c^2*x^2 + 1) + 1) + 5*integrate(1/5*(c^2*x^5*e^2 + 2
*c^2*d*x^3*e + c^2*d^2*x)*sqrt(x^2*e + d)/(c^2*x^2*e + (c^2*x^2*e + e)*sqrt
(c^2*x^2 + 1) + e), x) - 5*integrate(1/5*(c^2*x^5*(5*log(c) + 1)*e^2 + ((5*
d*log(c) + 2*d)*c^2*e + 5*e^2*log(c))*x^3 + (c^2*d^2 + 5*d*e*log(c))*x + 5*
(c^2*x^5*e^2 + (c^2*d*e + e^2)*x^3 + d*x*e)*log(x))*sqrt(x^2*e + d)/(c^2*x^
2*e + e), x))*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(228) = 456$.
time = 0.86, size = 1487, normalized size = 5.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] [1/160*(8*b*c^5*d^(5/2)*log((c^4*d^2*x^4 + 8*c^2*d^2*x^2 + x^4*cosh(1)^2 +
x^4*sinh(1)^2 - 4*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*sqr
t(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*
d^2 + 2*(3*c^2*d*x^4 + 4*d*x^2)*cosh(1) + 2*(3*c^2*d*x^4 + x^4*cosh(1) + 4*
d*x^2)*sinh(1))/x^4) + (15*b*c^4*d^2 - 10*b*c^2*d*cosh(1) + 3*b*cosh(1)^2 +
 3*b*sinh(1)^2 - 2*(5*b*c^2*d - 3*b*cosh(1))*sinh(1))*sqrt(cosh(1) + sinh(1
))*log(c^4*d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 + 8*c^2
```

```
*x^2 + 1)*sinh(1)^2 + 4*(c^4*d*x + (2*c^4*x^3 + c^2*x)*cosh(1) + (2*c^4*x^3
 + c^2*x)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt((c^2*x^2 + 1)/(
c^2*x^2))*sqrt(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1) + 2*(
4*c^4*d*x^2 + 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) + 32*
(b*c^5*x^4*cosh(1)^2 + b*c^5*x^4*sinh(1)^2 + 2*b*c^5*d*x^2*cosh(1) + b*c^5*
d^2 + 2*(b*c^5*x^4*cosh(1) + b*c^5*d*x^2)*sinh(1))*sqrt(x^2*cosh(1) + x^2*s
inh(1) + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(8*a*c^5
*x^4*cosh(1)^2 + 8*a*c^5*x^4*sinh(1)^2 + 16*a*c^5*d*x^2*cosh(1) + 8*a*c^5*d
^2 + 16*(a*c^5*x^4*cosh(1) + a*c^5*d*x^2)*sinh(1) + (9*b*c^4*d*x*cosh(1) +
(2*b*c^4*x^3 - 3*b*c^2*x)*cosh(1)^2 + (2*b*c^4*x^3 - 3*b*c^2*x)*sinh(1)^2 +
 (9*b*c^4*d*x + 2*(2*b*c^4*x^3 - 3*b*c^2*x)*cosh(1))*sinh(1))*sqrt((c^2*x^2
 + 1)/(c^2*x^2)))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(c^5*cosh(1) + c^5*s
inh(1)), 1/160*(16*b*c^5*sqrt(-d)*d^2*arctan(1/2*(c^3*d*x^3 + c*x^3*cosh(1)
 + c*x^3*sinh(1) + 2*c*d*x)*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(-d)*sq
rt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d^2*x^2 + d^2 + (c^2*d*x^4 + d*x^2)*cosh(1
) + (c^2*d*x^4 + d*x^2)*sinh(1))) + (15*b*c^4*d^2 - 10*b*c^2*d*cosh(1) + 3*
b*cosh(1)^2 + 3*b*sinh(1)^2 - 2*(5*b*c^2*d - 3*b*cosh(1))*sinh(1))*sqrt(cos
h(1) + sinh(1))*log(c^4*d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^
4*x^4 + 8*c^2*x^2 + 1)*sinh(1)^2 + 4*(c^4*d*x + (2*c^4*x^3 + c^2*x)*cosh(1)
 + (2*c^4*x^3 + c^2*x)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt((c
^2*x^2 + 1)/(c^2*x^2))*sqrt(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*
cosh(1) + 2*(4*c^4*d*x^2 + 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1))*s
inh(1)) + 32*(b*c^5*x^4*cosh(1)^2 + b*c^5*x^4*sinh(1)^2 + 2*b*c^5*d*x^2*cos
h(1) + b*c^5*d^2 + 2*(b*c^5*x^4*cosh(1) + b*c^5*d*x^2)*sinh(1))*sqrt(x^2*co
sh(1) + x^2*sinh(1) + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))
 + 4*(8*a*c^5*x^4*cosh(1)^2 + 8*a*c^5*x^4*sinh(1)^2 + 16*a*c^5*d*x^2*cosh(1
) + 8*a*c^5*d^2 + 16*(a*c^5*x^4*cosh(1) + a*c^5*d*x^2)*sinh(1) + (9*b*c^4*d
*x*cosh(1) + (2*b*c^4*x^3 - 3*b*c^2*x)*cosh(1)^2 + (2*b*c^4*x^3 - 3*b*c^2*x
)*sinh(1)^2 + (9*b*c^4*d*x + 2*(2*b*c^4*x^3 - 3*b*c^2*x)*cosh(1))*sinh(1))*
sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(c^5*co
sh(1) + c^5*sinh(1))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + b \operatorname{acsch}(cx)) \left(d + ex^2\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))*(d + e*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x\left(e\,x^2+d\right)^{3/2}\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))),x)
```

```
[Out] int(x*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))), x)
```

3.130 $\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x,x]
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x, x]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx=\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$$

Mathematica [A]
time = 4.04, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x, x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\left(a+b\,\text{arccsch}(cx)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/3*(3*d^(3/2)*arcsinh(sqrt(d)*e^(-1/2)/abs(x)) - (x^2*e + d)^(3/2) - 3*sq
rt(x^2*e + d)*d)*a + b*integrate((x^2*e + d)^(3/2)*log(sqrt(1/(c^2*x^2) + 1
) + 1/(c*x))/x, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccsch(c*x))*sqrt(x^2*e + d)/x,
x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsch}(cx))\left(d + ex^2\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**(3/2)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x, x)
```

3.131 $\int \frac{(d+ex^2)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{(d+ex^2)^{3/2}\,(a+b\mathrm{csch}^{-1}(cx))}{x^3},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\mathrm{csch}^{-1}(cx))}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^3,x]
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^3, x]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\mathrm{csch}^{-1}(cx))}{x^3}\,dx = \int \frac{(d+ex^2)^{3/2}\,(a+b\mathrm{csch}^{-1}(cx))}{x^3}\,dx$$

Mathematica [A]
time = 3.36, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\mathrm{csch}^{-1}(cx))}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^3,x]
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^3, x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arccsch}(cx))}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*sqrt(d)*arcsinh(sqrt(d)*e^(-1/2)/abs(x))*e - 3*sqrt(x^2*e + d)*e -
(x^2*e + d)^(3/2)*e/d + (x^2*e + d)^(5/2)/(d*x^2))*a + b*integrate((x^2*e +
 d)^(3/2)*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/x^3, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccsch(c*x))*sqrt(x^2*e + d)/x^3
, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acsch}(cx)) (d + ex^2)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**(3/2)/x**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^3,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^3, x)
```

3.132 $\int x^2(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(x^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Int[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Defer[Int][x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx=\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

Mathematica [A]
time = 6.25, size = 0, normalized size = 0.00

$$\int x^2\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Integrate[x^2*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int x^2\left(e\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\text{arccsch}(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

```
[Out] int(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] -1/48*(3*d^3*arcsinh(x*e^(1/2)/sqrt(d))*e^(-3/2) - 8*(x^2*e + d)^(5/2)*x*e^
(-1) + 2*(x^2*e + d)^(3/2)*d*x*e^(-1) + 3*sqrt(x^2*e + d)*d^2*x*e^(-1))*a +
 b*integrate((x^2*e + d)^(3/2)*x^2*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e + a*d*x^2 + (b*x^4*e + b*d*x^2)*arccsch(c*x))*sqrt(x^2*e
+ d), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int x^2 \left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))),x)
[Out] int(x^2*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))), x)
```

3.133 $\int (d+ex^2)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=23

$$\text{Int}\left(\left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx$$

Verification is not applicable to the result.

```
[In] Int[(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
[Out] Defer[Int][(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx = \int \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx$$

Mathematica [A]
time = 2.13, size = 0, normalized size = 0.00

$$\int \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
[Out] Integrate[(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b\,\text{arccsch}(cx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*(3*d^2*arcsinh(x*e^(1/2)/sqrt(d))*e^(-1/2) + 2*(x^2*e + d)^(3/2)*x + 3*
sqrt(x^2*e + d)*d*x)*a + b*integrate((x^2*e + d)^(3/2)*log(sqrt(1/(c^2*x^2)
 + 1) + 1/(c*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccsch(c*x))*sqrt(x^2*e + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{acsch}\left(cx\right)\right) \left(d + ex^{2}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))),x)
[Out] int((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))), x)
```

3.134 $\int \frac{(d+ex^2)^{3/2}\left(a+b\,\text{csch}^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2},x\right)$$

[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x)

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Verification is not applicable to the result.

[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^2,x]

[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^2, x]

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx=\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Mathematica [A]
time = 3.35, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^2}\,dx$$

Verification is not applicable to the result.

[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^2,x]

[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^2, x]

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\left(a+b\,\text{arccsch}(cx)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*(3*d*arcsinh(x*e^(1/2)/sqrt(d))*e^(1/2) + 3*sqrt(x^2*e + d)*x*e - 2*(x^
2*e + d)^(3/2)/x)*a + b*integrate((x^2*e + d)^(3/2)*log(sqrt(1/(c^2*x^2) +
1) + 1/(c*x))/x^2, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccsch(c*x))*sqrt(x^2*e + d)/x^2
, x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsch}(cx))\,(d + ex^2)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**2,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**(3/2)/x**2, x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^2,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^2, x)
```

3.135 $\int \frac{(d+ex^2)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{(d+ex^2)^{3/2}\,(a+b\,\mathrm{csch}^{-1}(cx))}{x^4},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\,\mathrm{csch}^{-1}(cx))}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] Defer[Int][((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^4, x]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\,\mathrm{csch}^{-1}(cx))}{x^4}\,dx = \int \frac{(d+ex^2)^{3/2}\,(a+b\,\mathrm{csch}^{-1}(cx))}{x^4}\,dx$$

Mathematica [A]
time = 9.26, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{3/2}\,(a+b\,\mathrm{csch}^{-1}(cx))}{x^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^4,x]
```

```
[Out] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^4, x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arccsch}(cx))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*arcsinh(x*e^(1/2)/sqrt(d))*e^(3/2) + 3*sqrt(x^2*e + d)*x*e^2/d - 2*(
x^2*e + d)^(3/2)*e/(d*x) - (x^2*e + d)^(5/2)/(d*x^3))*a + b*integrate((x^2*
e + d)^(3/2)*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/x^4, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccsch(c*x))*sqrt(x^2*e + d)/x^4
, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acsch}(cx))\,(d + ex^2)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acsch(c*x))*(d + e*x**2)**(3/2)/x**4, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^4, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^4,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^4, x)
```

3.136 $\int \frac{(d+ex^2)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=492

$$\frac{bc^3(8c^4d^2-23c^2de+23e^2)\,x^2\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}\,\sqrt{-1-c^2x^2}}+\frac{bc(8c^4d^2-23c^2de+23e^2)\,\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}}-\frac{4bc(c^2d-2e)}{75}$$

```
[Out] -1/5*(e*x^2+d)^(5/2)*(a+b*arccsch(c*x))/d/x^5+1/25*b*c*(e*x^2+d)^(3/2)*(-c^
2*x^2-1)^(1/2)/x^4/(-c^2*x^2)^(1/2)+1/75*b*c^3*(8*c^4*d^2-23*c^2*d*e+23*e^2
)*x^2*(e*x^2+d)^(1/2)/d/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)+1/75*b*c*(8*c^4
*d^2-23*c^2*d*e+23*e^2)*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/(-c^2*x^2)^(1/
2)-4/75*b*c*(c^2*d-2*e)*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/x^2/(-c^2*x^2)^(
1/2)-1/75*b*c^2*(8*c^4*d^2-23*c^2*d*e+23*e^2)*x*(1/(c^2*x^2+1))^(1/2)*(c^2*
x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(
1/2)/d/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)+
1/75*b*e*(4*c^4*d^2-11*c^2*d*e+15*e^2)*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^
(1/2)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^
2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 492, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {270, 6437, 12, 485, 594, 597, 545, 429, 506, 422}

$$-\frac{(d+ex^2)^{5/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{5dx^5}+\frac{bex(4c^4d^2-11c^2de+15e^2)\sqrt{d+ex^2}\,F\left(\mathrm{ArcTan}(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{75d^2\sqrt{-c^2x^2}\,\sqrt{-c^2x^2-1}\,\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}-\frac{bc^2x(8c^4d^2-23c^2de+23e^2)\sqrt{d+ex^2}\,E\left(\mathrm{ArcTan}(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{75d\sqrt{-c^2x^2}\,\sqrt{-c^2x^2-1}\,\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}-\frac{4bc\sqrt{-c^2x^2-1}\,(c^2d-2e)\sqrt{d+ex^2}}{75x^2\sqrt{-c^2x^2}}+\frac{bc\sqrt{-c^2x^2-1}\,(d+ex^2)^{3/2}}{25x^4\sqrt{-c^2x^2}}+\frac{bc\sqrt{-c^2x^2-1}\,(8c^4d^2-23c^2de+23e^2)\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}}+\frac{bc^3x^2(8c^4d^2-23c^2de+23e^2)\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}\,\sqrt{-c^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (b*c^3*(8*c^4*d^2 - 23*c^2*d*e + 23*e^2)*x^2*Sqrt[d + e*x^2])/(75*d*Sqrt[-(
c^2*x^2)]*Sqrt[-1 - c^2*x^2]) + (b*c*(8*c^4*d^2 - 23*c^2*d*e + 23*e^2)*Sqrt
[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(75*d*Sqrt[-(c^2*x^2)]) - (4*b*c*(c^2*d - 2
*e)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(75*x^2*Sqrt[-(c^2*x^2)]) + (b*c*Sq
rt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(25*x^4*Sqrt[-(c^2*x^2)]) - ((d + e*x^2
)^(5/2)*(a + b*ArcCsch[c*x]))/(5*d*x^5) - (b*c^2*(8*c^4*d^2 - 23*c^2*d*e +
23*e^2)*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(75*d*Sqrt
[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]) + (b*e
*(4*c^4*d^2 - 11*c^2*d*e + 15*e^2)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x],
 1 - e/(c^2*d)])/(75*d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^
2)/(d*(1 + c^2*x^2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 485

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^
(q - 1)/(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^6}\,dx = -\frac{(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5dx^5} - \frac{(bcx)\int -\frac{(d+ex^2)^{5/2}}{5dx^6\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= -\frac{(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5dx^5} + \frac{(bcx)\int \frac{(d+ex^2)^{5/2}}{x^6\sqrt{-1-c^2x^2}}\,dx}{5d\sqrt{-c^2x^2}}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{25x^4\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{5dx^5} - \frac{(bc}{}$$

$$= -\frac{4bc(c^2d-2e)\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{75x^2\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\,(d+ex^2)}{25x^4\sqrt{-c^2x^2}}$$

$$= \frac{bc(8c^4d^2-23c^2de+23e^2)\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}} - \frac{4bc(c^2d-2e)}{75}$$

$$= \frac{bc(8c^4d^2-23c^2de+23e^2)\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}} - \frac{4bc(c^2d-2e)}{75}$$

$$= \frac{bc^3(8c^4d^2-23c^2de+23e^2)\,x^2\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}\,\sqrt{-1-c^2x^2}} + \frac{bc(8c^4d^2-23c^2de+23e}{75d\sqrt{}}$$

$$= \frac{bc^3(8c^4d^2-23c^2de+23e^2)\,x^2\sqrt{d+ex^2}}{75d\sqrt{-c^2x^2}\,\sqrt{-1-c^2x^2}} + \frac{bc(8c^4d^2-23c^2de+23e}{75d\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.19, size = 291, normalized size = 0.59

$$\frac{\sqrt{d+ex^2}\left(-15a(d+ex^2)^2+bc\sqrt{1+\frac{1}{c^2x^2}}\,x(23e^2x^4+dex^2(11-23c^2x^2)+d^2(3-4c^2x^2+8c^4x^4))-15b(d+ex^2)^2\,\text{csch}^{-1}(cx)\right)}{75dx^5} + \frac{ibc\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(8c^4d^2-23c^2de+23e^2)\,E\left(i\sinh^{-1}\left(\sqrt{c^2}\,x\right)|\frac{e}{c^2d}\right)+(-8c^6d^3+27c^4d^2e-34c^2de^2+15e^3)\,F\left(i\sinh^{-1}\left(\sqrt{c^2}\,x\right)|\frac{e}{c^2d}\right)\right)}{75\sqrt{c^2}\,d\sqrt{1+c^2x^2}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^6,x]
```

```
[Out] (Sqrt[d + e*x^2]*(-15*a*(d + e*x^2)^2 + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(23*e^2
*x^4 + d*e*x^2*(11 - 23*c^2*x^2) + d^2*(3 - 4*c^2*x^2 + 8*c^4*x^4)) - 15*b*
(d + e*x^2)^2*ArcCsch[c*x]))/(75*d*x^5) + ((I/75)*b*c*Sqrt[1 + 1/(c^2*x^2)]
*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(8*c^4*d^2 - 23*c^2*d*e + 23*e^2)*EllipticE[I
*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] + (-8*c^6*d^3 + 27*c^4*d^2*e - 34*c^2*d*e
```

```
^2 + 15*e^3)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d*Sq
rt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [**F**]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^{\frac{3}{2}}\,(a+b\,\mathrm{arccsch}(cx))}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x, algorithm="maxima")
```

```
[Out] -1/5*b*((x^4*e^2 + 2*d*x^2*e + d^2)*sqrt(x^2*e + d)*log(sqrt(c^2*x^2 + 1) +
 1)/(d*x^5) + 5*integrate(-1/5*(c^2*x^6*e^2 - (5*d*log(c) - 2*d)*c^2*x^4*e
- ((5*d^2*log(c) - d^2)*c^2 + 5*d*e*log(c))*x^2 - 5*d^2*log(c) - 5*(c^2*d*x
^4*e + (c^2*d^2 + d*e)*x^2 + d^2)*log(x))*sqrt(x^2*e + d)/(c^2*d*x^8 + d*x^
6), x) + 5*integrate(1/5*(c^2*x^4*e^2 + 2*c^2*d*x^2*e + c^2*d^2)*sqrt(x^2*e
 + d)/(c^2*d*x^6 + d*x^4 + (c^2*d*x^6 + d*x^4)*sqrt(c^2*x^2 + 1)), x)) - 1/
5*(x^2*e + d)^(5/2)*a/(d*x^5)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**6,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^6,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^6, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^6,x)
```

```
[Out] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^6, x)
```

3.137 $\int \frac{(d+ex^2)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{x^8}\,dx$

Optimal. Leaf size=643

$$-\frac{bc^3(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\,x^2\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}\,\sqrt{-1-c^2x^2}}-\frac{bc(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\,\sqrt{-}}{3675d^2\sqrt{-c^2x^2}}$$

[Out] -1/7*(e*x^2+d)^(5/2)*(a+b*arccsch(c*x))/d/x^7+2/35*e*(e*x^2+d)^(5/2)*(a+b*arccsch(c*x))/d^2/x^5-1/1225*b*c*(30*c^2*d-11*e)*(e*x^2+d)^(3/2)*(-c^2*x^2-1)^(1/2)/d/x^4/(-c^2*x^2)^(1/2)+1/49*b*c*(e*x^2+d)^(5/2)*(-c^2*x^2-1)^(1/2)/d/x^6/(-c^2*x^2)^(1/2)-1/3675*b*c^3*(240*c^6*d^3-528*c^4*d^2*e+193*c^2*d*e^2+247*e^3)*x^2*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)-1/3675*b*c*(240*c^6*d^3-528*c^4*d^2*e+193*c^2*d*e^2+247*e^3)*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)+1/3675*b*c*(120*c^4*d^2-159*c^2*d*e-37*e^2)*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/x^2/(-c^2*x^2)^(1/2)+1/3675*b*c^2*(240*c^6*d^3-528*c^4*d^2*e+193*c^2*d*e^2+247*e^3)*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)-1/3675*b*e*(120*c^6*d^3-249*c^4*d^2*e+71*c^2*d*e^2+210*e^3)*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^3/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)

Rubi [A]
time = 0.55, antiderivative size = 643, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {277, 270, 6437, 12, 594, 597, 545, 429, 506, 422}

$$\frac{2e(d+ex^2)^{5/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{35d^2x^5}-\frac{(d+ex^2)^{5/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{7dx^7}-\frac{bex(120c^6d^3-249c^4d^2e+71c^2de^2+210e^3)\sqrt{d+ex^2}\,F\left(\mathrm{ArcTan}(cx)\middle|1-\frac{e}{c^2d}\right)}{3675d^3\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}+\frac{bc^2x(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\sqrt{d+ex^2}\,E\left(\mathrm{ArcTan}(cx)\middle|1-\frac{e}{c^2d}\right)}{3675d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}+\frac{bc\sqrt{-c^2x^2-1}(d+ex^2)^{5/2}}{49dx^6\sqrt{-c^2x^2}}-\frac{bc\sqrt{-c^2x^2-1}(30c^2d-11e)(d+ex^2)^{3/2}}{1225dx^4\sqrt{-c^2x^2}}+\frac{bc\sqrt{-c^2x^2-1}(120c^4d^2-159c^2de-37e^2)\sqrt{d+ex^2}}{3675dx^2\sqrt{-c^2x^2}}-\frac{bc\sqrt{-c^2x^2-1}(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}}-\frac{bc^3x^2(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}$$

Antiderivative was successfully verified.

[In] Int[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^8,x]

[Out] -1/3675*(b*c^3*(240*c^6*d^3 - 528*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*x^2*Sqrt[d + e*x^2])/(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]) - (b*c*(240*c^6*d^3 - 528*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(3675*d^2*Sqrt[-(c^2*x^2)]) + (b*c*(120*c^4*d^2 - 159*c^2*d*e - 37*e^2)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(3675*d*x^2*Sqrt[-(c^2*x^2)]) - (b*c*(30*c^2*d - 11*e)*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(1225*d*x^4*Sqrt[-(c^2*x^2)]) + (b*c*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(5/2))/(49*d*x^6*Sqrt[-(c^2*x^2)]) - ((d + e*x^2)^(5/2)*(a + b*ArcCsch[c*x]))/(7*d*x^7) + (2*e*(d + e*x^2)^(5/2)*(a + b*ArcCsch[c*x]))/(35*d^2*x^5) + (b*c^2*(240*c^6*d^3 - 5

```
28*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[
c*x], 1 - e/(c^2*d)])/(3675*d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d
 + e*x^2)/(d*(1 + c^2*x^2))]) - (b*e*(120*c^6*d^3 - 249*c^4*d^2*e + 71*c^2*
d*e^2 + 210*e^3)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(
3675*d^3*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x
^2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{x^8}\,dx = -\frac{(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7dx^7} + \frac{2e(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx\right.}{35d^2x^5}$$

$$= -\frac{(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7dx^7} + \frac{2e(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx\right.}{35d^2x^5}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\,(d+ex^2)^{5/2}}{49dx^6\sqrt{-c^2x^2}} - \frac{(d+ex^2)^{5/2}\left(a+b\text{csch}^{-1}(cx)\right)}{7dx^7} + \frac{2e}{}$$

$$= -\frac{bc(30c^2d-11e)\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{1225dx^4\sqrt{-c^2x^2}} + \frac{bc\sqrt{-1-c^2x^2}\,(d+}{49dx^6\sqrt{-c^2x^2}}$$

$$= \frac{bc(120c^4d^2-159c^2de-37e^2)\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{3675dx^2\sqrt{-c^2x^2}} - \frac{bc(30c^2d-}{}$$

$$= -\frac{bc(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\sqrt{-1-c^2x^2}\,\sqrt{d+ex}}{3675d^2\sqrt{-c^2x^2}}$$

$$= -\frac{bc(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\sqrt{-1-c^2x^2}\,\sqrt{d+ex}}{3675d^2\sqrt{-c^2x^2}}$$

$$= -\frac{bc^3(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\,x^2\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}\,\sqrt{-1-c^2x^2}} - \frac{bc(2}{}$$

$$= -\frac{bc^3(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\,x^2\sqrt{d+ex^2}}{3675d^2\sqrt{-c^2x^2}\,\sqrt{-1-c^2x^2}} - \frac{bc(2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.53, size = 372, normalized size = 0.58

$$-\frac{\sqrt{d+ex^2}\left(105a(5d-2ex^2)(d+ex^2)^2+bc\sqrt{1+\frac{1}{c^2x^2}}\,x(247e^3x^6+de^2x^4(-71+193c^2x^2)-3d^2ex^2(61-83c^2x^2+176c^4x^4)+15d^3(-5+6c^2x^2-8c^4x^4+16c^6x^6))+105b(5d-2ex^2)(d+ex^2)^2\,\text{csch}^{-1}(cx)\right)}{3675d^2x^7} - \frac{ibc\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(240c^6d^3-528c^4d^2e+193c^2de^2+247e^3)\,E\left(i\sinh^{-1}\left(\sqrt{c^2}\,x\right)|\frac{e}{c^2d}\right)-2(120c^8d^4-324c^6d^3e+221c^4d^2e^2+88c^2de^3-105e^4)\,F\left(i\sinh^{-1}\left(\sqrt{c^2}\,x\right)|\frac{e}{c^2d}\right)\right)}{3675\sqrt{c^2}\,d^2\sqrt{1+c^2x^2}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/x^8,x]
```

```
[Out] -1/3675*(Sqrt[d + e*x^2]*(105*a*(5*d - 2*e*x^2)*(d + e*x^2)^2 + b*c*Sqrt[1
+ 1/(c^2*x^2)]*x*(247*e^3*x^6 + d*e^2*x^4*(-71 + 193*c^2*x^2) - 3*d^2*e*x^2
```

```
*(61 - 83*c^2*x^2 + 176*c^4*x^4) + 15*d^3*(-5 + 6*c^2*x^2 - 8*c^4*x^4 + 16*
c^6*x^6)) + 105*b*(5*d - 2*e*x^2)*(d + e*x^2)^2*ArcCsch[c*x]))/(d^2*x^7) -
((I/3675)*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(240*c^6*d
^3 - 528*c^4*d^2*e + 193*c^2*d*e^2 + 247*e^3)*EllipticE[I*ArcSinh[Sqrt[c^2]
*x], e/(c^2*d)] - 2*(120*c^8*d^4 - 324*c^6*d^3*e + 221*c^4*d^2*e^2 + 88*c^2
*d*e^3 - 105*e^4)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]
*d^2*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccsch}\left(c x\right)\right)}{x^{8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x)
```

```
[Out] int((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x, algorithm="maxima")
```

```
[Out] 1/35*a*(2*(x^2*e + d)^(5/2)*e/(d^2*x^5) - 5*(x^2*e + d)^(5/2)/(d*x^7)) + 1/
35*b*((2*x^6*e^3 - d*x^4*e^2 - 8*d^2*x^2*e - 5*d^3)*sqrt(x^2*e + d)*log(sqr
t(c^2*x^2 + 1) + 1)/(d^2*x^7) - 35*integrate(1/35*(2*c^2*x^8*e^3 - c^2*d*x^
6*e^2 + (35*d^2*log(c) - 8*d^2)*c^2*x^4*e + 35*d^3*log(c) + 5*(7*d^2*e*log(
c) + (7*d^3*log(c) - d^3)*c^2)*x^2 + 35*(c^2*d^2*x^4*e + d^3 + (c^2*d^3 + d
^2*e)*x^2)*log(x))*sqrt(x^2*e + d)/(c^2*d^2*x^10 + d^2*x^8), x) + 35*integr
ate(1/35*(2*c^2*x^6*e^3 - c^2*d*x^4*e^2 - 8*c^2*d^2*x^2*e - 5*c^2*d^3)*sqrt
(x^2*e + d)/(c^2*d^2*x^8 + d^2*x^6 + (c^2*d^2*x^8 + d^2*x^6)*sqrt(c^2*x^2 +
 1)), x))
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)*(a+b*acsch(c*x))/x**8,x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/x^8,x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)/x^8, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^{3/2}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^8,x)
[Out] int(((d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))))/x^8, x)
```

3.138 $\int \frac{x^5\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=329

$$-\frac{b(19c^2d+9e)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}}+\frac{bx\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{20ce^2\sqrt{-c^2x^2}}+\frac{d^2\sqrt{d+ex^2}\,\left(a+b\mathrm{csch}^{-1}(cx)\right)}{e^3}-\frac{2d}{}$$

```
[Out] -2/3*d*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/e^3+1/5*(e*x^2+d)^(5/2)*(a+b*arcc
sch(c*x))/e^3+1/120*b*(45*c^4*d^2+10*c^2*d*e+9*e^2)*x*arctan(e^(1/2)*(-c^2*
x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^4/e^(5/2)/(-c^2*x^2)^(1/2)+8/15*b*c*d^(5/
2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^3/(-c^2*x^2)^(1/2
)+1/20*b*x*(e*x^2+d)^(3/2)*(-c^2*x^2-1)^(1/2)/c/e^2/(-c^2*x^2)^(1/2)+d^2*(a
+b*arccsch(c*x))*(e*x^2+d)^(1/2)/e^3-1/120*b*(19*c^2*d+9*e)*x*(-c^2*x^2-1)^
(1/2)*(e*x^2+d)^(1/2)/c^3/e^2/(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.77, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {272, 45, 6437, 12, 1629, 159, 163, 65, 223, 209, 95, 210}

$$\frac{d^2\sqrt{d+ex^2}\,\left(a+b\mathrm{csch}^{-1}(cx)\right)}{e^3}-\frac{2d(d+ex^2)^{3/2}\left(a+b\mathrm{csch}^{-1}(cx)\right)}{3e^3}+\frac{(d+ex^2)^{5/2}\left(a+b\mathrm{csch}^{-1}(cx)\right)}{5e^3}+\frac{8bcd^{5/2}x\,\mathrm{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{15e^3\sqrt{-c^2x^2}}+\frac{bx(45c^4d^2+10c^2de+9e^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{120c^4e^{5/2}\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}\,(d+ex^2)^{3/2}}{20ce^2\sqrt{-c^2x^2}}-\frac{bx\sqrt{-c^2x^2-1}\,(19c^2d+9e)\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] -1/120*(b*(19*c^2*d + 9*e)*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(c^3*e^2*S
qrt[-(c^2*x^2)]) + (b*x*Sqrt[-1 - c^2*x^2]*(d + e*x^2)^(3/2))/(20*c*e^2*Sqr
t[-(c^2*x^2)]) + (d^2*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/e^3 - (2*d*(d +
 e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) + ((d + e*x^2)^(5/2)*(a + b*Arc
Csch[c*x]))/(5*e^3) + (b*(45*c^4*d^2 + 10*c^2*d*e + 9*e^2)*x*ArcTan[(Sqrt[e
]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(120*c^4*e^(5/2)*Sqrt[-(c^2*x^2
)]) + (8*b*c*d^(5/2)*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])]
)/(15*e^3*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
```

```
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{2d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \cdots$$

$$= \frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{2d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \cdots$$

$$= \frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \frac{2d(d+ex^2)^{3/2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3} + \cdots$$

$$= \frac{bx\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{20ce^2\sqrt{-c^2x^2}} + \frac{d^2\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} - \cdots$$

$$= -\frac{b(19c^2d+9e)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{20ce^2\sqrt{-c^2x^2}} + \cdots$$

$$= -\frac{b(19c^2d+9e)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{20ce^2\sqrt{-c^2x^2}} + \cdots$$

$$= -\frac{b(19c^2d+9e)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{20ce^2\sqrt{-c^2x^2}} + \cdots$$

$$= -\frac{b(19c^2d+9e)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{20ce^2\sqrt{-c^2x^2}} + \cdots$$

$$= -\frac{b(19c^2d+9e)\,x\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{120c^3e^2\sqrt{-c^2x^2}} + \frac{bx\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}{20ce^2\sqrt{-c^2x^2}} + \cdots$$

<u>Mathematica</u> [A]
time = 2.14, size = 257, normalized size = 0.78

$$\frac{\sqrt{d+ex^2}\left(8ac^3(8d^2-4dex^2+3e^2x^4)+be\sqrt{1+\frac{1}{c^2x^2}}\,x(-9e+c^2(-13d+6ex^2))+8bc^3(8d^2-4dex^2+3e^2x^4)\operatorname{csch}^{-1}(cx)\right)}{120c^3e^3} + \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(-64c^5d^{5/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}\,(45c^4d^2+10c^2de+9e^2)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{120c^4e^3\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(8*a*c^3*(8*d^2 - 4*d*e*x^2 + 3*e^2*x^4) + b*e*Sqrt[1 + 1/
(c^2*x^2)]*x*(-9*e + c^2*(-13*d + 6*e*x^2)) + 8*b*c^3*(8*d^2 - 4*d*e*x^2 +
3*e^2*x^4)*ArcCsch[c*x]))/(120*c^3*e^3) + (b*Sqrt[1 + 1/(c^2*x^2)]*x*(-64*c
^5*d^(5/2)*ArcTanh[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2]] + Sqrt[e]*(
45*c^4*d^2 + 10*c^2*d*e + 9*e^2)*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqr
t[d + e*x^2])]))/(120*c^4*e^3*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^5(a + b\,\mathrm{arccsch}(cx))}{\sqrt{e\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*sqrt(x^2*e + d)*x^4*e^(-1) - 4*sqrt(x^2*e + d)*d*x^2*e^(-2) + 8*sqr
t(x^2*e + d)*d^2*e^(-3))*a + 1/15*((3*x^6*e^3 - d*x^4*e^2 + 4*d^2*x^2*e + 8
*d^3)*e^(-3)*log(sqrt(c^2*x^2 + 1) + 1)/sqrt(x^2*e + d) + 15*integrate(1/15
*(3*c^2*x^7*e^3 - c^2*d*x^5*e^2 + 4*c^2*d^2*x^3*e + 8*c^2*d^3*x)/((c^2*x^2*
e^3 + e^3)*sqrt(c^2*x^2 + 1)*sqrt(x^2*e + d) + (c^2*x^2*e^3 + e^3)*sqrt(x^2
*e + d)), x) - 15*integrate(1/15*(3*c^2*x^7*(5*log(c) + 1)*e^3 + 4*c^2*d^2*
x^3*e + 8*c^2*d^3*x - (c^2*d*e^2 - 15*e^3*log(c))*x^5 + 15*(c^2*x^7*e^3 + x
^5*e^3)*log(x))/((c^2*x^2*e^3 + e^3)*sqrt(x^2*e + d)), x))*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 761 vs. $2(279) = 558$.
time = 0.90, size = 1559, normalized size = 4.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/480*(64*b*c^5*d^(5/2)*log((c^4*d^2*x^4 + 8*c^2*d^2*x^2 + x^4*cosh(1)^2 +
 x^4*sinh(1)^2 - 4*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*sq
```

```
rt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8
*d^2 + 2*(3*c^2*d*x^4 + 4*d*x^2)*cosh(1) + 2*(3*c^2*d*x^4 + x^4*cosh(1) + 4
*d*x^2)*sinh(1))/x^4) + (45*b*c^4*d^2 + 10*b*c^2*d*cosh(1) + 9*b*cosh(1)^2
+ 9*b*sinh(1)^2 + 2*(5*b*c^2*d + 9*b*cosh(1))*sinh(1))*sqrt(cosh(1) + sinh(
1))*log(c^4*d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 + 8*c^
2*x^2 + 1)*sinh(1)^2 + 4*(c^4*d*x + (2*c^4*x^3 + c^2*x)*cosh(1) + (2*c^4*x^
3 + c^2*x)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt((c^2*x^2 + 1)/
(c^2*x^2))*sqrt(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1) + 2*
(4*c^4*d*x^2 + 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) + 32
*(3*b*c^5*x^4*cosh(1)^2 + 3*b*c^5*x^4*sinh(1)^2 - 4*b*c^5*d*x^2*cosh(1) + 8
*b*c^5*d^2 + 2*(3*b*c^5*x^4*cosh(1) - 2*b*c^5*d*x^2)*sinh(1))*sqrt(x^2*cosh
(1) + x^2*sinh(1) + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) +
 4*(24*a*c^5*x^4*cosh(1)^2 + 24*a*c^5*x^4*sinh(1)^2 - 32*a*c^5*d*x^2*cosh(1
) + 64*a*c^5*d^2 + 16*(3*a*c^5*x^4*cosh(1) - 2*a*c^5*d*x^2)*sinh(1) - (13*b
*c^4*d*x*cosh(1) - 3*(2*b*c^4*x^3 - 3*b*c^2*x)*cosh(1)^2 - 3*(2*b*c^4*x^3 -
 3*b*c^2*x)*sinh(1)^2 + (13*b*c^4*d*x - 6*(2*b*c^4*x^3 - 3*b*c^2*x)*cosh(1)
)*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(x^2*cosh(1) + x^2*sinh(1) +
d))/(c^5*cosh(1)^3 + 3*c^5*cosh(1)^2*sinh(1) + 3*c^5*cosh(1)*sinh(1)^2 + c^
5*sinh(1)^3), 1/480*(128*b*c^5*sqrt(-d)*d^2*arctan(1/2*(c^3*d*x^3 + c*x^3*c
osh(1) + c*x^3*sinh(1) + 2*c*d*x)*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(
-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d^2*x^2 + d^2 + (c^2*d*x^4 + d*x^2)*
cosh(1) + (c^2*d*x^4 + d*x^2)*sinh(1))) + (45*b*c^4*d^2 + 10*b*c^2*d*cosh(1
) + 9*b*cosh(1)^2 + 9*b*sinh(1)^2 + 2*(5*b*c^2*d + 9*b*cosh(1))*sinh(1))*sq
rt(cosh(1) + sinh(1))*log(c^4*d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 +
 (8*c^4*x^4 + 8*c^2*x^2 + 1)*sinh(1)^2 + 4*(c^4*d*x + (2*c^4*x^3 + c^2*x)*c
osh(1) + (2*c^4*x^3 + c^2*x)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*s
qrt((c^2*x^2 + 1)/(c^2*x^2))*sqrt(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2 + 3*c
^2*d)*cosh(1) + 2*(4*c^4*d*x^2 + 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh
(1))*sinh(1)) + 32*(3*b*c^5*x^4*cosh(1)^2 + 3*b*c^5*x^4*sinh(1)^2 - 4*b*c^5
*d*x^2*cosh(1) + 8*b*c^5*d^2 + 2*(3*b*c^5*x^4*cosh(1) - 2*b*c^5*d*x^2)*sinh
(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x
^2)) + 1)/(c*x)) + 4*(24*a*c^5*x^4*cosh(1)^2 + 24*a*c^5*x^4*sinh(1)^2 - 32*
a*c^5*d*x^2*cosh(1) + 64*a*c^5*d^2 + 16*(3*a*c^5*x^4*cosh(1) - 2*a*c^5*d*x^
2)*sinh(1) - (13*b*c^4*d*x*cosh(1) - 3*(2*b*c^4*x^3 - 3*b*c^2*x)*cosh(1)^2
- 3*(2*b*c^4*x^3 - 3*b*c^2*x)*sinh(1)^2 + (13*b*c^4*d*x - 6*(2*b*c^4*x^3 -
3*b*c^2*x)*cosh(1))*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(x^2*cosh(1
) + x^2*sinh(1) + d))/(c^5*cosh(1)^3 + 3*c^5*cosh(1)^2*sinh(1) + 3*c^5*cosh
(1)*sinh(1)^2 + c^5*sinh(1)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(a + b\,\mathrm{acsch}\,(cx))}{\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^5/sqrt(e*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.139 $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=229

$$\frac{bx\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}}-\frac{d\sqrt{d+ex^2}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{b(3c^2d+e)\,x\text{A}}{6}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/e^2-1/6*b*(3*c^2*d+e)*x*arctan(e^(1/
2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(3/2)/(-c^2*x^2)^(1/2)-2/3*b
*c*d^(3/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^2/(-c^2*x
^2)^(1/2)-d*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/e^2+1/6*b*x*(-c^2*x^2-1)^(1/
2)*(e*x^2+d)^(1/2)/c/e/(-c^2*x^2)^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {272, 45, 6437, 12, 587, 159, 163, 65, 223, 209, 95, 210}

$$-\frac{d\sqrt{d+ex^2}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2}-\frac{2bcd^{3/2}x\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{-c^2x^2-1}}\right)}{3e^2\sqrt{-c^2x^2}}-\frac{bx(3c^2d+e)\,\text{ArcTan}\left(\frac{\sqrt{e}\,\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{3/2}\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}\,\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (b*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(6*c*e*Sqrt[-(c^2*x^2)]) - (d*Sqrt
[d + e*x^2]*(a + b*ArcCsch[c*x]))/e^2 + ((d + e*x^2)^(3/2)*(a + b*ArcCsch[c
*x]))/(3*e^2) - (b*(3*c^2*d + e)*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*S
qrt[d + e*x^2])])/(6*c^2*e^(3/2)*Sqrt[-(c^2*x^2)]) - (2*b*c*d^(3/2)*x*ArcTa
n[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(3*e^2*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 159

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[h*(a + b*x)^m*(c + d*x)^(n +
 1)*((e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = -\frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2} - \frac{(bcx)\int \frac{(-}{3}}{}$$

$$= -\frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2} - \frac{(bcx)\int \frac{(-}{}}{3}$$

$$= -\frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2} - \frac{(bcx)\text{Subs}}{}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}(a-}{3e}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}(a-}{3e}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}(a-}{3e}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}(a-}{3e}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce\sqrt{-c^2x^2}} - \frac{d\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2} + \frac{\left(d+ex^2\right)^{3/2}(a-}{3e}$$

<u>Mathematica</u> [A]
time = 1.17, size = 198, normalized size = 0.86

$$\frac{\sqrt{d+ex^2}\left(-4acd+be\sqrt{1+\frac{1}{c^2x^2}}\,x+2acex^2+2bc(-2d+ex^2)\operatorname{csch}^{-1}(cx)\right)}{6ce^2}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(-4c^3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}\,(3c^2d+e)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{6c^2e^2\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(-4*a*c*d + b*e*Sqrt[1 + 1/(c^2*x^2)]*x + 2*a*c*e*x^2 + 2*
b*c*(-2*d + e*x^2)*ArcCsch[c*x]))/(6*c*e^2) - (b*Sqrt[1 + 1/(c^2*x^2)]*x*(-
4*c^3*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2]] + Sqrt[e
```

```
]*(3*c^2*d + e)*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]))/
(6*c^2*e^2*Sqrt[1 + c^2*x^2])
```

Maple [**F**]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\,\mathrm{arccsch}(cx))}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(x^2*e + d)*x^2*e^(-1) - 2*sqrt(x^2*e + d)*d*e^(-2))*a + 1/3*((x^4
*e^2 - d*x^2*e - 2*d^2)*e^(-2)*log(sqrt(c^2*x^2 + 1) + 1)/sqrt(x^2*e + d) +
 3*integrate(1/3*(c^2*x^5*e^2 - c^2*d*x^3*e - 2*c^2*d^2*x)/((c^2*x^2*e^2 +
e^2)*sqrt(c^2*x^2 + 1)*sqrt(x^2*e + d) + (c^2*x^2*e^2 + e^2)*sqrt(x^2*e + d
)), x) - 3*integrate(1/3*(c^2*x^5*(3*log(c) + 1)*e^2 - 2*c^2*d^2*x - (c^2*d
*e - 3*e^2*log(c))*x^3 + 3*(c^2*x^5*e^2 + x^3*e^2)*log(x))/((c^2*x^2*e^2 +
e^2)*sqrt(x^2*e + d)), x))*b
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. 2(191) = 382.
time = 0.59, size = 1136, normalized size = 4.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*b*c^3*d^(3/2)*log((c^4*d^2*x^4 + 8*c^2*d^2*x^2 + x^4*cosh(1)^2 + x
^4*sinh(1)^2 + 4*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*sqrt
(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d
^2 + 2*(3*c^2*d*x^4 + 4*d*x^2)*cosh(1) + 2*(3*c^2*d*x^4 + x^4*cosh(1) + 4*d
*x^2)*sinh(1))/x^4) + (3*b*c^2*d + b*cosh(1) + b*sinh(1))*sqrt(cosh(1) + si
nh(1))*log(c^4*d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 + 8
*c^2*x^2 + 1)*sinh(1)^2 - 4*(c^4*d*x + (2*c^4*x^3 + c^2*x)*cosh(1) + (2*c^4
```

```
*x^3 + c^2*x)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt((c^2*x^2 +
1)/(c^2*x^2))*sqrt(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1) +
 2*(4*c^4*d*x^2 + 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) +
 8*(b*c^3*x^2*cosh(1) + b*c^3*x^2*sinh(1) - 2*b*c^3*d)*sqrt(x^2*cosh(1) + x
^2*sinh(1) + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(2*a
*c^3*x^2*cosh(1) + 2*a*c^3*x^2*sinh(1) - 4*a*c^3*d + (b*c^2*x*cosh(1) + b*c
^2*x*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(x^2*cosh(1) + x^2*sinh(1)
 + d))/(c^3*cosh(1)^2 + 2*c^3*cosh(1)*sinh(1) + c^3*sinh(1)^2), -1/24*(8*b*
c^3*sqrt(-d)*d*arctan(1/2*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*
d*x)*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x
^2))/(c^2*d^2*x^2 + d^2 + (c^2*d*x^4 + d*x^2)*cosh(1) + (c^2*d*x^4 + d*x^2)
*sinh(1))) - (3*b*c^2*d + b*cosh(1) + b*sinh(1))*sqrt(cosh(1) + sinh(1))*lo
g(c^4*d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 + 8*c^2*x^2
+ 1)*sinh(1)^2 - 4*(c^4*d*x + (2*c^4*x^3 + c^2*x)*cosh(1) + (2*c^4*x^3 + c^
2*x)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt((c^2*x^2 + 1)/(c^2*x
^2))*sqrt(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1) + 2*(4*c^4
*d*x^2 + 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) - 8*(b*c^3
*x^2*cosh(1) + b*c^3*x^2*sinh(1) - 2*b*c^3*d)*sqrt(x^2*cosh(1) + x^2*sinh(1
) + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 4*(2*a*c^3*x^2*
cosh(1) + 2*a*c^3*x^2*sinh(1) - 4*a*c^3*d + (b*c^2*x*cosh(1) + b*c^2*x*sinh
(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(c
^3*cosh(1)^2 + 2*c^3*cosh(1)*sinh(1) + c^3*sinh(1)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\,\mathrm{acsch}\,(cx))}{\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/sqrt(e*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.140 $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=135

$$\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e}+\frac{bx\text{ArcTan}\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{-c^2x^2}}+\frac{bc\sqrt{d}\,x\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{e\sqrt{-c^2x^2}}$$

```
[Out] b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))*d^(1/2)/e/(-c^2*x^
2)^(1/2)+b*x*arctan(e^(1/2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(1/2)/(
-c^2*x^2)^(1/2)+(a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/e
```

Rubi [A]
time = 0.10, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6435, 457, 132, 65, 223, 209, 12, 95, 210}

$$\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e}+\frac{bx\text{ArcTan}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{-c^2x^2}}+\frac{bc\sqrt{d}\,x\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{e\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
```

```
[Out] (Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/e + (b*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c
^2*x^2])/(c*Sqrt[d + e*x^2])])/(Sqrt[e]*Sqrt[-(c^2*x^2)]) + (b*c*Sqrt[d]*x*
ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(e*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_), x_Symbol] :> Dist[b*d^(m + n)*f^p, Int[(a + b*x)^(m - 1)/(c + d*x
)^m, x], x] + Int[(a + b*x)^(m - 1)*((e + f*x)^p/(c + d*x)^m)*ExpandToSum[(
a + b*x)*(c + d*x)^(-p - 1) - (b*d^(-p - 1)*f^p)/(e + f*x)^p, x], x] /; Fre
eQ[{a, b, c, d, e, f, m, n}, x] && EqQ[m + n + p + 1, 0] && ILtQ[p, 0] && (
GtQ[m, 0] || SumSimplerQ[m, -1] ||  !(GtQ[n, 0] || SumSimplerQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6435

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2])), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rubi steps

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx = \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e} - \frac{(bcx)\int \frac{\sqrt{d+ex^2}}{x\sqrt{-1-c^2x^2}}\,dx}{e\sqrt{-c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e} - \frac{(bcx)\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x\sqrt{-1-c^2x}}\,dx,x,x^2\right)}{2e\sqrt{-c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e} - \frac{(bcx)\text{Subst}\left(\int \frac{1}{\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{2\sqrt{-c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e} + \frac{(bx)\text{Subst}\left(\int \frac{1}{\sqrt{d-\frac{e}{c^2}-\frac{ex^2}{c^2}}}\,dx,x,\sqrt{-1-c^2}\right.}{c\sqrt{-c^2x^2}}$$

$$= \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e} + \frac{bc\sqrt{d}\,x\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{e\sqrt{-c^2x^2}} + \frac{(bx)\text{Subs}}{}$$

$$= \frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e} + \frac{bx\tan^{-1}\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{\sqrt{e}\sqrt{-c^2x^2}} + \frac{bc\sqrt{d}\,x\tan^{-1}}{e}$$

Mathematica [A]
time = 0.54, size = 137, normalized size = 1.01

$$\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e} - \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(c\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)-\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{e\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
[Out] (Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/e - (b*Sqrt[1 + 1/(c^2*x^2)]*x*(c*Sq
rt[d]*ArcTanh[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2]] - Sqrt[e]*ArcTan
h[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]))/(e*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\,\text{arccsch}(cx))}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^2*e + d)*a*e^(-1) + (sqrt(x^2*e + d)*e^(-1)*log(sqrt(c^2*x^2 + 1) +
1) + integrate((c^2*x^3*e + c^2*d*x)/((c^2*x^2*e + e)*sqrt(c^2*x^2 + 1)*sqr
t(x^2*e + d) + (c^2*x^2*e + e)*sqrt(x^2*e + d)), x) - integrate((c^2*x^3*(l
og(c) + 1)*e + (c^2*d + e*log(c))*x + (c^2*x^3*e + x*e)*log(x))/((c^2*x^2*e
 + e)*sqrt(x^2*e + d)), x))*b
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 423 vs. $2(112) = 224$.
time = 0.52, size = 885, normalized size = 6.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*b*c*log((c*x*sqrt((c^2*x^2 + 1)
/(c^2*x^2)) + 1)/(c*x)) + b*c*sqrt(d)*log((c^4*d^2*x^4 + 8*c^2*d^2*x^2 + x^
4*cosh(1)^2 + x^4*sinh(1)^2 - 4*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1)
+ 2*c*d*x)*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(
c^2*x^2)) + 8*d^2 + 2*(3*c^2*d*x^4 + 4*d*x^2)*cosh(1) + 2*(3*c^2*d*x^4 + x^
4*cosh(1) + 4*d*x^2)*sinh(1))/x^4) + 4*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*
a*c + b*sqrt(cosh(1) + sinh(1))*log(c^4*d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*c
osh(1)^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*sinh(1)^2 + 4*(c^4*d*x + (2*c^4*x^3
+ c^2*x)*cosh(1) + (2*c^4*x^3 + c^2*x)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh
(1) + d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))*sqrt(cosh(1) + sinh(1)) + 2*(4*c^4*d
*x^2 + 3*c^2*d)*cosh(1) + 2*(4*c^4*d*x^2 + 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2
 + 1)*cosh(1))*sinh(1)))/(c*cosh(1) + c*sinh(1)), 1/4*(2*b*c*sqrt(-d)*arcta
n(1/2*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*sqrt(x^2*cosh(1
) + x^2*sinh(1) + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d^2*x^2 +
d^2 + (c^2*d*x^4 + d*x^2)*cosh(1) + (c^2*d*x^4 + d*x^2)*sinh(1))) + 4*sqrt(
x^2*cosh(1) + x^2*sinh(1) + d)*b*c*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c*x)) + 4*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*a*c + b*sqrt(cosh(1) + s
inh(1))*log(c^4*d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 +
8*c^2*x^2 + 1)*sinh(1)^2 + 4*(c^4*d*x + (2*c^4*x^3 + c^2*x)*cosh(1) + (2*c^
```

```
4*x^3 + c^2*x)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt((c^2*x^2 +
 1)/(c^2*x^2))*sqrt(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1)
+ 2*(4*c^4*d*x^2 + 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1))*sinh(1)))
/(c*cosh(1) + c*sinh(1))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a + b\,\text{acsch}\,(cx))}{\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(x*(a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x/sqrt(e*x^2 + d), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a + b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.141 $\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}},x\right)$$

`[Out] Unintegrable((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x)`

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

`[In] Int[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x^2]),x]`

`[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x^2]), x]`

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Mathematica [A]
time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

`[In] Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x^2]),x]`

`[Out] Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[d + e*x^2]), x]`

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsch}(cx)}{x\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x)
[Out] int((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(sqrt(x^2*e + d)*x), x) -
a*arcsinh(sqrt(d)*e^(-1/2)/abs(x))/sqrt(d)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(x^2*e + d)*(b*arccsch(c*x) + a)/(x^3*e + d*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x\sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*acsch(c*x))/(x*sqrt(d + e*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(e*x^2 + d)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^(1/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^(1/2)), x)
```

3.142 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^3*Sqrt[d + e*x^2]),x]
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Mathematica [A]
time = 15.79, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^3*Sqrt[d + e*x^2]),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^3*Sqrt[d + e*x^2]), x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsch}(cx)}{x^3\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(arcsinh(sqrt(d)*e^(-1/2)/abs(x))*e/d^(3/2) - sqrt(x^2*e + d)/(d*x^2)
) + b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(sqrt(x^2*e + d)*x^3),
 x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsch(c*x) + a)/(x^5*e + d*x^3), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x^3 \sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**3/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x**3*sqrt(d + e*x**2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(e*x^2 + d)*x^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x^3\,\sqrt{e\,x^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x^3*(d + e*x^2)^(1/2)), x)
```

3.143 $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
[Out] Defer[Int][(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A]
time = 3.80, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
[Out] Integrate[(x^2*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2], x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^2(a+b\,\text{arccsch}(cx))}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(d*arcsinh(x*e^(1/2)/sqrt(d))*e^(-3/2) - sqrt(x^2*e + d)*x*e^(-1))*a +
 b*integrate(x^2*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/sqrt(x^2*e + d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)/sqrt(x^2*e + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\,\mathrm{acsch}\,(cx))}{\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/sqrt(e*x^2 + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2 \left(a + b \operatorname{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2 + d}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.144 $\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/Sqrt[d + e*x^2],x]
[Out] Defer[Int][(a + b*ArcCsch[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A]
time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/Sqrt[d + e*x^2],x]
[Out] Integrate[(a + b*ArcCsch[c*x])/Sqrt[d + e*x^2], x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsch}(cx)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] a*arcsinh(x*e^(1/2)/sqrt(d))*e^(-1/2) + b*integrate(log(sqrt(1/(c^2*x^2) +
1) + 1/(c*x))/sqrt(x^2*e + d), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)/sqrt(x^2*e + d), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{\sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/sqrt(e*x^2 + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(1/2),x)
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(1/2), x)
```

3.145 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=294

$$\frac{bc^3x^2\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{dx}-\frac{bc^2x\sqrt{d+ex^2}\,E(\text{Arc}}{d\sqrt{-c^2x^2}\sqrt{-1-c^2x}}$$

```
[Out] -(a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/d/x+b*c^3*x^2*(e*x^2+d)^(1/2)/d/(-c^2*x
^2)^(1/2)/(-c^2*x^2-1)^(1/2)+b*c*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/(-c^2
*x^2)^(1/2)-b*c^2*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(
c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d/(-c^2*x^2)^(1/2)/(-c^
2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)+b*e*x*(1/(c^2*x^2+1))^(1/2)*
(c^2*x^2+1)^(1/2)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2
+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))
^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {270, 6437, 12, 486, 21, 433, 429, 506, 422}

$$-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{dx}+\frac{bex\sqrt{d+ex^2}\,F\left(\text{ArcTan}(cx)\left|1-\frac{e}{c^2d}\right.\right)}{d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}-\frac{bc^2x\sqrt{d+ex^2}\,E\left(\text{ArcTan}(cx)\left|1-\frac{e}{c^2d}\right.\right)}{d\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}+\frac{bc\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}}+\frac{bc^3x^2\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] (b*c^3*x^2*Sqrt[d + e*x^2])/(d*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]) + (b*c*
Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(d*Sqrt[-(c^2*x^2)]) - (Sqrt[d + e*x^2]
*(a + b*ArcCsch[c*x]))/(d*x) - (b*c^2*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*
x], 1 - e/(c^2*d)])/(d*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)
/(d*(1 + c^2*x^2))]) + (b*e*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/
(c^2*d)])/(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 +
 c^2*x^2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 486

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/
(a*e*(m + 1))), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] && !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] && !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] && !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{dx}+\frac{(bcx)\int\frac{\sqrt{d+ex^2}}{dx^2\sqrt{-1-c^2x^2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= -\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{dx}+\frac{(bcx)\int\frac{\sqrt{d+ex^2}}{x^2\sqrt{-1-c^2x^2}}\,dx}{d\sqrt{-c^2x^2}}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{dx}-\frac{(bcx)\int\frac{-e-c^2}{\sqrt{-1-c^2x^2}}}{d\sqrt{-c^2x}}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{dx}-\frac{(bcex)\int\frac{\sqrt{-1-c^2x}}{\sqrt{d+ex^2}}}{d\sqrt{-c^2x^2}}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{dx}+\frac{(bcex)\int\frac{}{\sqrt{-1-c^2x}}}{d\sqrt{-c^2x}}$$

$$= \frac{bc^3x^2\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(c\right.}{dx}$$

$$= \frac{bc^3x^2\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(c\right.}{dx}$$

Mathematica [A]
time = 0.89, size = 139, normalized size = 0.47

$$\frac{\sqrt{d+ex^2}\left(-a+bc\sqrt{1+\frac{1}{c^2x^2}}\,x-b\text{csch}^{-1}(cx)\right)}{dx}-\frac{bce\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\,E\left(\text{ArcSin}\left(\sqrt{-\frac{e}{d}}\,x\right)|\frac{c^2d}{e}\right)}{d\sqrt{-\frac{e}{d}}\sqrt{1+c^2x^2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*Sqrt[d + e*x^2]),x]
```

```
[Out] (Sqrt[d + e*x^2]*(-a + b*c*Sqrt[1 + 1/(c^2*x^2)]*x - b*ArcCsch[c*x]))/(d*x)
 - (b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqrt
[-(e/d)]*x], (c^2*d)/e])/(d*Sqrt[-(e/d)]*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsch}(cx)}{x^2\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -b*(sqrt(x^2*e + d)*log(sqrt(c^2*x^2 + 1) + 1)/(d*x) + integrate((c^2*x^2*e
 + c^2*d)/((c^2*d*x^2 + d)*sqrt(c^2*x^2 + 1)*sqrt(x^2*e + d) + (c^2*d*x^2 +
 d)*sqrt(x^2*e + d)), x) + integrate(-(c^2*x^4*e - (d*log(c) - d)*c^2*x^2 -
 d*log(c) - (c^2*d*x^2 + d)*log(x))/((c^2*d*x^4 + d*x^2)*sqrt(x^2*e + d)),
x)) - sqrt(x^2*e + d)*a/(d*x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{acsch}\,(cx)}{x^2\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x**2+d)**(1/2),x)
[Out] Integral((a + b*acsch(c*x))/(x**2*sqrt(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(e*x^2 + d)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x^2\,\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)^(1/2)), x)
```

3.146 $\int \frac{a+b\mathbf{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=425

$$-\frac{bc^3(2c^2d+5e)\,x^2\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}\,\sqrt{-1-c^2x^2}}-\frac{bc(2c^2d+5e)\,\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{9dx^2\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex}}{}$$

```
[Out] -1/3*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2)/d/x^3+2/3*e*(a+b*arccsch(c*x))*(e*x
^2+d)^(1/2)/d^2/x-1/9*b*c^3*(2*c^2*d+5*e)*x^2*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2
)^(1/2)/(-c^2*x^2-1)^(1/2)-1/9*b*c*(2*c^2*d+5*e)*(-c^2*x^2-1)^(1/2)*(e*x^2+
d)^(1/2)/d^2/(-c^2*x^2)^(1/2)+1/9*b*c*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d/
x^2/(-c^2*x^2)^(1/2)+1/9*b*c^2*(2*c^2*d+5*e)*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x
^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1
/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
-1/9*b*e*(c^2*d+6*e)*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticF(c*
x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^3/(-c^2*x^2)^(1/2)
/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {277, 270, 6437, 12, 594, 597, 545, 429, 506, 422}

$$\frac{2e\sqrt{d+ex^2}\,(a+b\text{csch}^{-1}(cx))}{3d^2x}-\frac{\sqrt{d+ex^2}\,(a+b\text{csch}^{-1}(cx))}{3dx^3}-\frac{bex(c^2d+6e)\,\sqrt{d+ex^2}\,F\left(\text{ArcTan}(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{9d^3\sqrt{-c^2x^2}\,\sqrt{-c^2x^2-1}\,\sqrt{\frac{d+ex^2}{d\,(c^2x^2+1)}}}+\frac{bc^2x(2c^2d+5e)\,\sqrt{d+ex^2}\,E\left(\text{ArcTan}(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{9d^2\sqrt{-c^2x^2}\,\sqrt{-c^2x^2-1}\,\sqrt{\frac{d+ex^2}{d\,(c^2x^2+1)}}}-\frac{bc\sqrt{-c^2x^2-1}\,(2c^2d+5e)\,\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}}+\frac{bc\sqrt{-c^2x^2-1}\,\sqrt{d+ex^2}}{9dx^2\sqrt{-c^2x^2}}-\frac{bc^3x^2(2c^2d+5e)\,\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}\,\sqrt{-c^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] -1/9*(b*c^3*(2*c^2*d + 5*e)*x^2*Sqrt[d + e*x^2])/(d^2*Sqrt[-(c^2*x^2)]*Sqrt
[-1 - c^2*x^2]) - (b*c*(2*c^2*d + 5*e)*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/
(9*d^2*Sqrt[-(c^2*x^2)]) + (b*c*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(9*d*x^
2*Sqrt[-(c^2*x^2)]) - (Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/(3*d*x^3) + (2
*e*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]))/(3*d^2*x) + (b*c^2*(2*c^2*d + 5*e)
*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(9*d^2*Sqrt[-(c^2
*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]) - (b*e*(c^2*
d + 6*e)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(9*d^3*Sq
rt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 594

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^q/(a*g*(m + 1))), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
```

```
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^4\sqrt{d+ex^2}}\,dx = -\frac{\sqrt{d+ex^2}\,\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\,\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2x}-\frac{(bcx)\int\frac{\sqrt{d+\cdots}}{3d^2x^4\cdots}}{\sqrt{-\cdots}}$$

$$= -\frac{\sqrt{d+ex^2}\,\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\,\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2x}-\frac{(bcx)\int\frac{\sqrt{d+\cdots}}{x^4\sqrt{\cdots}}}{3d^2\sqrt{\cdots}}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{9dx^2\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\,\left(a+b\text{csch}^{-1}(cx)\right)}{3dx^3}+\frac{2e\sqrt{d+ex^2}\,\left(a+b\text{cs}\cdots\right.}{3d^2x}$$

$$= -\frac{bc(2c^2d+5e)\,\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{9dx^2\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\cdots}{\cdots}$$

$$= -\frac{bc(2c^2d+5e)\,\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{9dx^2\sqrt{-c^2x^2}}-\frac{\sqrt{d+ex^2}\cdots}{\cdots}$$

$$= -\frac{bc^3(2c^2d+5e)\,x^2\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}\,\sqrt{-1-c^2x^2}}-\frac{bc(2c^2d+5e)\,\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}}+\frac{bc\sqrt{-1\cdots}}{9d\cdots}$$

$$= -\frac{bc^3(2c^2d+5e)\,x^2\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}\,\sqrt{-1-c^2x^2}}-\frac{bc(2c^2d+5e)\,\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{9d^2\sqrt{-c^2x^2}}+\frac{bc\sqrt{-1\cdots}}{9d\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.51, size = 239, normalized size = 0.56

$$-\frac{\sqrt{d+ex^2}\left(3a(d-2ex^2)+bc\sqrt{1+\frac{1}{c^2x^2}}\,x(-d+2c^2dx^2+5ex^2)+3b(d-2ex^2)\,\text{csch}^{-1}(cx)\right)}{9d^2x^3}-\frac{ibc\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\left(c^2d(2c^2d+5e)\,E\left(i\sinh^{-1}\left(\sqrt{c^2}\,x\right)|\frac{e}{c^2d}\right)-2(c^4d^2+2c^2de-3e^2)\,F\left(i\sinh^{-1}\left(\sqrt{c^2}\,x\right)|\frac{e}{c^2d}\right)\right)}{9\sqrt{c^2}\,d^2\sqrt{1+c^2x^2}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^4*Sqrt[d + e*x^2]),x]
```

```
[Out] -1/9*(Sqrt[d + e*x^2]*(3*a*(d - 2*e*x^2) + b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(-d
+ 2*c^2*d*x^2 + 5*e*x^2) + 3*b*(d - 2*e*x^2)*ArcCsch[c*x]))/(d^2*x^3) - ((I
/9)*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c^2*d*(2*c^2*d + 5*e)*
EllipticE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] - 2*(c^4*d^2 + 2*c^2*d*e - 3*e
^2)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d^2*Sqrt[1 +
c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsch}(cx)}{x^4\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*sqrt(x^2*e + d)*e/(d^2*x) - sqrt(x^2*e + d)/(d*x^3)) + 1/3*b*((2*x
^4*e^2 + d*x^2*e - d^2)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(x^2*e + d)*d^2*x^3
) + 3*integrate(1/3*(2*c^2*x^4*e^2 + c^2*d*x^2*e - c^2*d^2)/((c^2*d^2*x^4 +
 d^2*x^2)*sqrt(c^2*x^2 + 1)*sqrt(x^2*e + d) + (c^2*d^2*x^4 + d^2*x^2)*sqrt(
x^2*e + d)), x) - 3*integrate(1/3*(2*c^2*x^6*e^2 + c^2*d*x^4*e + (3*d^2*log
(c) - d^2)*c^2*x^2 + 3*d^2*log(c) + 3*(c^2*d^2*x^2 + d^2)*log(x))/((c^2*d^2
*x^6 + d^2*x^4)*sqrt(x^2*e + d)), x))
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acsch}(cx)}{x^4\sqrt{d + ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**4/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x**4*sqrt(d + e*x**2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^4/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(e*x^2 + d)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x^4\,\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x^4*(d + e*x^2)^(1/2)), x)
```

3.147 $\int \frac{x^5\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=256

$$\frac{bx\sqrt{-1-c^2x^2}\,\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}}-\frac{d^2\left(a+b\mathrm{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\,\left(a+b\mathrm{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\,\left(a+b\mathrm{csch}^{-1}\right.}{3e^3}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x))/e^3-1/6*b*(9*c^2*d+e)*x*arctan(e^(1/
2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/c^2/e^(5/2)/(-c^2*x^2)^(1/2)-8/3*b
*c*d^(3/2)*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^3/(-c^2*x
^2)^(1/2)-d^2*(a+b*arccsch(c*x))/e^3/(e*x^2+d)^(1/2)-2*d*(a+b*arccsch(c*x))
*(e*x^2+d)^(1/2)/e^3+1/6*b*x*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/c/e^2/(-c^2
*x^2)^(1/2)
```

Rubi [A]
time = 0.77, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {272, 45, 6437, 12, 1629, 163, 65, 223, 209, 95, 210}

$$-\frac{d^2\left(a+b\mathrm{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}-\frac{2d\sqrt{d+ex^2}\,\left(a+b\mathrm{csch}^{-1}(cx)\right)}{e^3}+\frac{\left(d+ex^2\right)^{3/2}\,\left(a+b\mathrm{csch}^{-1}(cx)\right)}{3e^3}-\frac{8bcd^{3/2}x\mathrm{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{-c^2x^2-1}}\right)}{3e^3\sqrt{-c^2x^2}}-\frac{bx(9c^2d+e)\,\mathrm{ArcTan}\left(\frac{\sqrt{e}\,\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{6c^2e^{5/2}\sqrt{-c^2x^2}}+\frac{bx\sqrt{-c^2x^2-1}\,\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (b*x*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(6*c*e^2*Sqrt[-(c^2*x^2)]) - (d^2*
(a + b*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x^2]) - (2*d*Sqrt[d + e*x^2]*(a + b*A
rcCsch[c*x]))/e^3 + ((d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]))/(3*e^3) - (b*(
9*c^2*d + e)*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(6
*c^2*e^(5/2)*Sqrt[-(c^2*x^2)]) - (8*b*c*d^(3/2)*x*ArcTan[Sqrt[d + e*x^2]/(S
qrt[d]*Sqrt[-1 - c^2*x^2])])/(3*e^3*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1629

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> With[{q = Expon[Px, x], k = Coeff[Px, x, Expo
n[Px, x]]}, Simp[k*(a + b*x)^(m + q - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p +
1)/(d*f*b^(q - 1)*(m + n + p + q + 1))), x] + Dist[1/(d*f*b^q*(m + n + p +
q + 1)), Int[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*ExpandToSum[d*f*b^q*(m + n
 + p + q + 1)*Px - d*f*k*(m + n + p + q + 1)*(a + b*x)^q + k*(a + b*x)^(q -
 2)*(a^2*d*f*(m + n + p + q + 1) - b*(b*c*e*(m + q - 1) + a*(d*e*(n + 1) +
c*f*(p + 1))) + b*(a*d*f*(2*(m + q) + n + p) - b*(d*e*(m + q + n) + c*f*(m
+ q + p)))*x), x], x], x] /; NeQ[m + n + p + q + 1, 0]] /; FreeQ[{a, b, c,
d, e, f, m, n, p}, x] && PolyQ[Px, x]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{3/2}\left(a+\right.}{3e^3}$$

$$= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{3/2}\left(a+\right.}{3e^3}$$

$$= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3} + \frac{\left(d+ex^2\right)^{3/2}\left(a+\right.}{3e^3}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{csch}\right.}{e^3}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{csch}\right.}{e^3}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{csch}\right.}{e^3}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{csch}\right.}{e^3}$$

$$= \frac{bx\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{6ce^2\sqrt{-c^2x^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} - \frac{2d\sqrt{d+ex^2}\left(a+b\operatorname{csch}\right.}{e^3}$$

Mathematica [A]
time = 1.29, size = 229, normalized size = 0.89

$$\frac{be\sqrt{1+\frac{1}{c^2x^2}}\,x(d+ex^2)-2ac(8d^2+4dex^2-e^2x^4)-2bc(8d^2+4dex^2-e^2x^4)\operatorname{csch}^{-1}(cx)}{6ce^3\sqrt{d+ex^2}} - \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(-16c^3d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)+\sqrt{e}\,(9c^2d+e)\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{6c^2e^3\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (b*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2) - 2*a*c*(8*d^2 + 4*d*e*x^2 - e^2*x
^4) - 2*b*c*(8*d^2 + 4*d*e*x^2 - e^2*x^4)*ArcCsch[c*x])/(6*c*e^3*Sqrt[d + e
*x^2]) - (b*Sqrt[1 + 1/(c^2*x^2)]*x*(-16*c^3*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[
1 + c^2*x^2])/Sqrt[d + e*x^2]] + Sqrt[e]*(9*c^2*d + e)*ArcTanh[(Sqrt[e]*Sqr
t[1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]))/(6*c^2*e^3*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x^5(a + b\,\mathrm{arccsch}(cx))}{(e\,x^2 + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^4*e^(-1)/sqrt(x^2*e + d) - 4*d*x^2*e^(-2)/sqrt(x^2*e + d) - 8*d^2*e^
(-3)/sqrt(x^2*e + d))*a + 1/3*((x^4*e^2 - 4*d*x^2*e - 8*d^2)*e^(-3)*log(sqr
t(c^2*x^2 + 1) + 1)/sqrt(x^2*e + d) + 3*integrate(1/3*(c^2*x^5*e^2 - 4*c^2*
d*x^3*e - 8*c^2*d^2*x)/((c^2*x^2*e^3 + e^3)*sqrt(c^2*x^2 + 1)*sqrt(x^2*e +
d) + (c^2*x^2*e^3 + e^3)*sqrt(x^2*e + d)), x) - 3*integrate(1/3*(c^2*x^7*(3
*log(c) + 1)*e^3 - 12*c^2*d^2*x^3*e - 8*c^2*d^3*x - 3*(c^2*d*e^2 - e^3*log(
c))*x^5 + 3*(c^2*x^7*e^3 + x^5*e^3)*log(x))/((c^2*x^4*e^4 + (c^2*d*e^3 + e^
4)*x^2 + d*e^3)*sqrt(x^2*e + d)), x))*b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 833 vs. 2(216) = 432.
time = 0.54, size = 1705, normalized size = 6.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*((9*b*c^2*d^2 + b*x^2*cosh(1)^2 + b*x^2*sinh(1)^2 + (9*b*c^2*d*x^2 +
b*d)*cosh(1) + (9*b*c^2*d*x^2 + 2*b*x^2*cosh(1) + b*d)*sinh(1))*sqrt(cosh(1
) + sinh(1))*log(c^4*d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x
^4 + 8*c^2*x^2 + 1)*sinh(1)^2 - 4*(c^4*d*x + (2*c^4*x^3 + c^2*x)*cosh(1) +
(2*c^4*x^3 + c^2*x)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt((c^2*
x^2 + 1)/(c^2*x^2))*sqrt(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*cos
h(1) + 2*(4*c^4*d*x^2 + 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1))*sinh
(1)) + 8*(b*c^3*x^4*cosh(1)^2 + b*c^3*x^4*sinh(1)^2 - 4*b*c^3*d*x^2*cosh(1)
```

```
 - 8*b*c^3*d^2 + 2*(b*c^3*x^4*cosh(1) - 2*b*c^3*d*x^2)*sinh(1))*sqrt(x^2*co
sh(1) + x^2*sinh(1) + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x))
 + 16*(b*c^3*d*x^2*cosh(1) + b*c^3*d*x^2*sinh(1) + b*c^3*d^2)*sqrt(d)*log((
c^4*d^2*x^4 + 8*c^2*d^2*x^2 + x^4*cosh(1)^2 + x^4*sinh(1)^2 + 4*(c^3*d*x^3
+ c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*sqrt(x^2*cosh(1) + x^2*sinh(1) +
 d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2 + 2*(3*c^2*d*x^4 + 4*d*x^
2)*cosh(1) + 2*(3*c^2*d*x^4 + x^4*cosh(1) + 4*d*x^2)*sinh(1))/x^4) + 4*(2*a
*c^3*x^4*cosh(1)^2 + 2*a*c^3*x^4*sinh(1)^2 - 8*a*c^3*d*x^2*cosh(1) - 16*a*c
^3*d^2 + 4*(a*c^3*x^4*cosh(1) - 2*a*c^3*d*x^2)*sinh(1) + (b*c^2*x^3*cosh(1)
^2 + b*c^2*x^3*sinh(1)^2 + b*c^2*d*x*cosh(1) + (2*b*c^2*x^3*cosh(1) + b*c^2
*d*x)*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(x^2*cosh(1) + x^2*sinh(1
) + d))/(c^3*x^2*cosh(1)^4 + c^3*x^2*sinh(1)^4 + c^3*d*cosh(1)^3 + (4*c^3*x
^2*cosh(1) + c^3*d)*sinh(1)^3 + 3*(2*c^3*x^2*cosh(1)^2 + c^3*d*cosh(1))*sin
h(1)^2 + (4*c^3*x^2*cosh(1)^3 + 3*c^3*d*cosh(1)^2)*sinh(1)), -1/24*(32*(b*c
^3*d*x^2*cosh(1) + b*c^3*d*x^2*sinh(1) + b*c^3*d^2)*sqrt(-d)*arctan(1/2*(c^
3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*sqrt(x^2*cosh(1) + x^2*s
inh(1) + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d^2*x^2 + d^2 + (c^
2*d*x^4 + d*x^2)*cosh(1) + (c^2*d*x^4 + d*x^2)*sinh(1))) - (9*b*c^2*d^2 + b
*x^2*cosh(1)^2 + b*x^2*sinh(1)^2 + (9*b*c^2*d*x^2 + b*d)*cosh(1) + (9*b*c^2
*d*x^2 + 2*b*x^2*cosh(1) + b*d)*sinh(1))*sqrt(cosh(1) + sinh(1))*log(c^4*d^
2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*sin
h(1)^2 - 4*(c^4*d*x + (2*c^4*x^3 + c^2*x)*cosh(1) + (2*c^4*x^3 + c^2*x)*sin
h(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))*sqr
t(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1) + 2*(4*c^4*d*x^2 +
 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) - 8*(b*c^3*x^4*cos
h(1)^2 + b*c^3*x^4*sinh(1)^2 - 4*b*c^3*d*x^2*cosh(1) - 8*b*c^3*d^2 + 2*(b*c
^3*x^4*cosh(1) - 2*b*c^3*d*x^2)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d
)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 4*(2*a*c^3*x^4*cosh(
1)^2 + 2*a*c^3*x^4*sinh(1)^2 - 8*a*c^3*d*x^2*cosh(1) - 16*a*c^3*d^2 + 4*(a*
c^3*x^4*cosh(1) - 2*a*c^3*d*x^2)*sinh(1) + (b*c^2*x^3*cosh(1)^2 + b*c^2*x^3
*sinh(1)^2 + b*c^2*d*x*cosh(1) + (2*b*c^2*x^3*cosh(1) + b*c^2*d*x)*sinh(1))
*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(c^3*x
^2*cosh(1)^4 + c^3*x^2*sinh(1)^4 + c^3*d*cosh(1)^3 + (4*c^3*x^2*cosh(1) + c
^3*d)*sinh(1)^3 + 3*(2*c^3*x^2*cosh(1)^2 + c^3*d*cosh(1))*sinh(1)^2 + (4*c^
3*x^2*cosh(1)^3 + 3*c^3*d*cosh(1)^2)*sinh(1))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^5/(e*x^2 + d)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.148 $\int \frac{x^3\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=160

$$\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{bx\text{ArcTan}\left(\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{-c^2x^2}}+\frac{2bc\sqrt{d}\,x\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{e^2\sqrt{-c^2x^2}}$$

```
[Out] b*x*arctan(e^(1/2)*(-c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(3/2)/(-c^2*x^2)
^(1/2)+2*b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))*d^(1/2)/e
^2/(-c^2*x^2)^(1/2)+d*(a+b*arccsch(c*x))/e^2/(e*x^2+d)^(1/2)+(a+b*arccsch(c
*x))*(e*x^2+d)^(1/2)/e^2
```

Rubi [A]
time = 0.18, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {272, 45, 6437, 12, 587, 163, 65, 223, 209, 95, 210}

$$\frac{\sqrt{d+ex^2}\left(a+b\text{csch}^{-1}(cx)\right)}{e^2}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{bx\text{ArcTan}\left(\frac{\sqrt{e}\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{e^{3/2}\sqrt{-c^2x^2}}+\frac{2bc\sqrt{d}\,x\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{e^2\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] (d*(a + b*ArcCsch[c*x]))/(e^2*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*Ar
cCsch[c*x]))/e^2 + (b*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x
^2])])/(e^(3/2)*Sqrt[-(c^2*x^2)]) + (2*b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*x^2]
/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(e^2*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
```

```
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
```

```
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^3\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}-\frac{(bcx)\int\frac{2d+ex\ldots}{e^2x\sqrt{-1-c^2x^2}\ldots}}{\sqrt{-c^2x^2}\ldots}$$

$$= \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}-\frac{(bcx)\int\frac{2d+ex^2\ldots}{x\sqrt{-1-c^2x^2}\ldots}}{e^2\sqrt{-c^2x^2}\ldots}$$

$$= \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}-\frac{(bcx)\text{Subst}\left(\int\frac{\ldots}{x\sqrt{-1-\ldots}}\ldots\right.}{2e^2\sqrt{\ldots}}$$

$$= \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}-\frac{(bcdx)\text{Subst}\left(\int\frac{\ldots}{x\sqrt{-1\ldots}}\ldots\right.}{e^2\sqrt{\ldots}}$$

$$= \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}-\frac{(2bcdx)\text{Subst}\left(\int\frac{1}{-d-x^2\ldots}\ldots\right.}{e^2\sqrt{-\ldots}}$$

$$= \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}+\frac{2bc\sqrt{d}\,x\tan^{-1}\left(\frac{\sqrt{\ldots}}{\sqrt{d}\,\sqrt{\ldots}}\right.}{e^2\sqrt{-c^2x^2}\ldots}$$

$$= \frac{d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2}+\frac{bx\tan^{-1}\left(\frac{\sqrt{e}\,\sqrt{-1-\ldots}}{c\sqrt{d+e\ldots}}\right.}{e^{3/2}\sqrt{-c^2x^2}\ldots}$$

Mathematica [A]

time = 0.95, size = 147, normalized size = 0.92

$$\frac{(2d+ex^2)\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^2\sqrt{d+ex^2}}-\frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(2c\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)-\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{e^2\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] ((2*d + e*x^2)*(a + b*ArcCsch[c*x]))/(e^2*Sqrt[d + e*x^2]) - (b*Sqrt[1 + 1/
(c^2*x^2)]*x*(2*c*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^
2]] - Sqrt[e]*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]))/(e
^2*Sqrt[1 + c^2*x^2])
```

Maple [**F**]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\,\mathrm{arccsch}(cx))}{(e\,x^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] (x^2*e^(-1)/sqrt(x^2*e + d) + 2*d*e^(-2)/sqrt(x^2*e + d))*a + ((x^2*e + 2*d
)*e^(-2)*log(sqrt(c^2*x^2 + 1) + 1)/sqrt(x^2*e + d) + integrate((c^2*x^3*e
+ 2*c^2*d*x)/((c^2*x^2*e^2 + e^2)*sqrt(c^2*x^2 + 1)*sqrt(x^2*e + d) + (c^2*
x^2*e^2 + e^2)*sqrt(x^2*e + d)), x) - integrate((c^2*x^5*(log(c) + 1)*e^2 +
 2*c^2*d^2*x + (3*c^2*d*e + e^2*log(c))*x^3 + (c^2*x^5*e^2 + x^3*e^2)*log(x
))/((c^2*x^4*e^3 + (c^2*d*e^2 + e^3)*x^2 + d*e^2)*sqrt(x^2*e + d)), x))*b
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 553 vs. 2(135) = 270.
time = 0.45, size = 1146, normalized size = 7.16

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*x^2*cosh(1) + b*x^2*sinh(1) + b*d)*sqrt(cosh(1) + sinh(1))*log(c^4
*d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*
sinh(1)^2 + 4*(c^4*d*x + (2*c^4*x^3 + c^2*x)*cosh(1) + (2*c^4*x^3 + c^2*x)*
sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))*
sqrt(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1) + 2*(4*c^4*d*x^
2 + 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) + 4*(b*c*x^2*co
sh(1) + b*c*x^2*sinh(1) + 2*b*c*d)*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*log(
(c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 2*(b*c*x^2*cosh(1) + b*c*x
^2*sinh(1) + b*c*d)*sqrt(d)*log((c^4*d^2*x^4 + 8*c^2*d^2*x^2 + x^4*cosh(1)^
2 + x^4*sinh(1)^2 - 4*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)
*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))
+ 8*d^2 + 2*(3*c^2*d*x^4 + 4*d*x^2)*cosh(1) + 2*(3*c^2*d*x^4 + x^4*cosh(1)
+ 4*d*x^2)*sinh(1))/x^4) + 4*(a*c*x^2*cosh(1) + a*c*x^2*sinh(1) + 2*a*c*d)*
sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(c*x^2*cosh(1)^3 + c*x^2*sinh(1)^3 + c
*d*cosh(1)^2 + (3*c*x^2*cosh(1) + c*d)*sinh(1)^2 + (3*c*x^2*cosh(1)^2 + 2*c
*d*cosh(1))*sinh(1)), 1/4*(4*(b*c*x^2*cosh(1) + b*c*x^2*sinh(1) + b*c*d)*sq
rt(-d)*arctan(1/2*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*sqr
t(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^
2*d^2*x^2 + d^2 + (c^2*d*x^4 + d*x^2)*cosh(1) + (c^2*d*x^4 + d*x^2)*sinh(1)
)) + (b*x^2*cosh(1) + b*x^2*sinh(1) + b*d)*sqrt(cosh(1) + sinh(1))*log(c^4*
d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*s
inh(1)^2 + 4*(c^4*d*x + (2*c^4*x^3 + c^2*x)*cosh(1) + (2*c^4*x^3 + c^2*x)*s
inh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))*s
qrt(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1) + 2*(4*c^4*d*x^2
 + 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) + 4*(b*c*x^2*cos
h(1) + b*c*x^2*sinh(1) + 2*b*c*d)*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*log((
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(a*c*x^2*cosh(1) + a*c*x^
2*sinh(1) + 2*a*c*d)*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(c*x^2*cosh(1)^3
+ c*x^2*sinh(1)^3 + c*d*cosh(1)^2 + (3*c*x^2*cosh(1) + c*d)*sinh(1)^2 + (3*
c*x^2*cosh(1)^2 + 2*c*d*cosh(1))*sinh(1))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\,\mathrm{acsch}\,(cx))}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*acsch(c*x))/(d + e*x**2)**(3/2), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.149 $\int \frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=82

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{bcx\operatorname{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{\sqrt{d}\,e\sqrt{-c^2x^2}}$$

```
[Out] -b*c*x*arctan((e*x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e/d^(1/2)/(-c^2*x
^2)^(1/2)+(-a-b*arccsch(c*x))/e/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {6435, 457, 95, 210}

$$-\frac{a+b\operatorname{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{bcx\operatorname{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{\sqrt{d}\,e\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
```

```
[Out] -((a + b*ArcCsch[c*x])/(e*Sqrt[d + e*x^2])) - (b*c*x*ArcTan[Sqrt[d + e*x^2]
/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(Sqrt[d]*e*Sqrt[-(c^2*x^2)])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6435

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2])), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx &= -\frac{a+b\text{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{(bcx)\int \frac{1}{x\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{e\sqrt{-c^2x^2}}\\
&= -\frac{a+b\text{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{(bcx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\sqrt{d+ex}}\,dx,x,x^2\right)}{2e\sqrt{-c^2x^2}}\\
&= -\frac{a+b\text{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{(bcx)\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx,x,\frac{\sqrt{d+ex^2}}{\sqrt{-1-c^2x^2}}\right)}{e\sqrt{-c^2x^2}}\\
&= -\frac{a+b\text{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}-\frac{bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{\sqrt{d}\,e\sqrt{-c^2x^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.50, size = 94, normalized size = 1.15

$$
-\frac{a+b\text{csch}^{-1}(cx)}{e\sqrt{d+ex^2}}+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}\,x\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{\sqrt{d}\,e\sqrt{1+c^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
[Out] -((a + b*ArcCsch[c*x])/(e*Sqrt[d + e*x^2])) + (b*c*Sqrt[1 + 1/(c^2*x^2)]*x*
ArcTanh[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2]])/(Sqrt[d]*e*Sqrt[1 + c
^2*x^2])
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$
\int \frac{x\left(a+b\,\mathrm{arccsch}(cx)\right)}{\left(e\,x^2+d\right)^{\frac{3}{2}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -(c^2*integrate(x/((c^2*x^2*e + e)*sqrt(c^2*x^2 + 1)*sqrt(x^2*e + d) + (c^2
*x^2*e + e)*sqrt(x^2*e + d)), x) + e^(-1)*log(sqrt(c^2*x^2 + 1) + 1)/sqrt(x
^2*e + d) + integrate((c^2*x^3*(log(c) - 1)*e - (c^2*d - e*log(c))*x + (c^2
*x^3*e + x*e)*log(x))/((c^2*x^4*e^2 + (c^2*d*e + e^2)*x^2 + d*e)*sqrt(x^2*e
 + d)), x))*b - a*e^(-1)/sqrt(x^2*e + d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(70) = 140.
time = 0.43, size = 561, normalized size = 6.84

$$\left[-\frac{4 \sqrt{x^{2} \cosh\left(1\right) + x^{2} \sinh\left(1\right) + d}\, b d \log\left(\frac{c x \sqrt{\frac{c^{2} x^{2} + 1}{c^{2} x^{2}}} + 1}{c x}\right) + 4 \sqrt{x^{2} \cosh\left(1\right) + x^{2} \sinh\left(1\right) + d}\, a d - \left(b x^{2} \cosh\left(1\right) + b x^{2} \sinh\left(1\right) + b d\right) \sqrt{d} \log\left(\frac{c^{4} d^{2} x^{4} + 8 c^{2} d^{2} x^{2} + x^{4} \cosh\left(1\right)^{2} + x^{4} \sinh\left(1\right)^{2} + 4 \left(c^{3} d x^{3} + c x^{3} \cosh\left(1\right) + c x^{3} \sinh\left(1\right) + 2 c d x\right) \sqrt{x^{2} \cosh\left(1\right) + x^{2} \sinh\left(1\right) + d} \sqrt{d} \sqrt{\frac{c^{2} x^{2} + 1}{c^{2} x^{2}}} + 8 d^{2} + 2 \left(3 c^{2} d x^{4} + 4 d x^{2}\right) \cosh\left(1\right) + 2 \left(3 c^{2} d x^{4} + x^{4} \cosh\left(1\right) + 4 d x^{2}\right) \sinh\left(1\right)}{x^{4}}\right)}{4 \left(d x^{2} \cosh\left(1\right)^{2} + d x^{2} \sinh\left(1\right)^{2} + d^{2} \cosh\left(1\right) + \left(2 d x^{2} \cosh\left(1\right) + d^{2}\right) \sinh\left(1\right)\right)}, -\frac{2 \sqrt{x^{2} \cosh\left(1\right) + x^{2} \sinh\left(1\right) + d}\, b d \log\left(\frac{c x \sqrt{\frac{c^{2} x^{2} + 1}{c^{2} x^{2}}} + 1}{c x}\right) + 2 \sqrt{x^{2} \cosh\left(1\right) + x^{2} \sinh\left(1\right) + d}\, a d + \left(b x^{2} \cosh\left(1\right) + b x^{2} \sinh\left(1\right) + b d\right) \sqrt{-d} \arctan\left(\frac{\left(c^{3} d x^{3} + c x^{3} \cosh\left(1\right) + c x^{3} \sinh\left(1\right) + 2 c d x\right) \sqrt{x^{2} \cosh\left(1\right) + x^{2} \sinh\left(1\right) + d} \sqrt{-d} \sqrt{\frac{c^{2} x^{2} + 1}{c^{2} x^{2}}}}{2 \left(c^{2} d^{2} x^{2} + d^{2} + \left(c^{2} d x^{4} + d x^{2}\right) \cosh\left(1\right) + \left(c^{2} d x^{4} + d x^{2}\right) \sinh\left(1\right)\right)}\right)}{2 \left(d x^{2} \cosh\left(1\right)^{2} + d x^{2} \sinh\left(1\right)^{2} + d^{2} \cosh\left(1\right) + \left(2 d x^{2} \cosh\left(1\right) + d^{2}\right) \sinh\left(1\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*b*d*log((c*x*sqrt((c^2*x^2 + 1
)/(c^2*x^2)) + 1)/(c*x)) + 4*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*a*d - (b*x
^2*cosh(1) + b*x^2*sinh(1) + b*d)*sqrt(d)*log((c^4*d^2*x^4 + 8*c^2*d^2*x^2
+ x^4*cosh(1)^2 + x^4*sinh(1)^2 + 4*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh
(1) + 2*c*d*x)*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(d)*sqrt((c^2*x^2 +
1)/(c^2*x^2)) + 8*d^2 + 2*(3*c^2*d*x^4 + 4*d*x^2)*cosh(1) + 2*(3*c^2*d*x^4
+ x^4*cosh(1) + 4*d*x^2)*sinh(1))/x^4))/(d*x^2*cosh(1)^2 + d*x^2*sinh(1)^2
+ d^2*cosh(1) + (2*d*x^2*cosh(1) + d^2)*sinh(1)), -1/2*(2*sqrt(x^2*cosh(1)
+ x^2*sinh(1) + d)*b*d*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) +
 2*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*a*d + (b*x^2*cosh(1) + b*x^2*sinh(1)
 + b*d)*sqrt(-d)*arctan(1/2*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*
c*d*x)*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2
*x^2))/(c^2*d^2*x^2 + d^2 + (c^2*d*x^4 + d*x^2)*cosh(1) + (c^2*d*x^4 + d*x^
2)*sinh(1))))/(d*x^2*cosh(1)^2 + d*x^2*sinh(1)^2 + d^2*cosh(1) + (2*d*x^2*c
osh(1) + d^2)*sinh(1))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\operatorname{acsch}(cx))}{(d+ex^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x*(a + b*acsch(c*x))/(d + e*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.150 $\int \frac{a+b\text{csch}^{-1}(cx)}{x(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}},x\right)$$

[Out] Unintegrable((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x)

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(3/2)),x]

[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(3/2)), x]

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A]
time = 22.13, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(3/2)),x]

[Out] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(3/2)), x]

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsch}(cx)}{x\left(e\,x^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(arcsinh(sqrt(d)*e^(-1/2)/abs(x))/d^(3/2) - 1/(sqrt(x^2*e + d)*d)) + b*i
ntegrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/((x^2*e + d)^(3/2)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsch(c*x) + a)/(x^5*e^2 + 2*d*x^3*e + d^2*x)
, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acsch}(cx)}{x (d + ex^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x*(d + e*x**2)**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(3/2)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x\,{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^(3/2)), x)
```

3.151 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^3(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

```
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A]
time = 31.97, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(3/2)),x]
```

```
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(3/2)), x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsch}(cx)}{x^3\left(e\,x^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(3*arcsinh(sqrt(d)*e^(-1/2)/abs(x))*e/d^(5/2) - 3*e/(sqrt(x^2*e + d)*
d^2) - 1/(sqrt(x^2*e + d)*d*x^2)) + b*integrate(log(sqrt(1/(c^2*x^2) + 1) +
 1/(c*x))/((x^2*e + d)^(3/2)*x^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsch(c*x) + a)/(x^7*e^2 + 2*d*x^5*e + d^2*x^
3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**3/(e*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(3/2)*x^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x^3\,{\left(e\,x^2+d\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x^3*(d + e*x^2)^(3/2)), x)
```

3.152 $\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Defer[Int][(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A]
time = 5.37, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+b\,\text{arccsch}(cx))}{(e\,x^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(x^3*e^(-1)/sqrt(x^2*e + d) - 3*d*arcsinh(x*e^(1/2)/sqrt(d))*e^(-5/2) +
 3*d*x*e^(-2)/sqrt(x^2*e + d))*a + b*integrate(x^4*log(sqrt(1/(c^2*x^2) + 1
) + 1/(c*x))/(x^2*e + d)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccsch(c*x) + a*x^4)*sqrt(x^2*e + d)/(x^4*e^2 + 2*d*x^2*e
+ d^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(a + b\operatorname{acsch}(cx))}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*acsch(c*x))/(d + e*x**2)**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^4/(e*x^2 + d)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^4\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.153 $\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Defer[Int][(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx = \int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A]
time = 2.98, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+b\,\mathrm{arccsch}(cx)\right)}{\left(e\,x^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] (arcsinh(x*e^(1/2)/sqrt(d))*e^(-3/2) - x*e^(-1)/sqrt(x^2*e + d))*a + b*inte
grate(x^2*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(x^2*e + d)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccsch(c*x) + a*x^2)*sqrt(x^2*e + d)/(x^4*e^2 + 2*d*x^2*e
+ d^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\operatorname{acsch}(cx))}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*acsch(c*x))/(d + e*x**2)**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.154 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=111

$$\frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{bx\sqrt{d+ex^2}\,F\left(\operatorname{ArcTan}(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\,\sqrt{-1-c^2x^2}\,\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}$$

```
[Out] x*(a+b*arccsch(c*x))/d/(e*x^2+d)^(1/2)-b*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1
)^(1/2)*EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/
d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {197, 6427, 12, 429}

$$\frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{bx\sqrt{d+ex^2}\,F\left(\operatorname{ArcTan}(cx)\,\middle|\,1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\,\sqrt{-c^2x^2-1}\,\sqrt{\frac{d+ex^2}{d(c^2x^2+1)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCsch[c*x]))/(d*Sqrt[d + e*x^2]) - (b*x*Sqrt[d + e*x^2]*Ellipti
cF[ArcTan[c*x], 1 - e/(c^2*d)])/(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sq
rt[(d + e*x^2)/(d*(1 + c^2*x^2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 6427

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x
] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2
*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p +
 1/2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^{3/2}}\,dx &= \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{1}{d\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{\sqrt{-c^2x^2}}\\
&= \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{1}{\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d\sqrt{-c^2x^2}}\\
&= \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}} - \frac{bx\sqrt{d+ex^2}\,F\left(\tan^{-1}(cx)|1-\frac{e}{c^2d}\right)}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}\sqrt{\frac{d+ex^2}{d(1+c^2x^2)}}}
\end{aligned}$$

Mathematica [A]
time = 0.67, size = 113, normalized size = 1.02

$$\frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{d\sqrt{d+ex^2}}+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\,F\left(\text{ArcSin}\left(\sqrt{-c^2}\,x\right)|\frac{e}{c^2d}\right)}{\sqrt{-c^2}\,d\sqrt{1+c^2x^2}\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2)^(3/2),x]
[Out] (x*(a + b*ArcCsch[c*x]))/(d*Sqrt[d + e*x^2]) + (b*c*Sqrt[1 + 1/(c^2*x^2)]*x
*Sqrt[1 + (e*x^2)/d]*EllipticF[ArcSin[Sqrt[-c^2]*x], e/(c^2*d)])/(Sqrt[-c^2
]*d*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsch}(cx)}{(e\,x^2+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(x^2*e + d)^(3/2), x) + a*
x/(sqrt(x^2*e + d)*d)
```

Fricas [A]
time = 0.11, size = 165, normalized size = 1.49

$$\frac{\sqrt{x^2\cosh(1)+x^2\sinh(1)+d}\,bc^2dx\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\sqrt{x^2\cosh(1)+x^2\sinh(1)+d}\,ac^2dx-(bx^2\cosh(1)+bx^2\sinh(1)+bd)\sqrt{-c^2}\,\sqrt{d}\,\mathrm{ellipticF}\left(\sqrt{-c^2}\,x,\frac{\cosh(1)+\sinh(1)}{c^2d}\right)}{c^2d^2x^2\cosh(1)+c^2d^2x^2\sinh(1)+c^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] (sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*b*c^2*d*x*log((c*x*sqrt((c^2*x^2 + 1)/
(c^2*x^2)) + 1)/(c*x)) + sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*a*c^2*d*x - (b
*x^2*cosh(1) + b*x^2*sinh(1) + b*d)*sqrt(-c^2)*sqrt(d)*ellipticF(sqrt(-c^2)
*x, (cosh(1) + sinh(1))/(c^2*d)))/(c^2*d^2*x^2*cosh(1) + c^2*d^2*x^2*sinh(1
) + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acsch}\,(cx)}{(d + ex^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(d + e*x**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{\left(e\,x^2+d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(3/2), x)
```

3.155 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^2(d+ex^2)^{3/2}}\,dx$

Optimal. Leaf size=321

$$\frac{bc^3x^2\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex\left(a+b\text{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{bc^2x\sqrt{d+}}{d^2\sqrt{-c^2x}}$$

```
[Out] (-a-b*arccsch(c*x))/d/x/(e*x^2+d)^(1/2)-2*e*x*(a+b*arccsch(c*x))/d^2/(e*x^2
+d)^(1/2)+b*c^3*x^2*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)
+b*c*(-c^2*x^2-1)^(1/2)*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)-b*c^2*x*(1/(c^
2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/
d)^(1/2))*(e*x^2+d)^(1/2)/d^2/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d
)/d/(c^2*x^2+1))^(1/2)+2*b*e*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*Elli
pticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^3/(-c^2*x^
2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {277, 197, 6437, 12, 597, 545, 429, 506, 422}

$$-\frac{2ex\left(a+b\text{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}+\frac{2bex\sqrt{d+ex^2}\,F\left(\text{ArcTan}(cx)\left|1-\frac{e}{c^2d}\right.\right)}{d^3\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}-\frac{bc^2x\sqrt{d+ex^2}\,E\left(\text{ArcTan}(cx)\left|1-\frac{e}{c^2d}\right.\right)}{d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}+\frac{bc\sqrt{-c^2x^2-1}\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}}+\frac{bc^3x^2\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] (b*c^3*x^2*Sqrt[d + e*x^2])/(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]) + (b*
c*Sqrt[-1 - c^2*x^2]*Sqrt[d + e*x^2])/(d^2*Sqrt[-(c^2*x^2)]) - (a + b*ArcCs
ch[c*x])/(d*x*Sqrt[d + e*x^2]) - (2*e*x*(a + b*ArcCsch[c*x]))/(d^2*Sqrt[d +
 e*x^2]) - (b*c^2*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/
(d^2*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))
]) + (2*b*e*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(d^3*S
qrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 597

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b
*x^n)^(p + 1)*((c + d*x^n)^(q + 1)/(a*c*g*(m + 1))), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^2\left(d+ex^2\right)^{3/2}}\,dx = -\frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex\left(a+b\text{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{(bcx)\int\frac{-d-2ex^2}{d^2x^2\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\cdots}{\sqrt{-c^2x^2}}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex\left(a+b\text{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{(bcx)\int\frac{-d-2ex^2}{x^2\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}\,dx}{d^2\sqrt{-c^2x^2}}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex\left(a+b\text{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}-\frac{(bcx)\int\cdots}{\cdots}$$

$$= \frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex\left(a+b\text{csch}^{-1}(cx)\right)}{d^2\sqrt{d+ex^2}}+\frac{(2bcex)\cdots}{\cdots}$$

$$= \frac{bc^3x^2\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex(\cdots}{\cdots}$$

$$= \frac{bc^3x^2\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}+\frac{bc\sqrt{-1-c^2x^2}\sqrt{d+ex^2}}{d^2\sqrt{-c^2x^2}}-\frac{a+b\text{csch}^{-1}(cx)}{dx\sqrt{d+ex^2}}-\frac{2ex(\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.89, size = 201, normalized size = 0.63

$$\frac{bc\sqrt{1+\frac{1}{c^2x^2}}\,x(d+ex^2)-a(d+2ex^2)-b(d+2ex^2)\,\text{csch}^{-1}(cx)}{d^2x\sqrt{d+ex^2}}+\frac{ibc\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\left(c^2dE\left(i\sinh^{-1}\left(\sqrt{c^2}\,x\right)|\frac{e}{c^2d}\right)+(-c^2d+2e)\,F\left(i\sinh^{-1}\left(\sqrt{c^2}\,x\right)|\frac{e}{c^2d}\right)\right)}{\sqrt{c^2}\,d^2\sqrt{1+c^2x^2}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^2*(d + e*x^2)^(3/2)),x]
```

```
[Out] (b*c*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2) - a*(d + 2*e*x^2) - b*(d + 2*e*x^2
)*ArcCsch[c*x])/(d^2*x*Sqrt[d + e*x^2]) + (I*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sq
```

```
rt[1 + (e*x^2)/d]*(c^2*d*EllipticE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] + (-(
c^2*d) + 2*e)*EllipticF[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d^2
*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arccsch}(cx)}{x^2\,(e\,x^2 + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(2*x*e/(sqrt(x^2*e + d)*d^2) + 1/(sqrt(x^2*e + d)*d*x)) - b*((2*x^2*e +
d)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(x^2*e + d)*d^2*x) + integrate((2*c^2*x^
2*e + c^2*d)/((c^2*d^2*x^2 + d^2)*sqrt(c^2*x^2 + 1)*sqrt(x^2*e + d) + (c^2*
d^2*x^2 + d^2)*sqrt(x^2*e + d)), x) + integrate(-(2*c^2*x^6*e^2 + 3*c^2*d*x
^4*e - (d^2*log(c) - d^2)*c^2*x^2 - d^2*log(c) - (c^2*d^2*x^2 + d^2)*log(x)
)/((c^2*d^2*x^6*e + d^3*x^2 + (c^2*d^3 + d^2*e)*x^4)*sqrt(x^2*e + d)), x))
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acsch}\,(cx)}{x^2\,(d + ex^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**2/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x**2*(d + e*x**2)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x^2\,{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)),x)
```

```
[Out] int((a + b*asinh(1/(c*x)))/(x^2*(d + e*x^2)^(3/2)), x)
```

3.156 $\int \frac{x^5\left(a+b\,\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=251

$$\frac{bcdx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e^2\sqrt{-c^2x^2}\,\sqrt{d+ex^2}}-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}$$

```
[Out] -1/3*d^2*(a+b*arccsch(c*x))/e^3/(e*x^2+d)^(3/2)+b*x*arctan(e^(1/2)*(-c^2*x^
2-1)^(1/2)/c/(e*x^2+d)^(1/2))/e^(5/2)/(-c^2*x^2)^(1/2)+8/3*b*c*x*arctan((e*
x^2+d)^(1/2)/d^(1/2)/(-c^2*x^2-1)^(1/2))*d^(1/2)/e^3/(-c^2*x^2)^(1/2)+2*d*(
a+b*arccsch(c*x))/e^3/(e*x^2+d)^(1/2)+1/3*b*c*d*x*(-c^2*x^2-1)^(1/2)/(c^2*d
-e)/e^2/(-c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)+(a+b*arccsch(c*x))*(e*x^2+d)^(1/2)
/e^3
```

Rubi [A]
time = 0.85, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {272, 45, 6437, 12, 1628, 163, 65, 223, 209, 95, 210}

$$-\frac{d^2\left(a+b\text{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}}+\frac{2d\left(a+b\text{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}}+\frac{\sqrt{d+ex^2}\,\left(a+b\text{csch}^{-1}(cx)\right)}{e^3}+\frac{bx\text{ArcTan}\left(\frac{\sqrt{e}\,\sqrt{-c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{e^{5/2}\sqrt{-c^2x^2}}+\frac{8bc\sqrt{d}\,x\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\,\sqrt{-c^2x^2-1}}\right)}{3e^3\sqrt{-c^2x^2}}+\frac{bcdx\sqrt{-c^2x^2-1}}{3e^2\sqrt{-c^2x^2}\,\left(c^2d-e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*d*x*Sqrt[-1 - c^2*x^2])/(3*(c^2*d - e)*e^2*Sqrt[-(c^2*x^2)]*Sqrt[d + e
*x^2]) - (d^2*(a + b*ArcCsch[c*x]))/(3*e^3*(d + e*x^2)^(3/2)) + (2*d*(a + b
*ArcCsch[c*x]))/(e^3*Sqrt[d + e*x^2]) + (Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x
]))/e^3 + (b*x*ArcTan[(Sqrt[e]*Sqrt[-1 - c^2*x^2])/(c*Sqrt[d + e*x^2])])/(e
^(5/2)*Sqrt[-(c^2*x^2)]) + (8*b*c*Sqrt[d]*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]
*Sqrt[-1 - c^2*x^2])])/(3*e^3*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 163

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^
p, x], x] + Dist[(b*g - a*h)/b, Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1628

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_
.)*(x_))^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[Px, a + b*x, x],
 R = PolynomialRemainder[Px, a + b*x, x]}, Simp[b*R*(a + b*x)^(m + 1)*(c +
d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Di
st[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(
e + f*x)^p*ExpandToSum[(m + 1)*(b*c - a*d)*(b*e - a*f)*Qx + a*d*f*R*(m + 1)
 - b*R*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*R*(m + n + p + 3)*x, x],
 x], x]] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && PolyQ[Px, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^5\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}$$

$$= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}$$

$$= -\frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3\sqrt{d+ex^2}} + \frac{\sqrt{d+ex^2}\,\left(a+b\operatorname{csch}^{-1}(cx)\right)}{e^3}$$

$$= \frac{bcdx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e^2\sqrt{-c^2x^2}\,\sqrt{d+ex^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d(a+b\operatorname{csch}^{-1}(c}{e^3\sqrt{d+ex^2}}$$

$$= \frac{bcdx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e^2\sqrt{-c^2x^2}\,\sqrt{d+ex^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d(a+b\operatorname{csch}^{-1}(c}{e^3\sqrt{d+ex^2}}$$

$$= \frac{bcdx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e^2\sqrt{-c^2x^2}\,\sqrt{d+ex^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d(a+b\operatorname{csch}^{-1}(c}{e^3\sqrt{d+ex^2}}$$

$$= \frac{bcdx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e^2\sqrt{-c^2x^2}\,\sqrt{d+ex^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d(a+b\operatorname{csch}^{-1}(c}{e^3\sqrt{d+ex^2}}$$

$$= \frac{bcdx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e^2\sqrt{-c^2x^2}\,\sqrt{d+ex^2}} - \frac{d^2\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3e^3\left(d+ex^2\right)^{3/2}} + \frac{2d(a+b\operatorname{csch}^{-1}(c}{e^3\sqrt{d+ex^2}}$$

<u>Mathematica</u> [A]
time = 1.16, size = 241, normalized size = 0.96

$$\frac{bcde\sqrt{1+\frac{1}{c^2x^2}}\,x\left(d+ex^2\right)+a\left(c^2d-e\right)\left(8d^2+12dex^2+3e^2x^4\right)+b\left(c^2d-e\right)\left(8d^2+12dex^2+3e^2x^4\right)\operatorname{csch}^{-1}(cx)}{3\left(c^2d-e\right)e^3\left(d+ex^2\right)^{3/2}} - \frac{b\sqrt{1+\frac{1}{c^2x^2}}\,x\left(8c\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d}\,\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)-3\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{1+c^2x^2}}{c\sqrt{d+ex^2}}\right)\right)}{3e^3\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*d*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2) + a*(c^2*d - e)*(8*d^2 + 12*d*
e*x^2 + 3*e^2*x^4) + b*(c^2*d - e)*(8*d^2 + 12*d*e*x^2 + 3*e^2*x^4)*ArcCsch
[c*x])/(3*(c^2*d - e)*e^3*(d + e*x^2)^(3/2)) - (b*Sqrt[1 + 1/(c^2*x^2)]*x*(
```

```
8*c*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2]] - 3*Sqrt[e
]*ArcTanh[(Sqrt[e]*Sqrt[1 + c^2*x^2])/(c*Sqrt[d + e*x^2])]))/(3*e^3*Sqrt[1
+ c^2*x^2])
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{x^5(a+b\,\mathrm{arccsch}(cx))}{(e\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*x^4*e^(-1)/(x^2*e + d)^(3/2) + 12*d*x^2*e^(-2)/(x^2*e + d)^(3/2) + 8
*d^2*e^(-3)/(x^2*e + d)^(3/2))*a + 1/3*b*((3*x^4*e^2 + 12*d*x^2*e + 8*d^2)*
log(sqrt(c^2*x^2 + 1) + 1)/((x^2*e^4 + d*e^3)*sqrt(x^2*e + d)) + 3*integrat
e(1/3*(3*c^2*x^5*e^2 + 12*c^2*d*x^3*e + 8*c^2*d^2*x)/((c^2*x^4*e^4 + (c^2*d
*e^3 + e^4)*x^2 + d*e^3)*sqrt(c^2*x^2 + 1)*sqrt(x^2*e + d) + (c^2*x^4*e^4 +
 (c^2*d*e^3 + e^4)*x^2 + d*e^3)*sqrt(x^2*e + d)), x) - 3*integrate(1/3*(3*c
^2*x^7*(log(c) + 1)*e^3 + 20*c^2*d^2*x^3*e + 8*c^2*d^3*x + 3*(5*c^2*d*e^2 +
 e^3*log(c))*x^5 + 3*(c^2*x^7*e^3 + x^5*e^3)*log(x))/((c^2*x^6*e^5 + (2*c^2
*d*e^4 + e^5)*x^4 + (c^2*d^2*e^3 + 2*d*e^4)*x^2 + d^2*e^3)*sqrt(x^2*e + d))
, x))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1531 vs. $2(213) = 426$.
time = 0.64, size = 3102, normalized size = 12.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(b*x^4*cosh(1)^3 + b*x^4*sinh(1)^3 - b*c^2*d^3 - (b*c^2*d*x^4 - 2*
b*d*x^2)*cosh(1)^2 - (b*c^2*d*x^4 - 3*b*x^4*cosh(1) - 2*b*d*x^2)*sinh(1)^2
- (2*b*c^2*d^2*x^2 - b*d^2)*cosh(1) - (2*b*c^2*d^2*x^2 - 3*b*x^4*cosh(1)^2
```

- b*d^2 + 2*(b*c^2*d*x^4 - 2*b*d*x^2)*cosh(1))*sinh(1))*sqrt(cosh(1) + sinh
(1))*log(c^4*d^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 + 8*c
^2*x^2 + 1)*sinh(1)^2 + 4*(c^4*d*x + (2*c^4*x^3 + c^2*x)*cosh(1) + (2*c^4*x
^3 + c^2*x)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt((c^2*x^2 + 1)
/(c^2*x^2))*sqrt(cosh(1) + sinh(1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1) + 2
*(4*c^4*d*x^2 + 3*c^2*d + (8*c^4*x^4 + 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) + 4
*(3*b*c*x^4*cosh(1)^3 + 3*b*c*x^4*sinh(1)^3 - 8*b*c^3*d^3 - 3*(b*c^3*d*x^4
- 4*b*c*d*x^2)*cosh(1)^2 - 3*(b*c^3*d*x^4 - 3*b*c*x^4*cosh(1) - 4*b*c*d*x^2
)*sinh(1)^2 - 4*(3*b*c^3*d^2*x^2 - 2*b*c*d^2)*cosh(1) - (12*b*c^3*d^2*x^2 -
9*b*c*x^4*cosh(1)^2 - 8*b*c*d^2 + 6*(b*c^3*d*x^4 - 4*b*c*d*x^2)*cosh(1))*s
inh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^
2*x^2)) + 1)/(c*x)) + 8*(b*c*x^4*cosh(1)^3 + b*c*x^4*sinh(1)^3 - b*c^3*d^3
- (b*c^3*d*x^4 - 2*b*c*d*x^2)*cosh(1)^2 - (b*c^3*d*x^4 - 3*b*c*x^4*cosh(1)
- 2*b*c*d*x^2)*sinh(1)^2 - (2*b*c^3*d^2*x^2 - b*c*d^2)*cosh(1) - (2*b*c^3*d
^2*x^2 - 3*b*c*x^4*cosh(1)^2 - b*c*d^2 + 2*(b*c^3*d*x^4 - 2*b*c*d*x^2)*cosh
(1))*sinh(1))*sqrt(d)*log((c^4*d^2*x^4 + 8*c^2*d^2*x^2 + x^4*cosh(1)^2 + x^
4*sinh(1)^2 - 4*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*sqrt(
x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^
2 + 2*(3*c^2*d*x^4 + 4*d*x^2)*cosh(1) + 2*(3*c^2*d*x^4 + x^4*cosh(1) + 4*d*
x^2)*sinh(1))/x^4) + 4*(3*a*c*x^4*cosh(1)^3 + 3*a*c*x^4*sinh(1)^3 - 8*a*c^3
d^3 - 3(a*c^3*d*x^4 - 4*a*c*d*x^2)*cosh(1)^2 - 3*(a*c^3*d*x^4 - 3*a*c*x^4
*cosh(1) - 4*a*c*d*x^2)*sinh(1)^2 - 4*(3*a*c^3*d^2*x^2 - 2*a*c*d^2)*cosh(1)
- (12*a*c^3*d^2*x^2 - 9*a*c*x^4*cosh(1)^2 - 8*a*c*d^2 + 6*(a*c^3*d*x^4 - 4
*a*c*d*x^2)*cosh(1))*sinh(1) - (b*c^2*d*x^3*cosh(1)^2 + b*c^2*d*x^3*sinh(1)
^2 + b*c^2*d^2*x*cosh(1) + (2*b*c^2*d*x^3*cosh(1) + b*c^2*d^2*x)*sinh(1))*s
qrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(c*x^4*c
osh(1)^6 + c*x^4*sinh(1)^6 - c^3*d^3*cosh(1)^3 - (c^3*d*x^4 - 2*c*d*x^2)*co
sh(1)^5 - (c^3*d*x^4 - 6*c*x^4*cosh(1) - 2*c*d*x^2)*sinh(1)^5 - (2*c^3*d^2*
x^2 - c*d^2)*cosh(1)^4 - (2*c^3*d^2*x^2 - 15*c*x^4*cosh(1)^2 - c*d^2 + 5*(c
^3*d*x^4 - 2*c*d*x^2)*cosh(1))*sinh(1)^4 + (20*c*x^4*cosh(1)^3 - c^3*d^3 -
10*(c^3*d*x^4 - 2*c*d*x^2)*cosh(1)^2 - 4*(2*c^3*d^2*x^2 - c*d^2)*cosh(1))*s
inh(1)^3 + (15*c*x^4*cosh(1)^4 - 3*c^3*d^3*cosh(1) - 10*(c^3*d*x^4 - 2*c*d*
x^2)*cosh(1)^3 - 6*(2*c^3*d^2*x^2 - c*d^2)*cosh(1)^2)*sinh(1)^2 + (6*c*x^4*
cosh(1)^5 - 3*c^3*d^3*cosh(1)^2 - 5*(c^3*d*x^4 - 2*c*d*x^2)*cosh(1)^4 - 4*(
2*c^3*d^2*x^2 - c*d^2)*cosh(1)^3)*sinh(1)), 1/12*(16*(b*c*x^4*cosh(1)^3 + b
*c*x^4*sinh(1)^3 - b*c^3*d^3 - (b*c^3*d*x^4 - 2*b*c*d*x^2)*cosh(1)^2 - (b*c
^3*d*x^4 - 3*b*c*x^4*cosh(1) - 2*b*c*d*x^2)*sinh(1)^2 - (2*b*c^3*d^2*x^2 -
b*c*d^2)*cosh(1) - (2*b*c^3*d^2*x^2 - 3*b*c*x^4*cosh(1)^2 - b*c*d^2 + 2*(b*
c^3*d*x^4 - 2*b*c*d*x^2)*cosh(1))*sinh(1))*sqrt(-d)*arctan(1/2*(c^3*d*x^3 +
c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*sqrt(x^2*cosh(1) + x^2*sinh(1) +
d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d^2*x^2 + d^2 + (c^2*d*x^4 +
d*x^2)*cosh(1) + (c^2*d*x^4 + d*x^2)*sinh(1))) + 3*(b*x^4*cosh(1)^3 + b*x^
4*sinh(1)^3 - b*c^2*d^3 - (b*c^2*d*x^4 - 2*b*d*x^2)*cosh(1)^2 - (b*c^2*d*x^
4 - 3*b*x^4*cosh(1) - 2*b*d*x^2)*sinh(1)^2 - (2*b*c^2*d^2*x^2 - b*d^2)*cosh
(1) - (2*b*c^2*d^2*x^2 - 3*b*x^4*cosh(1)^2 - b*d^2 + 2*(b*c^2*d*x^4 - 2*b*d

```
*x^2)*cosh(1))*sinh(1))*sqrt(cosh(1) + sinh(1))*log(c^4*d^2 + (8*c^4*x^4 +
8*c^2*x^2 + 1)*cosh(1)^2 + (8*c^4*x^4 + 8*c^2*x^2 + 1)*sinh(1)^2 + 4*(c^4*d
*x + (2*c^4*x^3 + c^2*x)*cosh(1) + (2*c^4*x^3 + c^2*x)*sinh(1))*sqrt(x^2*co
sh(1) + x^2*sinh(1) + d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))*sqrt(cosh(1) + sinh(
1)) + 2*(4*c^4*d*x^2 + 3*c^2*d)*cosh(1) + 2*(4*c^4*d*x^2 + 3*c^2*d + (8*c^4
*x^4 + 8*c^2*x^2 + 1)*cosh(1))*sinh(1)) + 4*(3*b*c*x^4*cosh(1)^3 + 3*b*c*x^
4*sinh(1)^3 - 8*b*c^3*d^3 - 3*(b*c^3*d*x^4 - 4*b*c*d*x^2)*cosh(1)^2 - 3*(b*
c^3*d*x^4 - 3*b*c*x^4*cosh(1) - 4*b*c*d*x^2)*sinh(1)^2 - 4*(3*b*c^3*d^2*x^2
 - 2*b*c*d^2)*cosh(1) - (12*b*c^3*d^2*x^2 - 9*b*c*x^4*cosh(1)^2 - 8*b*c*d^2
 + 6*(b*c^3*d*x^4 - 4*b*c*d*x^2)*cosh(1))*sinh(1))*sqrt(x^2*cosh(1) + x^2*s
inh(1) + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + 4*(3*a*c*x
^4*cosh(1)^3 + 3*a*c*x^4*sinh(1)^3 - 8*a*c^3*d^3 - 3*(a*c^3*d*x^4 - 4*a*c*d
*x^2)*cosh(1)^2 - 3*(a*c^3*d*x^4 - 3*a*c*x^4*cosh(1) - 4*a*c*d*x^2)*sinh(1)
^2 - 4*(3*a*c^3*d^2*x^2 - 2*a*c*d^2)*cosh(1) - (12*a*c^3*d^2*x^2 - 9*a*c*x^
4*cosh(1)^2 - 8*a*c*d^2 + 6*(a*c^3*d*x^4 - 4*a*c*d*x^2)*cosh(1))*sinh(1) -
(b*c^2*d*x^3*cosh(1)^2 + b*c^2*d*x^3*sinh(1)^2 + b*c^2*d^2*x*cosh(1) + (2*b
*c^2*d*x^3*cosh(1) + b*c^2*d^2*x)*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*s
qrt(x^2*cosh(1) + x^2*sinh(1) + d))/(c*x^4*cosh(1)^6 + c*x^4*sinh(1)^6 - c^
3*d^3*cosh(1)^3 - (c^3*d*x^4 - 2*c*d*x^2)*cosh(...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^5/(e*x^2 + d)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^5*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.157 $\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=169

$$-\frac{bcx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{d+ex^2}}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}-\frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}}-\frac{2bcx\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3\sqrt{d}\,e^2\sqrt{-c^2x^2}}$$

```
[Out] 1/3*d*(a+b*arccsch(c*x))/e^2/(e*x^2+d)^(3/2)-2/3*b*c*x*arctan((e*x^2+d)^(1/
2)/d^(1/2)/(-c^2*x^2-1)^(1/2))/e^2/d^(1/2)/(-c^2*x^2)^(1/2)+(-a-b*arccsch(c
*x))/e^2/(e*x^2+d)^(1/2)-1/3*b*c*x*(-c^2*x^2-1)^(1/2)/(c^2*d-e)/e/(-c^2*x^2
)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {272, 45, 6437, 12, 587, 157, 95, 210}

$$-\frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}}+\frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}}-\frac{2bcx\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3\sqrt{d}\,e^2\sqrt{-c^2x^2}}-\frac{bcx\sqrt{-c^2x^2-1}}{3e\sqrt{-c^2x^2}\left(c^2d-e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] -1/3*(b*c*x*Sqrt[-1 - c^2*x^2])/((c^2*d - e)*e*Sqrt[-(c^2*x^2)]*Sqrt[d + e*
x^2]) + (d*(a + b*ArcCsch[c*x]))/(3*e^2*(d + e*x^2)^(3/2)) - (a + b*ArcCsch
[c*x])/(e^2*Sqrt[d + e*x^2]) - (2*b*c*x*ArcTan[Sqrt[d + e*x^2]/(Sqrt[d]*Sqr
t[-1 - c^2*x^2])])/(3*Sqrt[d]*e^2*Sqrt[-(c^2*x^2)])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
```

```
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 157

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1
)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 587

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simpl
ify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /;
 FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[Simplify[(m + 1)/n
]]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{-2d-3ex^2}{3e^2x\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{-2d-3ex^2}{x\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}\,dx}{3e^2\sqrt{-c^2x^2}}$$

$$= \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}} - \frac{(bcx)\text{Subst}\left(\int \frac{-2d-3ex}{x\sqrt{-1-c^2x}\,(d+ex)^{3/2}}\right.}{6e^2\sqrt{-c^2x^2}}$$

$$= -\frac{bcx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}}$$

$$= -\frac{bcx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}}$$

$$= -\frac{bcx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}}$$

$$= -\frac{bcx\sqrt{-1-c^2x^2}}{3\left(c^2d-e\right)e\sqrt{-c^2x^2}\sqrt{d+ex^2}} + \frac{d\left(a+b\text{csch}^{-1}(cx)\right)}{3e^2\left(d+ex^2\right)^{3/2}} - \frac{a+b\text{csch}^{-1}(cx)}{e^2\sqrt{d+ex^2}}$$

Mathematica [A]
time = 0.16, size = 173, normalized size = 1.02

$$\frac{bce\sqrt{1+\frac{1}{c^2x^2}}\,x(d+ex^2)+a(c^2d-e)\left(2d+3ex^2\right)+b(c^2d-e)\left(2d+3ex^2\right)\text{csch}^{-1}(cx)}{3e^2\left(-c^2d+e\right)\left(d+ex^2\right)^{3/2}} + \frac{2bc\sqrt{1+\frac{1}{c^2x^2}}\,x\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{3\sqrt{d}\,e^2\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2) + a*(c^2*d - e)*(2*d + 3*e*x^2)
+ b*(c^2*d - e)*(2*d + 3*e*x^2)*ArcCsch[c*x])/(3*e^2*(-(c^2*d) + e)*(d + e*
x^2)^(3/2)) + (2*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[(Sqrt[d]*Sqrt[1 + c^2*
x^2])/Sqrt[d + e*x^2]])/(3*Sqrt[d]*e^2*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+b\,\text{arccsch}(cx))}{(e\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*x^2*e^(-1)/(x^2*e + d)^(3/2) + 2*d*e^(-2)/(x^2*e + d)^(3/2))*a + b*
integrate(x^3*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(x^2*e + d)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 909 vs. $2(144) = 288$.
time = 0.56, size = 1858, normalized size = 10.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(2*b*c^2*d^3 - 3*b*d*x^2*cosh(1)^2 - 3*b*d*x^2*sinh(1)^2 + (3*b*c^2
*d^2*x^2 - 2*b*d^2)*cosh(1) + (3*b*c^2*d^2*x^2 - 6*b*d*x^2*cosh(1) - 2*b*d^
2)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*log((c*x*sqrt((c^2*x^2 + 1)
/(c^2*x^2)) + 1)/(c*x)) + (b*x^4*cosh(1)^3 + b*x^4*sinh(1)^3 - b*c^2*d^3 -
(b*c^2*d*x^4 - 2*b*d*x^2)*cosh(1)^2 - (b*c^2*d*x^4 - 3*b*x^4*cosh(1) - 2*b*
d*x^2)*sinh(1)^2 - (2*b*c^2*d^2*x^2 - b*d^2)*cosh(1) - (2*b*c^2*d^2*x^2 - 3
*b*x^4*cosh(1)^2 - b*d^2 + 2*(b*c^2*d*x^4 - 2*b*d*x^2)*cosh(1))*sinh(1))*sq
rt(d)*log((c^4*d^2*x^4 + 8*c^2*d^2*x^2 + x^4*cosh(1)^2 + x^4*sinh(1)^2 + 4*
(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*sqrt(x^2*cosh(1) + x^
2*sinh(1) + d)*sqrt(d)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 8*d^2 + 2*(3*c^2*d*x
^4 + 4*d*x^2)*cosh(1) + 2*(3*c^2*d*x^4 + x^4*cosh(1) + 4*d*x^2)*sinh(1))/x^
4) + 2*(2*a*c^2*d^3 - 3*a*d*x^2*cosh(1)^2 - 3*a*d*x^2*sinh(1)^2 + (3*a*c^2*
d^2*x^2 - 2*a*d^2)*cosh(1) + (3*a*c^2*d^2*x^2 - 6*a*d*x^2*cosh(1) - 2*a*d^2
)*sinh(1) + (b*c*d*x^3*cosh(1)^2 + b*c*d*x^3*sinh(1)^2 + b*c*d^2*x*cosh(1)
+ (2*b*c*d*x^3*cosh(1) + b*c*d^2*x)*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))
*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(d*x^4*cosh(1)^5 + d*x^4*sinh(1)^5 -
c^2*d^4*cosh(1)^2 - (c^2*d^2*x^4 - 2*d^2*x^2)*cosh(1)^4 - (c^2*d^2*x^4 - 5*
d*x^4*cosh(1) - 2*d^2*x^2)*sinh(1)^4 - (2*c^2*d^3*x^2 - d^3)*cosh(1)^3 - (2
*c^2*d^3*x^2 - 10*d*x^4*cosh(1)^2 - d^3 + 4*(c^2*d^2*x^4 - 2*d^2*x^2)*cosh(
1))*sinh(1)^3 + (10*d*x^4*cosh(1)^3 - c^2*d^4 - 6*(c^2*d^2*x^4 - 2*d^2*x^2)
```

```
*cosh(1)^2 - 3*(2*c^2*d^3*x^2 - d^3)*cosh(1))*sinh(1)^2 + (5*d*x^4*cosh(1)^
4 - 2*c^2*d^4*cosh(1) - 4*(c^2*d^2*x^4 - 2*d^2*x^2)*cosh(1)^3 - 3*(2*c^2*d^
3*x^2 - d^3)*cosh(1)^2)*sinh(1)), -1/3*((b*x^4*cosh(1)^3 + b*x^4*sinh(1)^3
- b*c^2*d^3 - (b*c^2*d*x^4 - 2*b*d*x^2)*cosh(1)^2 - (b*c^2*d*x^4 - 3*b*x^4*
cosh(1) - 2*b*d*x^2)*sinh(1)^2 - (2*b*c^2*d^2*x^2 - b*d^2)*cosh(1) - (2*b*c
^2*d^2*x^2 - 3*b*x^4*cosh(1)^2 - b*d^2 + 2*(b*c^2*d*x^4 - 2*b*d*x^2)*cosh(1
))*sinh(1))*sqrt(-d)*arctan(1/2*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3*sinh(1)
+ 2*c*d*x)*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/
(c^2*x^2))/(c^2*d^2*x^2 + d^2 + (c^2*d*x^4 + d*x^2)*cosh(1) + (c^2*d*x^4 +
d*x^2)*sinh(1))) - (2*b*c^2*d^3 - 3*b*d*x^2*cosh(1)^2 - 3*b*d*x^2*sinh(1)^2
 + (3*b*c^2*d^2*x^2 - 2*b*d^2)*cosh(1) + (3*b*c^2*d^2*x^2 - 6*b*d*x^2*cosh(
1) - 2*b*d^2)*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*log((c*x*sqrt((c
^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - (2*a*c^2*d^3 - 3*a*d*x^2*cosh(1)^2 - 3
*a*d*x^2*sinh(1)^2 + (3*a*c^2*d^2*x^2 - 2*a*d^2)*cosh(1) + (3*a*c^2*d^2*x^2
 - 6*a*d*x^2*cosh(1) - 2*a*d^2)*sinh(1) + (b*c*d*x^3*cosh(1)^2 + b*c*d*x^3*
sinh(1)^2 + b*c*d^2*x*cosh(1) + (2*b*c*d*x^3*cosh(1) + b*c*d^2*x)*sinh(1))*
sqrt((c^2*x^2 + 1)/(c^2*x^2)))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(d*x^4*
cosh(1)^5 + d*x^4*sinh(1)^5 - c^2*d^4*cosh(1)^2 - (c^2*d^2*x^4 - 2*d^2*x^2)
*cosh(1)^4 - (c^2*d^2*x^4 - 5*d*x^4*cosh(1) - 2*d^2*x^2)*sinh(1)^4 - (2*c^2
*d^3*x^2 - d^3)*cosh(1)^3 - (2*c^2*d^3*x^2 - 10*d*x^4*cosh(1)^2 - d^3 + 4*(
c^2*d^2*x^4 - 2*d^2*x^2)*cosh(1))*sinh(1)^3 + (10*d*x^4*cosh(1)^3 - c^2*d^4
 - 6*(c^2*d^2*x^4 - 2*d^2*x^2)*cosh(1)^2 - 3*(2*c^2*d^3*x^2 - d^3)*cosh(1))
*sinh(1)^2 + (5*d*x^4*cosh(1)^4 - 2*c^2*d^4*cosh(1) - 4*(c^2*d^2*x^4 - 2*d^
2*x^2)*cosh(1)^3 - 3*(2*c^2*d^3*x^2 - d^3)*cosh(1)^2)*sinh(1))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^3/(e*x^2 + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.158 $\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=144

$$\frac{bcx\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}}-\frac{a+b\text{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}-\frac{bcx\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c^2x^2}}\right)}{3d^{3/2}e\sqrt{-c^2x^2}}$$

```
[Out] 1/3*(-a-b*arccsch(c*x))/e/(e*x^2+d)^(3/2)-1/3*b*c*x*arctan((e*x^2+d)^(1/2)/
d^(1/2)/(-c^2*x^2-1)^(1/2))/d^(3/2)/e/(-c^2*x^2)^(1/2)+1/3*b*c*x*(-c^2*x^2-
1)^(1/2)/d/(c^2*d-e)/(-c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6435, 457, 98, 95, 210}

$$-\frac{a+b\text{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}-\frac{bcx\text{ArcTan}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-c^2x^2-1}}\right)}{3d^{3/2}e\sqrt{-c^2x^2}}+\frac{bcx\sqrt{-c^2x^2-1}}{3d\sqrt{-c^2x^2}\left(c^2d-e\right)\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*x*Sqrt[-1 - c^2*x^2])/(3*d*(c^2*d - e)*Sqrt[-(c^2*x^2)]*Sqrt[d + e*x^2
]) - (a + b*ArcCsch[c*x])/(3*e*(d + e*x^2)^(3/2)) - (b*c*x*ArcTan[Sqrt[d +
e*x^2]/(Sqrt[d]*Sqrt[-1 - c^2*x^2])])/(3*d^(3/2)*e*Sqrt[-(c^2*x^2)])
```

Rule 95

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x
)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
```

```
, 1])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6435

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(x_)*((d_.) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCsch[c*x])/(2*e*(p + 1))),
 x] - Dist[b*c*(x/(2*e*(p + 1)*Sqrt[(-c^2)*x^2])), Int[(d + e*x^2)^(p + 1)/
(x*Sqrt[-1 - c^2*x^2]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[p, -1
]
```

Rubi steps

$$\int \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = -\frac{a+b\text{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{(bcx)\int \frac{1}{x\sqrt{-1-c^2x^2}\left(d+ex^2\right)^{3/2}}\,dx}{3e\sqrt{-c^2x^2}}$$

$$= -\frac{a+b\text{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{(bcx)\text{Subst}\left(\int \frac{1}{x\sqrt{-1-c^2x}\,(d+ex)^{3/2}}\,dx,x,x^2\right)}{6e\sqrt{-c^2x^2}}$$

$$= \frac{bcx\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}}-\frac{a+b\text{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{(bcx)\text{Subst}\left(\int \frac{}{x\sqrt{-1-c}\ldots}\right.}{6de\sqrt{-}\ldots}$$

$$= \frac{bcx\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}}-\frac{a+b\text{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}+\frac{(bcx)\text{Subst}\left(\int \frac{1}{-d-x^2}\,dx,\ldots\right.}{3de\sqrt{-c^2x}\ldots}$$

$$= \frac{bcx\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{d+ex^2}}-\frac{a+b\text{csch}^{-1}(cx)}{3e\left(d+ex^2\right)^{3/2}}-\frac{bcx\tan^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{-1-c}\ldots}\right.}{3d^{3/2}e\sqrt{-c^2x^2}}$$

Mathematica [A]
time = 0.14, size = 157, normalized size = 1.09

$$\frac{ad(-c^2d+e)+bce\sqrt{1+\frac{1}{c^2x^2}}\,x(d+ex^2)+bd(-c^2d+e)\operatorname{csch}^{-1}(cx)}{3d\left(c^2d-e\right)e\left(d+ex^2\right)^{3/2}}+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}\,x\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{3d^{3/2}e\sqrt{1+c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (a*d*(-(c^2*d) + e) + b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2) + b*d*(-(c^
2*d) + e)*ArcCsch[c*x])/(3*d*(c^2*d - e)*e*(d + e*x^2)^(3/2)) + (b*c*Sqrt[1
 + 1/(c^2*x^2)]*x*ArcTanh[(Sqrt[d]*Sqrt[1 + c^2*x^2])/Sqrt[d + e*x^2]])/(3*
d^(3/2)*e*Sqrt[1 + c^2*x^2])
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x(a+b\operatorname{arccsch}(cx))}{(e\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(x^2*e + d)^(5/2), x) -
1/3*a*e^(-1)/(x^2*e + d)^(3/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 724 vs. $2(122) = 244$.
time = 0.50, size = 1488, normalized size = 10.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(b*c^2*d^3 - b*d^2*cosh(1) - b*d^2*sinh(1))*sqrt(x^2*cosh(1) + x^2
*sinh(1) + d)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (b*x^4*c
osh(1)^3 + b*x^4*sinh(1)^3 - b*c^2*d^3 - (b*c^2*d*x^4 - 2*b*d*x^2)*cosh(1)^
2 - (b*c^2*d*x^4 - 3*b*x^4*cosh(1) - 2*b*d*x^2)*sinh(1)^2 - (2*b*c^2*d^2*x^
2 - b*d^2)*cosh(1) - (2*b*c^2*d^2*x^2 - 3*b*x^4*cosh(1)^2 - b*d^2 + 2*(b*c^
2*d*x^4 - 2*b*d*x^2)*cosh(1))*sinh(1))*sqrt(d)*log((c^4*d^2*x^4 + 8*c^2*d^2
*x^2 + x^4*cosh(1)^2 + x^4*sinh(1)^2 + 4*(c^3*d*x^3 + c*x^3*cosh(1) + c*x^3
*sinh(1) + 2*c*d*x)*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*sqrt(d)*sqrt((c^2*x
^2 + 1)/(c^2*x^2)) + 8*d^2 + 2*(3*c^2*d*x^4 + 4*d*x^2)*cosh(1) + 2*(3*c^2*d
*x^4 + x^4*cosh(1) + 4*d*x^2)*sinh(1))/x^4) + 4*(a*c^2*d^3 - a*d^2*cosh(1)
- a*d^2*sinh(1) - (b*c*d*x^3*cosh(1)^2 + b*c*d*x^3*sinh(1)^2 + b*c*d^2*x*co
sh(1) + (2*b*c*d*x^3*cosh(1) + b*c*d^2*x)*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*
x^2)))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d))/(d^2*x^4*cosh(1)^4 + d^2*x^4*si
nh(1)^4 - c^2*d^5*cosh(1) - (c^2*d^3*x^4 - 2*d^3*x^2)*cosh(1)^3 - (c^2*d^3*
x^4 - 4*d^2*x^4*cosh(1) - 2*d^3*x^2)*sinh(1)^3 - (2*c^2*d^4*x^2 - d^4)*cosh
(1)^2 - (2*c^2*d^4*x^2 - 6*d^2*x^4*cosh(1)^2 - d^4 + 3*(c^2*d^3*x^4 - 2*d^3
*x^2)*cosh(1))*sinh(1)^2 + (4*d^2*x^4*cosh(1)^3 - c^2*d^5 - 3*(c^2*d^3*x^4
- 2*d^3*x^2)*cosh(1)^2 - 2*(2*c^2*d^4*x^2 - d^4)*cosh(1))*sinh(1)), -1/6*((
b*x^4*cosh(1)^3 + b*x^4*sinh(1)^3 - b*c^2*d^3 - (b*c^2*d*x^4 - 2*b*d*x^2)*c
osh(1)^2 - (b*c^2*d*x^4 - 3*b*x^4*cosh(1) - 2*b*d*x^2)*sinh(1)^2 - (2*b*c^2
*d^2*x^2 - b*d^2)*cosh(1) - (2*b*c^2*d^2*x^2 - 3*b*x^4*cosh(1)^2 - b*d^2 +
2*(b*c^2*d*x^4 - 2*b*d*x^2)*cosh(1))*sinh(1))*sqrt(-d)*arctan(1/2*(c^3*d*x^
3 + c*x^3*cosh(1) + c*x^3*sinh(1) + 2*c*d*x)*sqrt(x^2*cosh(1) + x^2*sinh(1)
 + d)*sqrt(-d)*sqrt((c^2*x^2 + 1)/(c^2*x^2))/(c^2*d^2*x^2 + d^2 + (c^2*d*x^
4 + d*x^2)*cosh(1) + (c^2*d*x^4 + d*x^2)*sinh(1))) - 2*(b*c^2*d^3 - b*d^2*c
osh(1) - b*d^2*sinh(1))*sqrt(x^2*cosh(1) + x^2*sinh(1) + d)*log((c*x*sqrt((
c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) - 2*(a*c^2*d^3 - a*d^2*cosh(1) - a*d^2*
sinh(1) - (b*c*d*x^3*cosh(1)^2 + b*c*d*x^3*sinh(1)^2 + b*c*d^2*x*cosh(1) +
(2*b*c*d*x^3*cosh(1) + b*c*d^2*x)*sinh(1))*sqrt((c^2*x^2 + 1)/(c^2*x^2)))*s
qrt(x^2*cosh(1) + x^2*sinh(1) + d))/(d^2*x^4*cosh(1)^4 + d^2*x^4*sinh(1)^4
- c^2*d^5*cosh(1) - (c^2*d^3*x^4 - 2*d^3*x^2)*cosh(1)^3 - (c^2*d^3*x^4 - 4*
d^2*x^4*cosh(1) - 2*d^3*x^2)*sinh(1)^3 - (2*c^2*d^4*x^2 - d^4)*cosh(1)^2 -
(2*c^2*d^4*x^2 - 6*d^2*x^4*cosh(1)^2 - d^4 + 3*(c^2*d^3*x^4 - 2*d^3*x^2)*co
sh(1))*sinh(1)^2 + (4*d^2*x^4*cosh(1)^3 - c^2*d^5 - 3*(c^2*d^3*x^4 - 2*d^3*
x^2)*cosh(1)^2 - 2*(2*c^2*d^4*x^2 - d^4)*cosh(1))*sinh(1))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x/(e*x^2 + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.159 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x(d+ex^2)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(5/2)),x]
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

Rubi steps

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A]
time = 38.23, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{csch}^{-1}(cx)}{x\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(5/2)),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*(d + e*x^2)^(5/2)), x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccsch}(cx)}{x\left(e\,x^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*arcsinh(sqrt(d)*e^(-1/2)/abs(x))/d^(5/2) - 3/(sqrt(x^2*e + d)*d^2
) - 1/((x^2*e + d)^(3/2)*d)) + b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c
*x))/((x^2*e + d)^(5/2)*x), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsch(c*x) + a)/(x^7*e^3 + 3*d*x^5*e^2 + 3*d^
2*x^3*e + d^3*x), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(5/2)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x\,\left(e\,x^2+d\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x*(d + e*x^2)^(5/2)), x)
```

3.160 $\int \frac{a+b\,\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

Rubi [A]
time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A]
time = 50.24, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^3\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(5/2)),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^3*(d + e*x^2)^(5/2)), x]
```

Maple [A]
time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsch}(cx)}{x^3\left(e\,x^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*(15*arcsinh(sqrt(d)*e^(-1/2)/abs(x))*e/d^(7/2) - 15*e/(sqrt(x^2*e + d
)*d^3) - 5*e/((x^2*e + d)^(3/2)*d^2) - 3/((x^2*e + d)^(3/2)*d*x^2)) + b*int
egrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/((x^2*e + d)^(5/2)*x^3), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsch(c*x) + a)/(x^9*e^3 + 3*d*x^7*e^2 + 3*d^
2*x^5*e + d^3*x^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**3/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^3/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/((e*x^2 + d)^(5/2)*x^3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x^3\,{\left(e\,x^2+d\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x^3*(d + e*x^2)^(5/2)), x)
```

3.161 $\int \frac{x^6\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}} dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{x^6\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Rubi [A]
time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^6\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Int[(x^6*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
[Out] Defer[Int][(x^6*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^6\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}} dx = \int \frac{x^6\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}} dx$$

Mathematica [A]
time = 8.53, size = 0, normalized size = 0.00

$$\int \frac{x^6\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^6*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
[Out] Integrate[(x^6*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2), x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^6(a+b\,\text{arccsch}(cx))}{(e\,x^2+d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(3*x^5*e^(-1)/(x^2*e + d)^(3/2) + 5*(3*x^2*e^(-1)/(x^2*e + d)^(3/2) + 2
*d*e^(-2)/(x^2*e + d)^(3/2))*d*x*e^(-1) - 15*d*arcsinh(x*e^(1/2)/sqrt(d))*e
^(-7/2) + 5*d*x*e^(-3)/sqrt(x^2*e + d))*a + b*integrate(x^6*log(sqrt(1/(c^2
*x^2) + 1) + 1/(c*x))/(x^2*e + d)^(5/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^6*arccsch(c*x) + a*x^6)*sqrt(x^2*e + d)/(x^6*e^3 + 3*d*x^4*e^
2 + 3*d^2*x^2*e + d^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^6/(e*x^2 + d)^(5/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^6\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^6*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.162 $\int \frac{x^4\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
[Out] Defer[Int][(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A]
time = 7.42, size = 0, normalized size = 0.00

$$\int \frac{x^4\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
[Out] Integrate[(x^4*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2), x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^4(a+b\,\mathrm{arccsch}(cx))}{(e\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*((3*x^2*e^(-1)/(x^2*e + d)^(3/2) + 2*d*e^(-2)/(x^2*e + d)^(3/2))*x - 3
*arcsinh(x*e^(1/2)/sqrt(d))*e^(-5/2) + x*e^(-2)/sqrt(x^2*e + d))*a + b*inte
grate(x^4*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(x^2*e + d)^(5/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccsch(c*x) + a*x^4)*sqrt(x^2*e + d)/(x^6*e^3 + 3*d*x^4*e^
2 + 3*d^2*x^2*e + d^3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*x^4/(e*x^2 + d)^(5/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^4\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^4*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.163 $\int \frac{x^2\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=359

$$\frac{bcx^2\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\,\sqrt{d+ex^2}}+\frac{bc^3x^2\sqrt{d+ex^2}}{3d\left(c^2d-e\right)e\sqrt{-c^2x^2}\,\sqrt{-1-c^2x^2}}+\frac{x^3\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bc^2x\sqrt{d}}{3d\left(c^2d-e\right)e}$$

```
[Out] 1/3*x^3*(a+b*arccsch(c*x))/d/(e*x^2+d)^(3/2)+1/3*b*c*x^2*(-c^2*x^2-1)^(1/2)
/d/(c^2*d-e)/(-c^2*x^2)^(1/2)/(e*x^2+d)^(1/2)+1/3*b*c^3*x^2*(e*x^2+d)^(1/2)
/d/(c^2*d-e)/e/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)-1/3*b*c^2*x*(1/(c^2*x^2+
1))^(1/2)*(c^2*x^2+1)^(1/2)*EllipticE(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/
2))*(e*x^2+d)^(1/2)/d/(c^2*d-e)/e/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x
^2+d)/d/(c^2*x^2+1))^(1/2)+1/3*b*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*
EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^2/(c^2
*d-e)/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {270, 6437, 12, 482, 433, 429, 506, 422}

$$\frac{x^3\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{bx\sqrt{d+ex^2}\,F\left(\mathrm{ArcTan}(cx)\left|1-\frac{e}{c^2d}\right.\right)}{3d^2\sqrt{-c^2x^2}\,\sqrt{-c^2x^2-1}\,\left(c^2d-e\right)\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}-\frac{bc^2x\sqrt{d+ex^2}\,E\left(\mathrm{ArcTan}(cx)\left|1-\frac{e}{c^2d}\right.\right)}{3de\sqrt{-c^2x^2}\,\sqrt{-c^2x^2-1}\,\left(c^2d-e\right)\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}+\frac{bcx^2\sqrt{-c^2x^2-1}}{3d\sqrt{-c^2x^2}\,\left(c^2d-e\right)\sqrt{d+ex^2}}+\frac{bc^3x^2\sqrt{d+ex^2}}{3de\sqrt{-c^2x^2}\,\sqrt{-c^2x^2-1}\,\left(c^2d-e\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*x^2*Sqrt[-1 - c^2*x^2])/(3*d*(c^2*d - e)*Sqrt[-(c^2*x^2)]*Sqrt[d + e*x
^2]) + (b*c^3*x^2*Sqrt[d + e*x^2])/(3*d*(c^2*d - e)*e*Sqrt[-(c^2*x^2)]*Sqrt
[-1 - c^2*x^2]) + (x^3*(a + b*ArcCsch[c*x]))/(3*d*(d + e*x^2)^(3/2)) - (b*c
^2*x*Sqrt[d + e*x^2]*EllipticE[ArcTan[c*x], 1 - e/(c^2*d)])/(3*d*(c^2*d - e
)*e*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^2))]
) + (b*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(3*d^2*(c^2
*d - e)*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^
2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
```

```
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int \frac{x^2\left(a+b\text{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} - \frac{(bcx)\int \frac{x^2}{3d\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} - \frac{(bcx)\int \frac{x^2}{\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}\,dx}{3d\sqrt{-c^2x^2}}$$

$$= \frac{bcx^2\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\,\sqrt{d+ex^2}} + \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{(bcx)\int \frac{\sqrt{-1-c^2x^2}}{\sqrt{d+ex^2}} \cdots}{3d\left(-c^2d+e\right)\sqrt{-c \cdots}}$$

$$= \frac{bcx^2\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\,\sqrt{d+ex^2}} + \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} - \frac{(bcx)\int \frac{1 \cdots}{\sqrt{-1-c^2x \cdots}}}{3d\left(-c^2d+e\right) \cdots}$$

$$= \frac{bcx^2\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\,\sqrt{d+ex^2}} + \frac{bc^3x^2\sqrt{d+ex^2}}{3d\left(c^2d-e\right)e\sqrt{-c^2x^2}\,\sqrt{-1-c^2x^2}} + \frac{x^3(a \cdots}{3 \cdots}$$

$$= \frac{bcx^2\sqrt{-1-c^2x^2}}{3d\left(c^2d-e\right)\sqrt{-c^2x^2}\,\sqrt{d+ex^2}} + \frac{bc^3x^2\sqrt{d+ex^2}}{3d\left(c^2d-e\right)e\sqrt{-c^2x^2}\,\sqrt{-1-c^2x^2}} + \frac{x^3(a \cdots}{3 \cdots}$$

Mathematica [A]
time = 0.19, size = 189, normalized size = 0.53

$$\frac{x^2\left(a(c^2d-e)\,x+bc\sqrt{1+\frac{1}{c^2x^2}}\,(d+ex^2)+b(c^2d-e)\,x\text{csch}^{-1}(cx)\right)}{3d\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}} - \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\,E\left(\text{ArcSin}\left(\sqrt{-\frac{e}{d}}\,x\right)|\frac{c^2d}{e}\right)}{3d\left(c^2d-e\right)\sqrt{-\frac{e}{d}}\,\sqrt{1+c^2x^2}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(5/2),x]
```

```
[Out] (x^2*(a*(c^2*d - e)*x + b*c*Sqrt[1 + 1/(c^2*x^2)]*(d + e*x^2) + b*(c^2*d -
e)*x*ArcCsch[c*x]))/(3*d*(c^2*d - e)*(d + e*x^2)^(3/2)) - (b*c*Sqrt[1 + 1/(
```

```
c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*EllipticE[ArcSin[Sqrt[-(e/d)]*x], (c^2*d)/e
])/(3*d*(c^2*d - e)*Sqrt[-(e/d)]*Sqrt[1 + c^2*x^2]*Sqrt[d + e*x^2])
```

Maple [**F**]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x^2(a + b\,\mathrm{arccsch}(cx))}{(e\,x^2 + d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(x*e^(-1)/(x^2*e + d)^(3/2) - x*e^(-1)/(sqrt(x^2*e + d)*d)) + b*inte
grate(x^2*log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(x^2*e + d)^(5/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^2/(e*x^2 + d)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2),x)
[Out] int((x^2*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(5/2), x)
```

3.164 $\int \frac{a+b\operatorname{csch}^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=278

$$\frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}-\frac{bc\sqrt{e}\,x\sqrt{-1-c^2x^2}\,E\left(\operatorname{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)|1-\frac{c^2d}{e}\right)}{3d^{3/2}\left(c^2d-e\right)\sqrt{-c^2x^2}\sqrt{\frac{d\left(1+c^2x^2\right)}{d+ex^2}}\sqrt{d+ex^2}}-\frac{b(3c^2d-}{3d^3\left(c^2d-\right.}$$

```
[Out] 1/3*x*(a+b*arccsch(c*x))/d/(e*x^2+d)^(3/2)+2/3*x*(a+b*arccsch(c*x))/d^2/(e*
x^2+d)^(1/2)-1/3*b*c*x*(1/(1+e*x^2/d))^(1/2)*(1+e*x^2/d)^(1/2)*EllipticE(x*
e^(1/2)/d^(1/2)/(1+e*x^2/d)^(1/2),(1-c^2*d/e)^(1/2))*e^(1/2)*(-c^2*x^2-1)^(
1/2)/d^(3/2)/(c^2*d-e)/(-c^2*x^2)^(1/2)/(d*(c^2*x^2+1)/(e*x^2+d))^(1/2)/(e*
x^2+d)^(1/2)-1/3*b*(3*c^2*d-2*e)*x*(1/(c^2*x^2+1))^(1/2)*(c^2*x^2+1)^(1/2)*
EllipticF(c*x/(c^2*x^2+1)^(1/2),(1-e/c^2/d)^(1/2))*(e*x^2+d)^(1/2)/d^3/(c^2
*d-e)/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)/((e*x^2+d)/d/(c^2*x^2+1))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {198, 197, 6427, 12, 539, 429, 422}

$$\frac{2x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\operatorname{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{bc\sqrt{e}\,x\sqrt{-c^2x^2-1}\,E\left(\operatorname{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)|1-\frac{c^2d}{e}\right)}{3d^{3/2}\sqrt{-c^2x^2}\left(c^2d-e\right)\sqrt{d+ex^2}\sqrt{\frac{d\left(c^2x^2+1\right)}{d+ex^2}}}-\frac{bx\left(3c^2d-2e\right)\sqrt{d+ex^2}\,F\left(\operatorname{ArcTan}(cx)|1-\frac{e}{c^2d}\right)}{3d^3\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}\left(c^2d-e\right)\sqrt{\frac{d+ex^2}{d\left(c^2x^2+1\right)}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCsch[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (x*(a + b*ArcCsch[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*ArcCsch[c*x]
))/(3*d^2*Sqrt[d + e*x^2]) - (b*c*Sqrt[e]*x*Sqrt[-1 - c^2*x^2]*EllipticE[Ar
cTan[(Sqrt[e]*x)/Sqrt[d]], 1 - (c^2*d)/e])/(3*d^(3/2)*(c^2*d - e)*Sqrt[-(c^
2*x^2)]*Sqrt[(d*(1 + c^2*x^2))/(d + e*x^2)]*Sqrt[d + e*x^2]) - (b*(3*c^2*d
- 2*e)*x*Sqrt[d + e*x^2]*EllipticF[ArcTan[c*x], 1 - e/(c^2*d)])/(3*d^3*(c^2
*d - e)*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2]*Sqrt[(d + e*x^2)/(d*(1 + c^2*x^
2))])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 198

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1],
0] && NeQ[p, -1]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 6427

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((d_.) + (e_.)*(x_)^2)^(p_.), x_Sym
bol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCsch[c*x], u, x
] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyIntegrand[u/(x*Sqrt[-1 - c^2
*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && (IGtQ[p, 0] || ILtQ[p +
 1/2, 0])
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{(d+ex^2)^{5/2}}\,dx = \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{3d+2ex^2}{3d^2\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}\,dx}{\sqrt{-c^2x^2}}$$

$$= \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{(bcx)\int \frac{3d+2ex^2}{\sqrt{-1-c^2x^2}\,(d+ex^2)^{3/2}}\,dx}{3d^2\sqrt{-c^2x^2}}$$

$$= \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{(bc(3c^2d-2e)\,x)\int \frac{1}{\sqrt{-1-c^2x^2}\,\sqrt{}}}{3d^2\left(c^2d-e\right)\sqrt{-c^2x^2}}$$

$$= \frac{x\left(a+b\text{csch}^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\text{csch}^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{bc\sqrt{e}\,x\sqrt{-1-c^2x^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.\right.}{3d^{3/2}\left(c^2d-e\right)\sqrt{-c^2x^2}\,\sqrt{\frac{d\left(1+c^2\right.}{d+ex}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.23, size = 248, normalized size = 0.89

$$\frac{x\left(-bce\sqrt{1+\frac{1}{c^2x^2}}\,x(d+ex^2)+a(c^2d-e)\left(3d+2ex^2\right)+b(c^2d-e)\left(3d+2ex^2\right)\text{csch}^{-1}(cx)\right)}{3d^2\left(c^2d-e\right)\left(d+ex^2\right)^{3/2}} - \frac{ibc\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{1+\frac{ex^2}{d}}\left(c^2dE\left(i\sinh^{-1}\left(\sqrt{c^2}\,x\right)|\frac{e}{c^2d}\right)+2(c^2d-e)\,F\left(i\sinh^{-1}\left(\sqrt{c^2}\,x\right)|\frac{e}{c^2d}\right)\right)}{3\sqrt{c^2}\,d^2\left(c^2d-e\right)\sqrt{1+c^2x^2}\,\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (x*(-(b*c*e*Sqrt[1 + 1/(c^2*x^2)]*x*(d + e*x^2)) + a*(c^2*d - e)*(3*d + 2*e
*x^2) + b*(c^2*d - e)*(3*d + 2*e*x^2)*ArcCsch[c*x]))/(3*d^2*(c^2*d - e)*(d
+ e*x^2)^(3/2)) - ((I/3)*b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 + (e*x^2)/d]*(c
^2*d*EllipticE[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)] + 2*(c^2*d - e)*EllipticF
[I*ArcSinh[Sqrt[c^2]*x], e/(c^2*d)]))/(Sqrt[c^2]*d^2*(c^2*d - e)*Sqrt[1 + c
^2*x^2]*Sqrt[d + e*x^2])
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsch}(cx)}{\left(e\,x^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] 1/3*a*(2*x/(sqrt(x^2*e + d)*d^2) + x/((x^2*e + d)^(3/2)*d)) + b*integrate(l
og(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(x^2*e + d)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(5/2),x)
[Out] int((a + b*asinh(1/(c*x)))/(d + e*x^2)^(5/2), x)
```

3.165 $\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

Optimal. Leaf size=596

$$\frac{be\left(e^2(15+8m+m^2)^2-3c^2de(3+m)^2\left(42+13m+m^2\right)+3c^4d^2(840+638m+179m^2+22m^3+m^4)\right)x}{c^5f(2+m)(3+m)(4+m)(5+m)(6+m)(7+m)\sqrt{-c^2x^2}}$$

```
[Out] d^3*(f*x)^(1+m)*(a+b*arccsch(c*x))/f/(1+m)+3*d^2*e*(f*x)^(3+m)*(a+b*arccsch
(c*x))/f^3/(3+m)+3*d*e^2*(f*x)^(5+m)*(a+b*arccsch(c*x))/f^5/(5+m)+e^3*(f*x)
^(7+m)*(a+b*arccsch(c*x))/f^7/(7+m)+b*e*(e^2*(m^2+8*m+15)^2-3*c^2*d*e*(3+m)
^2*(m^2+13*m+42)+3*c^4*d^2*(m^4+22*m^3+179*m^2+638*m+840))*x*(f*x)^(1+m)*(-
c^2*x^2-1)^(1/2)/c^5/f/(6+m)/(m^2+6*m+8)/(m^3+15*m^2+71*m+105)/(-c^2*x^2)^(
1/2)-b*e^2*(e*(5+m)^2-3*c^2*d*(m^2+13*m+42))*x*(f*x)^(3+m)*(-c^2*x^2-1)^(1/
2)/c^3/f^3/(4+m)/(6+m)/(m^2+12*m+35)/(-c^2*x^2)^(1/2)+b*e^3*x*(f*x)^(5+m)*(
-c^2*x^2-1)^(1/2)/c/f^5/(6+m)/(7+m)/(-c^2*x^2)^(1/2)-b*(c^6*d^3*(2+m)*(4+m)
*(6+m)/(1+m)-e*(1+m)*(e^2*(m^2+8*m+15)^2-3*c^2*d*e*(3+m)^2*(m^2+13*m+42)+3*
c^4*d^2*(m^4+22*m^3+179*m^2+638*m+840))/(m^3+15*m^2+71*m+105))*x*(f*x)^(1+m
)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/c^5/f/
(1+m)/(2+m)/(4+m)/(6+m)/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)
```

Rubi [A]
time = 1.59, antiderivative size = 577, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {276, 6437, 1823, 1281, 470, 372, 371}

$$\frac{d^3(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(a+b\text{csch}^{-1}(cx)\right)}{f^7(m+7)}+\frac{be^3x\sqrt{-c^2x^2-1}(fx)^{m+5}}{cf^5(m+6)(m+7)\sqrt{-c^2x^2}}-\frac{be^2x\sqrt{-c^2x^2-1}(fx)^{m+3}\left(e(m+5)^2-3c^2d(m^2+13m+42)\right)}{c^3f^3(m+4)(m+5)(m+6)(m+7)\sqrt{-c^2x^2}}-\frac{bcx\sqrt{c^2x^2+1}(fx)^{m+1}\left(\frac{d^3}{(m+1)^2}-\frac{e\left(3c^4d^2(m^4+22m^3+179m^2+638m+840)-3c^2de(m+3)^2(m^2+13m+42)+e^2(m^2+8m+15)^2\right)}{c^6(m+2)(m+3)(m+4)(m+5)(m+6)(m+7)}\right){}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)}{f\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}+\frac{bex\sqrt{-c^2x^2-1}(fx)^{m+1}\left(3c^4d^2(m^4+22m^3+179m^2+638m+840)-3c^2de(m+3)^2(m^2+13m+42)+e^2(m^2+8m+15)^2\right)}{c^5f(m+2)(m+3)(m+4)(m+5)(m+6)(m+7)\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*e*(e^2*(15 + 8*m + m^2)^2 - 3*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^
4*d^2*(840 + 638*m + 179*m^2 + 22*m^3 + m^4))*x*(f*x)^(1 + m)*Sqrt[-1 - c^2
*x^2])/(c^5*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[-(c^2*x^
2)]) - (b*e^2*(e*(5 + m)^2 - 3*c^2*d*(42 + 13*m + m^2))*x*(f*x)^(3 + m)*Sqr
t[-1 - c^2*x^2])/(c^3*f^3*(4 + m)*(5 + m)*(6 + m)*(7 + m)*Sqrt[-(c^2*x^2)])
 + (b*e^3*x*(f*x)^(5 + m)*Sqrt[-1 - c^2*x^2])/(c*f^5*(6 + m)*(7 + m)*Sqrt[-
(c^2*x^2)]) + (d^3*(f*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(f*(1 + m)) + (3*d^2
*e*(f*x)^(3 + m)*(a + b*ArcCsch[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 +
m)*(a + b*ArcCsch[c*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcCsch[
c*x]))/(f^7*(7 + m)) - (b*c*(d^3/(1 + m)^2 - (e*(e^2*(15 + 8*m + m^2)^2 - 3
*c^2*d*e*(3 + m)^2*(42 + 13*m + m^2) + 3*c^4*d^2*(840 + 638*m + 179*m^2 + 2
2*m^3 + m^4)))/(c^6*(2 + m)*(3 + m)*(4 + m)*(5 + m)*(6 + m)*(7 + m)))*x*(f*
x)^(1 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -
(c^2*x^2)])/(f*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1823

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
```

```
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{d^3(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{3d^2e(fx)^{3+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(3+m)}$$

$$= \frac{be^3x(fx)^{5+m}\sqrt{-1-c^2x^2}}{cf^5(6+m)(7+m)\sqrt{-c^2x^2}} + \frac{d^3(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)}$$

$$= -\frac{be^2\left(e(5+m)^2-3c^2d\left(42+13m+m^2\right)\right)x(fx)^{3+m}\sqrt{-1-c^2x^2}}{c^3f^3(4+m)(5+m)(6+m)(7+m)\sqrt{-c^2x^2}}$$

$$= \frac{be\left(e^2\left(15+8m+m^2\right)^2-3c^2de(3+m)^2\left(42+13m+m^2\right)+3\right.}{c^5f(2+m)(3+m)(4+m)(}$$

$$= \frac{be\left(e^2\left(15+8m+m^2\right)^2-3c^2de(3+m)^2\left(42+13m+m^2\right)+3\right.}{c^5f(2+m)(3+m)(4+m)(}$$

$$= \frac{be\left(e^2\left(15+8m+m^2\right)^2-3c^2de(3+m)^2\left(42+13m+m^2\right)+3\right.}{c^5f(2+m)(3+m)(4+m)(}$$

Mathematica [F]
time = 0.09, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCsch[c*x]), x]
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int (fx)^{m} \left(e\,x^{2}+d\right)^{3} \left(a+b\,\mathrm{arccsch}(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] a*f^m*x^7*e^(m*log(x) + 3)/(m + 7) + 3*a*d*f^m*x^5*e^(m*log(x) + 2)/(m + 5)
 + 3*a*d^2*f^m*x^3*e^(m*log(x) + 1)/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1
)) - (((m^3 + 9*m^2 + 23*m + 15)*b*f^m*x^7*e^3 + 3*(m^3 + 11*m^2 + 31*m + 2
1)*b*d*f^m*x^5*e^2 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*d^2*f^m*x^3*e + (m^3 +
15*m^2 + 71*m + 105)*b*d^3*f^m*x)*x^m*log(x) - ((m^3 + 9*m^2 + 23*m + 15)*b
*f^m*x^7*e^3 + 3*(m^3 + 11*m^2 + 31*m + 21)*b*d*f^m*x^5*e^2 + 3*(m^3 + 13*m
^2 + 47*m + 35)*b*d^2*f^m*x^3*e + (m^3 + 15*m^2 + 71*m + 105)*b*d^3*f^m*x)*
x^m*log(sqrt(c^2*x^2 + 1) + 1))/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105) + int
egrate(((m^3 + 9*m^2 + 23*m + 15)*b*c^2*f^m*x^8*e^3 + 3*(m^3 + 11*m^2 + 31*
m + 21)*b*c^2*d*f^m*x^6*e^2 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^2*d^2*f^m*x^
4*e + (m^3 + 15*m^2 + 71*m + 105)*b*c^2*d^3*f^m*x^2)*x^m/((m^4 + 16*m^3 + 8
6*m^2 + 176*m + 105)*c^2*x^2 + m^4 + 16*m^3 + 86*m^2 + ((m^4 + 16*m^3 + 86*
m^2 + 176*m + 105)*c^2*x^2 + m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*sqrt(c^2*
x^2 + 1) + 176*m + 105), x) - integrate(((m^4 + 16*m^3 + 86*m^2 + 176*m + 1
05)*b*c^2*f^m*x^8*e^3*log(c) + (3*(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*b*c
^2*d*f^m*e^2*log(c) + ((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*f^m*log(c) - (
m^3 + 9*m^2 + 23*m + 15)*f^m)*b*e^3)*x^6 + 3*((m^4 + 16*m^3 + 86*m^2 + 176*
m + 105)*b*c^2*d^2*f^m*e*log(c) + ((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*d*
f^m*log(c) - (m^3 + 11*m^2 + 31*m + 21)*d*f^m)*b*e^2)*x^4 + ((m^4 + 16*m^3
+ 86*m^2 + 176*m + 105)*b*c^2*d^3*f^m*log(c) + 3*((m^4 + 16*m^3 + 86*m^2 +
176*m + 105)*d^2*f^m*log(c) - (m^3 + 13*m^2 + 47*m + 35)*d^2*f^m)*b*e)*x^2
+ ((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*d^3*f^m*log(c) - (m^3 + 15*m^2 + 7
```

```
1*m + 105)*d^3*f^m)*b)*x^m/((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^2*x^2 +
 m^4 + 16*m^3 + 86*m^2 + 176*m + 105), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral((a*x^6*e^3 + 3*a*d*x^4*e^2 + 3*a*d^2*x^2*e + a*d^3 + (b*x^6*e^3 +
3*b*d*x^4*e^2 + 3*b*d^2*x^2*e + b*d^3)*arccsch(c*x))*(f*x)^m, x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**3*(a+b*acsch(c*x)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^3*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^m\,\left(e\,x^2+d\right)^3\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)^3*(a + b*asinh(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^3*(a + b*asinh(1/(c*x))), x)
```

3.166 $\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right) \, dx$

Optimal. Leaf size=379

$$-\frac{be\left(e(3+m)^2-2c^2d\left(20+9m+m^2\right)\right)x(fx)^{1+m}\sqrt{-1-c^2x^2}}{c^3f(2+m)(3+m)(4+m)(5+m)\sqrt{-c^2x^2}}+\frac{be^2x(fx)^{3+m}\sqrt{-1-c^2x^2}}{cf^3(4+m)(5+m)\sqrt{-c^2x^2}}+\frac{d^2(fx)^{1+m}\,(\ldots}{f(\ldots}$$

```
[Out] d^2*(f*x)^(1+m)*(a+b*arccsch(c*x))/f/(1+m)+2*d*e*(f*x)^(3+m)*(a+b*arccsch(c
*x))/f^3/(3+m)+e^2*(f*x)^(5+m)*(a+b*arccsch(c*x))/f^5/(5+m)-b*e*(e*(3+m)^2-
2*c^2*d*(m^2+9*m+20))*x*(f*x)^(1+m)*(-c^2*x^2-1)^(1/2)/c^3/f/(4+m)/(5+m)/(m
^2+5*m+6)/(-c^2*x^2)^(1/2)+b*e^2*x*(f*x)^(3+m)*(-c^2*x^2-1)^(1/2)/c/f^3/(4+
m)/(5+m)/(-c^2*x^2)^(1/2)-b*(c^4*d^2*(2+m)*(3+m)*(4+m)*(5+m)+e*(1+m)^2*(e*(
3+m)^2-2*c^2*d*(m^2+9*m+20)))*x*(f*x)^(1+m)*hypergeom([1/2, 1/2+1/2*m],[3/2
+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/c^3/f/(1+m)^2/(2+m)/(3+m)/(4+m)/(5+m)/(
-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 360, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {276, 6437, 12, 1281, 470, 372, 371}

$$\frac{d^2(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\text{csch}^{-1}(cx)\right)}{f^5(m+5)}+\frac{be^2x\sqrt{-c^2x^2-1}\,(fx)^{m+3}}{cf^3(m+4)(m+5)\sqrt{-c^2x^2}}-\frac{bcx\sqrt{c^2x^2+1}\,(fx)^{m+1}\left(\frac{e\left(e(m+3)^2-2c^2d\left(m^2+9m+20\right)\right)}{c^4(m+2)(m+3)(m+4)(m+5)}+\frac{d^2}{(m+1)^2}\right){}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)}{f\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}-\frac{bex\sqrt{-c^2x^2-1}\,(fx)^{m+1}\left(e(m+3)^2-2c^2d\left(m^2+9m+20\right)\right)}{c^3f(m+2)(m+3)(m+4)(m+5)\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
```

```
[Out] -((b*e*(e*(3 + m)^2 - 2*c^2*d*(20 + 9*m + m^2))*x*(f*x)^(1 + m)*Sqrt[-1 - c
^2*x^2])/(c^3*f*(2 + m)*(3 + m)*(4 + m)*(5 + m)*Sqrt[-(c^2*x^2)])) + (b*e^2
*x*(f*x)^(3 + m)*Sqrt[-1 - c^2*x^2])/(c*f^3*(4 + m)*(5 + m)*Sqrt[-(c^2*x^2)
]) + (d^2*(f*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3
 + m)*(a + b*ArcCsch[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcCs
ch[c*x]))/(f^5*(5 + m)) - (b*c*(d^2/(1 + m)^2 + (e*(e*(3 + m)^2 - 2*c^2*d*(
20 + 9*m + m^2)))/(c^4*(2 + m)*(3 + m)*(4 + m)*(5 + m)))*x*(f*x)^(1 + m)*Sq
rt[1 + c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, -(c^2*x^2)])/(
f*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{d^2(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(3+m)}$$

$$= \frac{d^2(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(3+m)}$$

$$= \frac{be^2x(fx)^{3+m}\sqrt{-1-c^2x^2}}{cf^3(4+m)(5+m)\sqrt{-c^2x^2}} + \frac{d^2(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} +$$

$$= -\frac{be(e(3+m)^2-2c^2d(20+9m+m^2))\,x(fx)^{1+m}\sqrt{-1-c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{-c^2x^2}}$$

$$= -\frac{be(e(3+m)^2-2c^2d(20+9m+m^2))\,x(fx)^{1+m}\sqrt{-1-c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{-c^2x^2}}$$

$$= -\frac{be(e(3+m)^2-2c^2d(20+9m+m^2))\,x(fx)^{1+m}\sqrt{-1-c^2x^2}}{c^3f(2+m)(4+m)\left(15+8m+m^2\right)\sqrt{-c^2x^2}}$$

Mathematica [F]
time = 0.07, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\text{csch}^{-1}(cx)\right)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsch[c*x]),x]
[Out] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCsch[c*x]), x]
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int (fx)^m \left(e\,x^2+d\right)^2 \left(a+b\,\text{arccsch}(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x)
[Out] int((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] a*f^m*x^5*e^(m*log(x) + 2)/(m + 5) + 2*a*d*f^m*x^3*e^(m*log(x) + 1)/(m + 3)
 + (f*x)^(m + 1)*a*d^2/(f*(m + 1)) - (((m^2 + 4*m + 3)*b*f^m*x^5*e^2 + 2*(m
^2 + 6*m + 5)*b*d*f^m*x^3*e + (m^2 + 8*m + 15)*b*d^2*f^m*x)*x^m*log(x) - ((
m^2 + 4*m + 3)*b*f^m*x^5*e^2 + 2*(m^2 + 6*m + 5)*b*d*f^m*x^3*e + (m^2 + 8*m
 + 15)*b*d^2*f^m*x)*x^m*log(sqrt(c^2*x^2 + 1) + 1))/(m^3 + 9*m^2 + 23*m + 1
5) + integrate(((m^2 + 4*m + 3)*b*c^2*f^m*x^6*e^2 + 2*(m^2 + 6*m + 5)*b*c^2
*d*f^m*x^4*e + (m^2 + 8*m + 15)*b*c^2*d^2*f^m*x^2)*x^m/((m^3 + 9*m^2 + 23*m
 + 15)*c^2*x^2 + m^3 + 9*m^2 + ((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 + m^3 + 9
*m^2 + 23*m + 15)*sqrt(c^2*x^2 + 1) + 23*m + 15), x) - integrate(((m^3 + 9*
m^2 + 23*m + 15)*b*c^2*f^m*x^6*e^2*log(c) + (2*(m^3 + 9*m^2 + 23*m + 15)*b*
c^2*d*f^m*e*log(c) + ((m^3 + 9*m^2 + 23*m + 15)*f^m*log(c) - (m^2 + 4*m + 3
)*f^m)*b*e^2)*x^4 + ((m^3 + 9*m^2 + 23*m + 15)*b*c^2*d^2*f^m*log(c) + 2*((m
^3 + 9*m^2 + 23*m + 15)*d*f^m*log(c) - (m^2 + 6*m + 5)*d*f^m)*b*e)*x^2 + ((
m^3 + 9*m^2 + 23*m + 15)*d^2*f^m*log(c) - (m^2 + 8*m + 15)*d^2*f^m)*b)*x^m/
((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 + m^3 + 9*m^2 + 23*m + 15), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*x^4*e^2 + 2*a*d*x^2*e + a*d^2 + (b*x^4*e^2 + 2*b*d*x^2*e + b*d^
2)*arccsch(c*x))*(f*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m (a + b\,\mathrm{acsch}\,(cx)) \left(d + ex^2\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**2*(a+b*acsch(c*x)),x)
```

```
[Out] Integral((f*x)**m*(a + b*acsch(c*x))*(d + e*x**2)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^2*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^m\,{\left(e\,x^2+d\right)}^2\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)^2*(a + b*asinh(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^2*(a + b*asinh(1/(c*x))), x)
```

3.167 $\int (fx)^m (d+ex^2) \left(a+b\text{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=220

$$\frac{bex(fx)^{1+m}\sqrt{-1-c^2x^2}}{cf(6+5m+m^2)\sqrt{-c^2x^2}}+\frac{d(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)}+\frac{e(fx)^{3+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(3+m)}+\frac{b(e(1+m)^2-c^2}{c}$$

```
[Out] d*(f*x)^(1+m)*(a+b*arccsch(c*x))/f/(1+m)+e*(f*x)^(3+m)*(a+b*arccsch(c*x))/f
^3/(3+m)+b*e*x*(f*x)^(1+m)*(-c^2*x^2-1)^(1/2)/c/f/(m^2+5*m+6)/(-c^2*x^2)^(1
/2)+b*(e*(1+m)^2-c^2*d*(2+m)*(3+m))*x*(f*x)^(1+m)*hypergeom([1/2, 1/2+1/2*m
],[3/2+1/2*m],-c^2*x^2)*(c^2*x^2+1)^(1/2)/c/f/(1+m)^2/(2+m)/(3+m)/(-c^2*x^2
)^(1/2)/(-c^2*x^2-1)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 208, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {14, 6437, 12, 470, 372, 371}

$$\frac{d(fx)^{m+1}\left(a+b\text{csch}^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(m+3)}-\frac{bcx\sqrt{c^2x^2+1}(fx)^{m+1}\left(\frac{d}{(m+1)^2}-\frac{e}{c^2(m+2)(m+3)}\right){}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};-c^2x^2\right)}{f\sqrt{-c^2x^2}\sqrt{-c^2x^2-1}}+\frac{bex\sqrt{-c^2x^2-1}(fx)^{m+1}}{cf(m^2+5m+6)\sqrt{-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] (b*e*x*(f*x)^(1 + m)*Sqrt[-1 - c^2*x^2])/(c*f*(6 + 5*m + m^2)*Sqrt[-(c^2*x^
2)]) + (d*(f*x)^(1 + m)*(a + b*ArcCsch[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m
)*(a + b*ArcCsch[c*x]))/(f^3*(3 + m)) - (b*c*(d/(1 + m)^2 - e/(c^2*(2 + m)*
(3 + m)))*x*(f*x)^(1 + m)*Sqrt[1 + c^2*x^2]*Hypergeometric2F1[1/2, (1 + m)/
2, (3 + m)/2, -(c^2*x^2)])/(f*Sqrt[-(c^2*x^2)]*Sqrt[-1 - c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 6437

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(
x_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Di
st[a + b*ArcCsch[c*x], u, x] - Dist[b*c*(x/Sqrt[(-c^2)*x^2]), Int[SimplifyI
ntegrand[u/(x*Sqrt[-1 - c^2*x^2]), x], x], x]] /; FreeQ[{a, b, c, d, e, f,
m, p}, x] && ((IGtQ[p, 0] &&  !(ILtQ[(m - 1)/2, 0] && GtQ[m + 2*p + 3, 0]))
 || (IGtQ[(m + 1)/2, 0] &&  !(ILtQ[p, 0] && GtQ[m + 2*p + 3, 0])) || (ILtQ[
(m + 2*p + 1)/2, 0] &&  !ILtQ[(m - 1)/2, 0]))
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)\left(a+b\text{csch}^{-1}(cx)\right)\,dx = \frac{d(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(3+m)} - \frac{(b}{}$$

$$= \frac{d(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f^3(3+m)} - \frac{(b}{}$$

$$= \frac{bex(fx)^{1+m}\sqrt{-1-c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{-c^2x^2}} + \frac{d(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{e(}{}$$

$$= \frac{bex(fx)^{1+m}\sqrt{-1-c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{-c^2x^2}} + \frac{d(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{e(}{}$$

$$= \frac{bex(fx)^{1+m}\sqrt{-1-c^2x^2}}{cf\left(6+5m+m^2\right)\sqrt{-c^2x^2}} + \frac{d(fx)^{1+m}\left(a+b\text{csch}^{-1}(cx)\right)}{f(1+m)} + \frac{e(}{}$$

Mathematica [F]
time = 0.06, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right) \left(a+b\text{csch}^{-1}(cx)\right) \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCsch[c*x]),x]
```

```
[Out] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCsch[c*x]), x]
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int (fx)^m \left(e\,x^2+d\right) \left(a+b\,\text{arccsch}(cx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="maxima")
```

```
[Out] a*f^m*x^3*e^(m*log(x) + 1)/(m + 3) + (f*x)^(m + 1)*a*d/(f*(m + 1)) - ((b*f^
m*(m + 1)*x^3*e + b*d*f^m*(m + 3)*x)*x^m*log(x) - (b*f^m*(m + 1)*x^3*e + b*
d*f^m*(m + 3)*x)*x^m*log(sqrt(c^2*x^2 + 1) + 1))/(m^2 + 4*m + 3) + integrat
e((b*c^2*f^m*(m + 1)*x^4*e + b*c^2*d*f^m*(m + 3)*x^2)*x^m/((m^2 + 4*m + 3)*
c^2*x^2 + m^2 + ((m^2 + 4*m + 3)*c^2*x^2 + m^2 + 4*m + 3)*sqrt(c^2*x^2 + 1)
 + 4*m + 3), x) - integrate(((m^2 + 4*m + 3)*b*c^2*f^m*x^4*e*log(c) + ((m^2
 + 4*m + 3)*b*c^2*d*f^m*log(c) + ((m^2 + 4*m + 3)*f^m*log(c) - f^m*(m + 1))
*b*e)*x^2 + ((m^2 + 4*m + 3)*d*f^m*log(c) - d*f^m*(m + 3))*b)*x^m/((m^2 + 4
*m + 3)*c^2*x^2 + m^2 + 4*m + 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccsch(c*x))*(f*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m (a + b\,\text{acsch}\,(cx)) \left(d + ex^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(a+b*acsch(c*x)),x)
[Out] Integral((f*x)**m*(a + b*acsch(c*x))*(d + e*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^m\,\left(e\,x^2+d\right)\,\left(a+b\,\text{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*asinh(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)*(a + b*asinh(1/(c*x))), x)
```

3.168 $\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx=\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$$

Mathematica [A]
time = 1.23, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{d+ex^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2), x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\text{arccsch}(cx)\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

```
[Out] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/(x^2*e + d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)*(f*x)^m/(x^2*e + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b\operatorname{acsch}(cx))}{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsch(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((f*x)**m*(a + b*acsch(c*x))/(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(f\,x)^m\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{e\,x^2+d} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2),x)
[Out] int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2), x)
```

3.169 $\int \frac{(fx)^m\left(a+b\mathbf{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{(fx)^m\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(fx)^m\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Mathematica [A]
time = 4.46, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\mathrm{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2,x]
[Out] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^2, x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\mathrm{arccsch}(cx)\right)}{\left(e\,x^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/(x^2*e + d)^2, x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)*(f*x)^m/(x^4*e^2 + 2*d*x^2*e + d^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsch(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(f\,x)^m\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2,x)
[Out] int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2)^2, x)
```

3.170 $\int (fx)^m (d+ex^2)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=28

$$\text{Int}\left((fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right), x\right)$$

```
[Out] Unintegrable((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
[Out] Defer[Int][(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx = \int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx$$

Mathematica [A]
time = 0.66, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right)^{3/2} \left(a+b\text{csch}^{-1}(cx)\right) dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]),x]
[Out] Integrate[(f*x)^m*(d + e*x^2)^(3/2)*(a + b*ArcCsch[c*x]), x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int (fx)^m \left(e\,x^2+d\right)^{\frac{3}{2}} \left(a+b\,\text{arccsch}(cx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
[Out] int((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="maxima")
[Out] integrate((x^2*e + d)^(3/2)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="fricas")
[Out] integral((a*x^2*e + a*d + (b*x^2*e + b*d)*arccsch(c*x))*sqrt(x^2*e + d)*(f*
x)^m, x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**(3/2)*(a+b*acsch(c*x)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^(3/2)*(a+b*arccsch(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)^(3/2)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (f\,x)^m\,{\left(e\,x^2+d\right)}^{3/2}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^(3/2)*(a + b*asinh(1/(c*x))), x)
```

3.171 $\int (fx)^m \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right) dx$

Optimal. Leaf size=28

$$\text{Int}\left((fx)^m \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (fx)^m \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right) dx$$

Verification is not applicable to the result.

```
[In] Int[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
[Out] Defer[Int][(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Rubi steps

$$\int (fx)^m \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right) dx = \int (fx)^m \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right) dx$$

Mathematica [A]
time = 0.06, size = 0, normalized size = 0.00

$$\int (fx)^m \sqrt{d+ex^2} \left(a+b\text{csch}^{-1}(cx)\right) dx$$

Verification is not applicable to the result.

```
[In] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]),x]
[Out] Integrate[(f*x)^m*Sqrt[d + e*x^2]*(a + b*ArcCsch[c*x]), x]
```

Maple [A]
time = 0.15, size = 0, normalized size = 0.00

$$\int (fx)^m (a+b\,\text{arccsch}(cx)) \sqrt{e\,x^2+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2*e + d)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(a + b\operatorname{acsch}(cx)\right)\sqrt{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsch(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acsch(c*x))*sqrt(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)*(b*arccsch(c*x) + a)*(f*x)^m, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (f\,x)^m\,\sqrt{e\,x^2+d}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))),x)
[Out] int((f*x)^m*(d + e*x^2)^(1/2)*(a + b*asinh(1/(c*x))), x)
```

3.172 $\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx=\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A]
time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{d+ex^2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2],x]
[Out] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/Sqrt[d + e*x^2], x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\text{arccsch}(cx)\right)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/sqrt(x^2*e + d), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*arccsch(c*x) + a)*(f*x)^m/sqrt(x^2*e + d), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\,(a+b\operatorname{acsch}{(cx)})}{\sqrt{d+ex^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsch(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acsch(c*x))/sqrt(d + e*x**2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/sqrt(e*x^2 + d), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(f\,x)^m\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2),x)
[Out] int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(1/2), x)
```

3.173 $\int \frac{(fx)^m\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=28

$$\operatorname{Int}\left(\frac{(fx)^m\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{(fx)^m\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A]
time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcCsch[c*x]))/(d + e*x^2)^(3/2), x]
```

Maple [A]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\,\mathrm{arccsch}(cx)\right)}{\left(e\,x^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/(x^2*e + d)^(3/2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2*e + d)*(b*arccsch(c*x) + a)*(f*x)^m/(x^4*e^2 + 2*d*x^2*e
+ d^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{acsch}(cx))}{(d + ex^2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acsch(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acsch(c*x))/(d + e*x**2)**(3/2), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccsch(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)*(f*x)^m/(e*x^2 + d)^(3/2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(f\,x)^m\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2),x)
[Out] int(((f*x)^m*(a + b*asinh(1/(c*x))))/(d + e*x^2)^(3/2), x)
```

3.174 $\int \frac{x^{11}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=395

$$-\frac{4b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{7b(1-c^2x^2)^{3/2}\,\sqrt{1+c^2x^2}}{90c^{13}\sqrt{1+\frac{1}{c^2x^2}}\,x}-\frac{13b(1-c^2x^2)^{5/2}\,\sqrt{1+c^2x^2}}{150c^{13}\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{3b(1-c^2x^2)^{7/2}\,\sqrt{1}}{70c^{13}\sqrt{1+\frac{1}{c^2}}}$$

```
[Out] 1/3*(-c^4*x^4+1)^(3/2)*(a+b*arccsch(c*x))/c^12-1/10*(-c^4*x^4+1)^(5/2)*(a+b
*arccsch(c*x))/c^12+7/90*b*(-c^2*x^2+1)^(3/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1+1
/c^2/x^2)^(1/2)-13/150*b*(-c^2*x^2+1)^(5/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1+1/c
^2/x^2)^(1/2)+3/70*b*(-c^2*x^2+1)^(7/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1+1/c^2/x
^2)^(1/2)-1/90*b*(-c^2*x^2+1)^(9/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1+1/c^2/x^2)^
(1/2)+4/15*b*arctanh((-c^2*x^2+1)^(1/2))*(c^2*x^2+1)^(1/2)/c^13/x/(1+1/c^2/
x^2)^(1/2)-4/15*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)^(1/2)/c^13/x/(1+1/c^2/x^2)
^(1/2)-1/2*(a+b*arccsch(c*x))*(-c^4*x^4+1)^(1/2)/c^12
```

Rubi [A]

time = 1.53, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {272, 45, 6445, 12, 6853, 6874, 862, 52, 65, 214, 797}

$$-\frac{(1-c^4x^4)^{5/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{10c^{12}}+\frac{(1-c^4x^4)^{3/2}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{3c^{12}}-\frac{\sqrt{1-c^4x^4}\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{2c^{12}}-\frac{b\sqrt{c^2x^2+1}\,(1-c^2x^2)^{9/2}}{90c^{13}x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{3b\sqrt{c^2x^2+1}\,(1-c^2x^2)^{7/2}}{70c^{13}x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{13b\sqrt{c^2x^2+1}\,(1-c^2x^2)^{5/2}}{150c^{13}x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{7b\sqrt{c^2x^2+1}\,(1-c^2x^2)^{3/2}}{90c^{13}x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{4b\sqrt{c^2x^2+1}\,\sqrt{1-c^2x^2}}{15c^{13}x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{4b\sqrt{c^2x^2+1}\,\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{15c^{13}x\sqrt{\frac{1}{c^2x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] (-4*b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(15*c^13*Sqrt[1 + 1/(c^2*x^2)]*x
) + (7*b*(1 - c^2*x^2)^(3/2)*Sqrt[1 + c^2*x^2])/(90*c^13*Sqrt[1 + 1/(c^2*x^
2)]*x) - (13*b*(1 - c^2*x^2)^(5/2)*Sqrt[1 + c^2*x^2])/(150*c^13*Sqrt[1 + 1/
(c^2*x^2)]*x) + (3*b*(1 - c^2*x^2)^(7/2)*Sqrt[1 + c^2*x^2])/(70*c^13*Sqrt[1
 + 1/(c^2*x^2)]*x) - (b*(1 - c^2*x^2)^(9/2)*Sqrt[1 + c^2*x^2])/(90*c^13*Sqr
t[1 + 1/(c^2*x^2)]*x) - (Sqrt[1 - c^4*x^4]*(a + b*ArcCsch[c*x]))/(2*c^12) +
 ((1 - c^4*x^4)^(3/2)*(a + b*ArcCsch[c*x]))/(3*c^12) - ((1 - c^4*x^4)^(5/2)
*(a + b*ArcCsch[c*x]))/(10*c^12) + (4*b*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 -
c^2*x^2]])/(15*c^13*Sqrt[1 + 1/(c^2*x^2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 797

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c/e)*x)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^{11}\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1-c^4x^4}\,\left(a+b\text{csch}^{-1}(cx)\right)}{2c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^{12}}-\frac{(1-c^4x}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\,\left(a+b\text{csch}^{-1}(cx)\right)}{2c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^{12}}-\frac{(1-c^4x}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\,\left(a+b\text{csch}^{-1}(cx)\right)}{2c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^{12}}-\frac{(1-c^4x}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\,\left(a+b\text{csch}^{-1}(cx)\right)}{2c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^{12}}-\frac{(1-c^4x}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\,\left(a+b\text{csch}^{-1}(cx)\right)}{2c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^{12}}-\frac{(1-c^4x}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\,\left(a+b\text{csch}^{-1}(cx)\right)}{2c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^{12}}-\frac{(1-c^4x}{}$$

$$= -\frac{\sqrt{1-c^4x^4}\,\left(a+b\text{csch}^{-1}(cx)\right)}{2c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{3c^{12}}-\frac{(1-c^4x}{}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1+\frac{1}{c^2x^2}}\,x}-\frac{\sqrt{1-c^4x^4}\,\left(a+b\text{csch}^{-1}(cx)\right)}{2c^{12}}+\frac{\left(1-c^4x^4\right)^{3/2}\left(a\right.}{3}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{7b\left(1-c^2x^2\right)^{3/2}\sqrt{1+c^2x^2}}{90c^{13}\sqrt{1+\frac{1}{c^2x^2}}\,x}-\frac{13b\left(1-c^2x^2\right)^{5/2}}{150c^{13}\sqrt{1+}}$$

$$= -\frac{4b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{15c^{13}\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{7b\left(1-c^2x^2\right)^{3/2}\sqrt{1+c^2x^2}}{90c^{13}\sqrt{1+\frac{1}{c^2x^2}}\,x}-\frac{13b\left(1-c^2x^2\right)^{5/2}}{150c^{13}\sqrt{1+}}$$

Mathematica [A]
time = 0.19, size = 214, normalized size = 0.54

$$-\frac{105a\sqrt{1-c^4x^4}\,(8+4c^4x^4+3c^8x^8)+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{1-c^4x^4}\,(768-36c^2x^2+78c^4x^4-5c^6x^6+35c^8x^8)}{1+c^2x^2}+105b\sqrt{1-c^4x^4}\,(8+4c^4x^4+3c^8x^8)\,\mathrm{csch}^{-1}(cx)+840b\log(x+c^2x^3)-840b\log\left(1+c^2x^2+c\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{1-c^4x^4}\right)}{3150c^{12}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/3150*(105*a*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 + 3*c^8*x^8) + (b*c*Sqrt[1
+ 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]*(768 - 36*c^2*x^2 + 78*c^4*x^4 - 5*c^6*x
^6 + 35*c^8*x^8))/(1 + c^2*x^2) + 105*b*Sqrt[1 - c^4*x^4]*(8 + 4*c^4*x^4 +
3*c^8*x^8)*ArcCsch[c*x] + 840*b*Log[x + c^2*x^3] - 840*b*Log[1 + c^2*x^2 +
c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]])/c^12
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{x^{11}(a+b\,\mathrm{arccsch}(cx))}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/30*a*(3*(-c^4*x^4 + 1)^(5/2)/c^12 - 10*(-c^4*x^4 + 1)^(3/2)/c^12 + 15*sq
rt(-c^4*x^4 + 1)/c^12) + 1/30*b*((3*c^12*x^12 + c^8*x^8 + 4*c^4*x^4 - 8)*lo
g(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^
12) - 30*integrate((x^11*log(c) + x^11*log(x))*e^(-1/2*log(c^2*x^2 + 1) - 1
/2*log(c*x + 1) - 1/2*log(-c*x + 1)), x) - 30*integrate(1/30*(3*c^10*x^11 -
 3*c^8*x^9 + 4*c^6*x^7 - 4*c^4*x^5 + 8*c^2*x^3 - 8*x)/(sqrt(c^2*x^2 + 1)*sq
rt(c*x + 1)*sqrt(-c*x + 1)*c^10 + sqrt(c*x + 1)*sqrt(-c*x + 1)*c^10), x))
```

Fricas [A]
time = 0.40, size = 382, normalized size = 0.97

$$-\frac{105\,(3\,bc^{10}x^{10}+3\,bc^8x^8+4\,bc^6x^6+4\,bc^4x^4+8\,bc^2x^2+8\,b)\sqrt{-c^4x^4+1}\,\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+(35\,bc^9x^9-5\,bc^7x^7+78\,bc^5x^5-36\,bc^3x^3+768\,bcx)\sqrt{-c^4x^4+1}\sqrt{\frac{c^2x^2+1}{c^2x^2}}-420\,(bc^2x^2+b)\log\left(\frac{c^2x^2+\sqrt{-c^4x^4+1}\,cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{c^2x^2+1}\right)+420\,(bc^2x^2+b)\log\left(-\frac{c^2x^2-\sqrt{-c^4x^4+1}\,cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{c^2x^2+1}\right)+105\,(3\,ac^{10}x^{10}+3\,ac^8x^8+4\,ac^6x^6+4\,ac^4x^4+8\,ac^2x^2+8\,a)\sqrt{-c^4x^4+1}}{3150\,(c^{14}x^2+c^{12})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3150*(105*(3*b*c^10*x^10 + 3*b*c^8*x^8 + 4*b*c^6*x^6 + 4*b*c^4*x^4 + 8*b
*c^2*x^2 + 8*b)*sqrt(-c^4*x^4 + 1)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c*x)) + (35*b*c^9*x^9 - 5*b*c^7*x^7 + 78*b*c^5*x^5 - 36*b*c^3*x^3 + 76
8*b*c*x)*sqrt(-c^4*x^4 + 1)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 420*(b*c^2*x^2
+ b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
1)/(c^2*x^2 + 1)) + 420*(b*c^2*x^2 + b)*log(-(c^2*x^2 - sqrt(-c^4*x^4 + 1)*
c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)) + 105*(3*a*c^10*x^10
+ 3*a*c^8*x^8 + 4*a*c^6*x^6 + 4*a*c^4*x^4 + 8*a*c^2*x^2 + 8*a)*sqrt(-c^4*x^
4 + 1))/(c^14*x^2 + c^12)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(a+b*acsch(c*x))/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4847 deep
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^{11}\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

```
[Out] int((x^11*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.175 $\int \frac{x^7\left(a+b\operatorname{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=264

$$-\frac{b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{3c^9\sqrt{1+\frac{1}{c^2x^2}}\,x}+\frac{b(1-c^2x^2)^{3/2}\,\sqrt{1+c^2x^2}}{18c^9\sqrt{1+\frac{1}{c^2x^2}}\,x}-\frac{b(1-c^2x^2)^{5/2}\,\sqrt{1+c^2x^2}}{30c^9\sqrt{1+\frac{1}{c^2x^2}}\,x}-\frac{\sqrt{1-c^4x^4}\,(a+b\operatorname{csch}^-}{2c^8}$$

```
[Out] 1/6*(-c^4*x^4+1)^(3/2)*(a+b*arccsch(c*x))/c^8+1/18*b*(-c^2*x^2+1)^(3/2)*(c^
2*x^2+1)^(1/2)/c^9/x/(1+1/c^2/x^2)^(1/2)-1/30*b*(-c^2*x^2+1)^(5/2)*(c^2*x^2
+1)^(1/2)/c^9/x/(1+1/c^2/x^2)^(1/2)+1/3*b*arctanh((-c^2*x^2+1)^(1/2))*(c^2*
x^2+1)^(1/2)/c^9/x/(1+1/c^2/x^2)^(1/2)-1/3*b*(-c^2*x^2+1)^(1/2)*(c^2*x^2+1)
^(1/2)/c^9/x/(1+1/c^2/x^2)^(1/2)-1/2*(a+b*arccsch(c*x))*(-c^4*x^4+1)^(1/2)/
c^8
```

Rubi [A]
time = 1.31, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {272, 45, 6445, 12, 6853, 6874, 862, 52, 65, 214, 797}

$$\frac{(1-c^4x^4)^{3/2}\,(a+b\operatorname{csch}^{-1}(cx))}{6c^8}-\frac{\sqrt{1-c^4x^4}\,(a+b\operatorname{csch}^{-1}(cx))}{2c^8}-\frac{b\sqrt{c^2x^2+1}\,(1-c^2x^2)^{5/2}}{30c^9x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{b\sqrt{c^2x^2+1}\,(1-c^2x^2)^{3/2}}{18c^9x\sqrt{\frac{1}{c^2x^2}+1}}-\frac{b\sqrt{c^2x^2+1}\,\sqrt{1-c^2x^2}}{3c^9x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{b\sqrt{c^2x^2+1}\,\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{3c^9x\sqrt{\frac{1}{c^2x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/3*(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(c^9*Sqrt[1 + 1/(c^2*x^2)]*x)
+ (b*(1 - c^2*x^2)^(3/2)*Sqrt[1 + c^2*x^2])/(18*c^9*Sqrt[1 + 1/(c^2*x^2)]*x
) - (b*(1 - c^2*x^2)^(5/2)*Sqrt[1 + c^2*x^2])/(30*c^9*Sqrt[1 + 1/(c^2*x^2)]
*x) - (Sqrt[1 - c^4*x^4]*(a + b*ArcCsch[c*x]))/(2*c^8) + ((1 - c^4*x^4)^(3/
2)*(a + b*ArcCsch[c*x]))/(6*c^8) + (b*Sqrt[1 + c^2*x^2]*ArcTanh[Sqrt[1 - c^
2*x^2]])/(3*c^9*Sqrt[1 + 1/(c^2*x^2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 797

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_
.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)*(a/d + (c/e)*x)^p, x] /; F
reeQ[{a, c, d, e, f, g, m}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
```

```
FreeQ[{a, b, c}, x]
```

Rule 6853

```
Int[(u_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[b^IntPart[p]*((
a + b*x^n)^FracPart[p]/(x^(n*FracPart[p])*(1 + a*(1/(x^n*b)))^FracPart[p]))
, Int[u*x^(n*p)*(1 + a*(1/(x^n*b)))^p, x], x] /; FreeQ[{a, b, p}, x] &&  !I
ntegerQ[p] && ILtQ[n, 0] &&  !RationalFunctionQ[u, x] && IntegerQ[p + 1/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{x^7\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1-c^4x^4}\,\left(a+b\text{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{6c^8} + \frac{b\int \frac{(-2-c^4 \cdots}{\cdots}}{6c^8\sqrt{\cdots}}$$

$$= -\frac{\sqrt{1-c^4x^4}\,\left(a+b\text{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{6c^8} + \frac{b\int \frac{(-2-c^4 \cdots}{\cdots}}{\sqrt{\cdots}}$$

$$= -\frac{\sqrt{1-c^4x^4}\,\left(a+b\text{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{6c^8} + \frac{\left(b\sqrt{1+c \cdots}\right.}{\cdots}$$

$$= -\frac{\sqrt{1-c^4x^4}\,\left(a+b\text{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1+c \cdots}\right.}{\cdots}$$

$$= -\frac{\sqrt{1-c^4x^4}\,\left(a+b\text{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1+c \cdots}\right.}{\cdots}$$

$$= -\frac{\sqrt{1-c^4x^4}\,\left(a+b\text{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1+c \cdots}\right.}{\cdots}$$

$$= -\frac{\sqrt{1-c^4x^4}\,\left(a+b\text{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a+b\text{csch}^{-1}(cx)\right)}{6c^8} - \frac{\left(b\sqrt{1+c \cdots}\right.}{\cdots}$$

$$= -\frac{b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{3c^9\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{\sqrt{1-c^4x^4}\,\left(a+b\text{csch}^{-1}(cx)\right)}{2c^8} + \frac{\left(1-c^4x^4\right)^{3/2}\left(a - \cdots\right.}{6c^8 \cdots}$$

$$= -\frac{b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{3c^9\sqrt{1+\frac{1}{c^2x^2}}\,x} + \frac{b(1-c^2x^2)^{3/2}\sqrt{1+c^2x^2}}{18c^9\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{b(1-c^2x^2)^{5/2}\sqrt{1+\cdots}}{30c^9\sqrt{1+\frac{1}{c^2x^2}}\,\cdots}$$

$$= -\frac{b\sqrt{1-c^2x^2}\,\sqrt{1+c^2x^2}}{3c^9\sqrt{1+\frac{1}{c^2x^2}}\,x} + \frac{b(1-c^2x^2)^{3/2}\sqrt{1+c^2x^2}}{18c^9\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{b(1-c^2x^2)^{5/2}\sqrt{1+\cdots}}{30c^9\sqrt{1+\frac{1}{c^2x^2}}\,\cdots}$$

Mathematica [A]
time = 0.26, size = 180, normalized size = 0.68

$$-\frac{15a\sqrt{1-c^4x^4}\,(2+c^4x^4)+\frac{bc\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{1-c^4x^4}\,(28-c^2x^2+3c^4x^4)}{1+c^2x^2}+15b\sqrt{1-c^4x^4}\,(2+c^4x^4)\,\operatorname{csch}^{-1}(cx)+30b\log\left(x+c^2x^3\right)-30b\log\left(1+c^2x^2+c\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{1-c^4x^4}\right)}{90c^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/90*(15*a*Sqrt[1 - c^4*x^4]*(2 + c^4*x^4) + (b*c*Sqrt[1 + 1/(c^2*x^2)]*x*
Sqrt[1 - c^4*x^4]*(28 - c^2*x^2 + 3*c^4*x^4))/(1 + c^2*x^2) + 15*b*Sqrt[1 -
 c^4*x^4]*(2 + c^4*x^4)*ArcCsch[c*x] + 30*b*Log[x + c^2*x^3] - 30*b*Log[1 +
 c^2*x^2 + c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]])/c^8
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{x^7(a+b\,\mathrm{arccsch}(cx))}{\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*a*((-c^4*x^4 + 1)^(3/2)/c^8 - 3*sqrt(-c^4*x^4 + 1)/c^8) + 1/6*b*((c^8*x
^8 + c^4*x^4 - 2)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*x +
1)*sqrt(-c*x + 1)*c^8) - 6*integrate((x^7*log(c) + x^7*log(x))*e^(-1/2*log(
c^2*x^2 + 1) - 1/2*log(c*x + 1) - 1/2*log(-c*x + 1)), x) - 6*integrate(1/6*
(c^6*x^7 - c^4*x^5 + 2*c^2*x^3 - 2*x)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt
(-c*x + 1)*c^6 + sqrt(c*x + 1)*sqrt(-c*x + 1)*c^6), x))
```

Fricas [A]
time = 0.36, size = 324, normalized size = 1.23

$$-\frac{15\left(bc^6x^6+bc^4x^4+2\,bc^2x^2+2\,b\right)\sqrt{-c^4x^4+1}\,\log\left(\frac{cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{cx}\right)+\left(3\,bc^5x^5-bc^3x^3+28\,bcx\right)\sqrt{-c^4x^4+1}\,\sqrt{\frac{c^2x^2+1}{c^2x^2}}-15\left(bc^2x^2+b\right)\log\left(\frac{c^2x^2+\sqrt{-c^4x^4+1}\,cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{c^2x^2+1}\right)+15\left(bc^2x^2+b\right)\log\left(-\frac{c^2x^2-\sqrt{-c^4x^4+1}\,cx\sqrt{\frac{c^2x^2+1}{c^2x^2}}+1}{c^2x^2+1}\right)+15\left(ac^6x^6+ac^4x^4+2\,ac^2x^2+2\,a\right)\sqrt{-c^4x^4+1}}{90\left(c^{10}x^2+c^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/90*(15*(b*c^6*x^6 + b*c^4*x^4 + 2*b*c^2*x^2 + 2*b)*sqrt(-c^4*x^4 + 1)*lo
g((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x)) + (3*b*c^5*x^5 - b*c^3*x^3
 + 28*b*c*x)*sqrt(-c^4*x^4 + 1)*sqrt((c^2*x^2 + 1)/(c^2*x^2)) - 15*(b*c^2*x
^2 + b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2))
 + 1)/(c^2*x^2 + 1)) + 15*(b*c^2*x^2 + b)*log(-(c^2*x^2 - sqrt(-c^4*x^4 + 1
)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)) + 15*(a*c^6*x^6 + a
*c^4*x^4 + 2*a*c^2*x^2 + 2*a)*sqrt(-c^4*x^4 + 1))/(c^10*x^2 + c^8)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(a+b*acsch(c*x))/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const in
dex_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
```

```
[Out] int((x^7*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.176 $\int \frac{x^3\left(a+b\,\mathrm{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=130

$$\frac{bx\sqrt{1-c^4x^4}}{2c^3\sqrt{-c^2x^2}\sqrt{-1-c^2x^2}}-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4}-\frac{bx\operatorname{ArcTan}\left(\frac{\sqrt{1-c^4x^4}}{\sqrt{-1-c^2x^2}}\right)}{2c^3\sqrt{-c^2x^2}}$$

```
[Out] -1/2*b*x*arctan((-c^4*x^4+1)^(1/2)/(-c^2*x^2-1)^(1/2))/c^3/(-c^2*x^2)^(1/2)
-1/2*(a+b*arccsch(c*x))*(-c^4*x^4+1)^(1/2)/c^4+1/2*b*x*(-c^4*x^4+1)^(1/2)/c
^3/(-c^2*x^2)^(1/2)/(-c^2*x^2-1)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 133, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {267, 6445, 12, 1586, 1266, 862, 52, 65, 214}

$$-\frac{\sqrt{1-c^4x^4}\left(a+b\operatorname{csch}^{-1}(cx)\right)}{2c^4}-\frac{b\sqrt{1-c^2x^2}\sqrt{c^2x^2+1}}{2c^5x\sqrt{\frac{1}{c^2x^2}+1}}+\frac{b\sqrt{c^2x^2+1}\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{2c^5x\sqrt{\frac{1}{c^2x^2}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/2*(b*Sqrt[1 - c^2*x^2]*Sqrt[1 + c^2*x^2])/(c^5*Sqrt[1 + 1/(c^2*x^2)]*x)
- (Sqrt[1 - c^4*x^4]*(a + b*ArcCsch[c*x]))/(2*c^4) + (b*Sqrt[1 + c^2*x^2]*A
rcTanh[Sqrt[1 - c^2*x^2]])/(2*c^5*Sqrt[1 + 1/(c^2*x^2)]*x)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1586

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(
p_.), x_Symbol] :> Dist[(e^IntPart[q]*((d + e*x^mn)^FracPart[q]/(1 + d*(1/(
x^mn*e)))^FracPart[q]))/x^(mn*FracPart[q]), Int[x^(m + mn*q)*(1 + d*(1/(x^m
n*e)))^q*(a + c*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, m, mn, p, q}, x] && E
qQ[n2, -2*mn] &&  !IntegerQ[p] &&  !IntegerQ[q] && PosQ[n2]
```

Rule 6445

```
Int[((a_.) + ArcCsch[(c_.)*(x_)]*(b_.))*(u_), x_Symbol] :> With[{v = IntHid
e[u, x]}, Dist[a + b*ArcCsch[c*x], v, x] + Dist[b/c, Int[SimplifyIntegrand[
v/(x^2*Sqrt[1 + 1/(c^2*x^2)]), x], x], x] /; InverseFunctionFreeQ[v, x]] /;
 FreeQ[{a, b, c}, x]
```

Rubi steps

$$\int \frac{x^3\left(a+b\text{csch}^{-1}(cx)\right)}{\sqrt{1-c^4x^4}}\,dx = -\frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^4} + \frac{b\int -\frac{\sqrt{1-c^4x^4}}{2c^4\sqrt{1+\frac{1}{c^2x^2}}\,x^2}\,dx}{c}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^4} - \frac{b\int \frac{\sqrt{1-c^4x^4}}{\sqrt{1+\frac{1}{c^2x^2}}\,x^2}\,dx}{2c^5}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^4} - \frac{\left(b\sqrt{1+c^2x^2}\right)\int \frac{\sqrt{1-c^4x^4}}{x\sqrt{1+c^2x^2}}\,dx}{2c^5\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^4} - \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1-c^4x^2}}{x\sqrt{1+c^2x}}\,dx, x\right.}{4c^5\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^4} - \frac{\left(b\sqrt{1+c^2x^2}\right)\text{Subst}\left(\int \frac{\sqrt{1-c^2x}}{x}\,dx, x,\right.}{4c^5\sqrt{1+\frac{1}{c^2x^2}}\,x}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^4} - \frac{\left(b\sqrt{1+c^2x^2}\right)}{}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^4} + \frac{\left(b\sqrt{1+c^2x^2}\right)}{}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{1+c^2x^2}}{2c^5\sqrt{1+\frac{1}{c^2x^2}}\,x} - \frac{\sqrt{1-c^4x^4}\left(a+b\text{csch}^{-1}(cx)\right)}{2c^4} + \frac{b\sqrt{1+c^2x^2}\,\tan}{2c^5\sqrt{1}}$$

Mathematica [A]
time = 0.22, size = 141, normalized size = 1.08

$$-\frac{a\sqrt{1-c^4x^4} + \frac{bc\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{1-c^4x^4}}{1+c^2x^2} + b\sqrt{1-c^4x^4}\,\text{csch}^{-1}(cx) + b\log\left(x+c^2x^3\right) - b\log\left(1+c^2x^2+c\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{1-c^4x^4}\right)}{2c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCsch[c*x]))/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -1/2*(a*Sqrt[1 - c^4*x^4] + (b*c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4])
/(1 + c^2*x^2) + b*Sqrt[1 - c^4*x^4]*ArcCsch[c*x] + b*Log[x + c^2*x^3] - b*
Log[1 + c^2*x^2 + c*Sqrt[1 + 1/(c^2*x^2)]*x*Sqrt[1 - c^4*x^4]])/c^4
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\left(a+b\,\mathrm{arccsch}\left(c x\right)\right)}{\sqrt{-c^{4} x^{4}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*b*((c^4*x^4 - 1)*log(sqrt(c^2*x^2 + 1) + 1)/(sqrt(c^2*x^2 + 1)*sqrt(c*x
 + 1)*sqrt(-c*x + 1)*c^4) - 2*integrate((x^3*log(c) + x^3*log(x))*e^(-1/2*l
og(c^2*x^2 + 1) - 1/2*log(c*x + 1) - 1/2*log(-c*x + 1)), x) - 2*integrate(1
/2*(c^2*x^3 - x)/(sqrt(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*c^2 + sqrt
(c*x + 1)*sqrt(-c*x + 1)*c^2), x)) - 1/2*sqrt(-c^4*x^4 + 1)*a/c^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(110) = 220.
time = 0.39, size = 265, normalized size = 2.04

$$-\frac{2\sqrt{-c^{4} x^{4}+1}\, b c x \sqrt{\frac{c^{2} x^{2}+1}{c^{2} x^{2}}} + 2\sqrt{-c^{4} x^{4}+1}\,(b c^{2} x^{2}+b) \log\left(\frac{c x \sqrt{\frac{c^{2} x^{2}+1}{c^{2} x^{2}}}+1}{c x}\right) - (b c^{2} x^{2}+b) \log\left(\frac{c^{2} x^{2}+\sqrt{-c^{4} x^{4}+1}\, c x \sqrt{\frac{c^{2} x^{2}+1}{c^{2} x^{2}}}+1}{c^{2} x^{2}+1}\right) + (b c^{2} x^{2}+b) \log\left(-\frac{c^{2} x^{2}-\sqrt{-c^{4} x^{4}+1}\, c x \sqrt{\frac{c^{2} x^{2}+1}{c^{2} x^{2}}}+1}{c^{2} x^{2}+1}\right) + 2\sqrt{-c^{4} x^{4}+1}\,(a c^{2} x^{2}+a)}{4\,(c^{6} x^{2}+c^{4})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*sqrt(-c^4*x^4 + 1)*b*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 2*sqrt(-c^
4*x^4 + 1)*(b*c^2*x^2 + b)*log((c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c*x
)) - (b*c^2*x^2 + b)*log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 +
1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)) + (b*c^2*x^2 + b)*log(-(c^2*x^2 - sqrt(-c
```

```
^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)) + 2*sqrt(
-c^4*x^4 + 1)*(a*c^2*x^2 + a))/(c^6*x^2 + c^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a + b\operatorname{acsch}(cx))}{\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acsch(c*x))/(-c**4*x**4+1)**(1/2),x)
[Out] Integral(x**3*(a + b*acsch(c*x))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccsch(c*x))/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccsch(c*x) + a)*x^3/sqrt(-c^4*x^4 + 1), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)\right)}{\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2),x)
[Out] int((x^3*(a + b*asinh(1/(c*x))))/(1 - c^4*x^4)^(1/2), x)
```

3.177 $\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx = \int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

Mathematica [A]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x\sqrt{1-c^4x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[1 - c^4*x^4]),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x*Sqrt[1 - c^4*x^4]), x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsch}(cx)}{x\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*a*(log(sqrt(-c^4*x^4 + 1) + 1) - log(sqrt(-c^4*x^4 + 1) - 1)) + b*inte
grate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x))/(sqrt(-(c^2*x^2 + 1)*(c*x + 1)*(
c*x - 1))*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arccsch(c*x) + a)/(c^4*x^5 - x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acsch}\,(cx)}{x\sqrt{-\left(cx-1\right)\left(cx+1\right)\left(c^{2}x^{2}+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))),
 x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x\,\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x*(1 - c^4*x^4)^(1/2)), x)
```

3.178 $\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}},x\right)$$

```
[Out] Unintegrable((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*ArcCsch[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
[Out] Defer[Int][(a + b*ArcCsch[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

Rubi steps

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx=\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

Mathematica [A]
time = 2.29, size = 0, normalized size = 0.00

$$\int \frac{a+b\text{csch}^{-1}(cx)}{x^5\sqrt{1-c^4x^4}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*ArcCsch[c*x])/(x^5*Sqrt[1 - c^4*x^4]),x]
[Out] Integrate[(a + b*ArcCsch[c*x])/(x^5*Sqrt[1 - c^4*x^4]), x]
```

Maple [A]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\text{arccsch}(cx)}{x^5\sqrt{-c^4x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

```
[Out] int((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(c^4*log(sqrt(-c^4*x^4 + 1) + 1) - c^4*log(sqrt(-c^4*x^4 + 1) - 1) + 2
*sqrt(-c^4*x^4 + 1)/x^4)*a + b*integrate(log(sqrt(1/(c^2*x^2) + 1) + 1/(c*x
))/(sqrt(-(c^2*x^2 + 1)*(c*x + 1)*(c*x - 1))*x^5), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^4*x^4 + 1)*(b*arccsch(c*x) + a)/(c^4*x^9 - x^5), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acsch}(cx)}{x^5\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acsch(c*x))/x**5/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral((a + b*acsch(c*x))/(x**5*sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1)
)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccsch(c*x))/x^5/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccsch(c*x) + a)/(sqrt(-c^4*x^4 + 1)*x^5), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{a+b\,\mathrm{asinh}\left(\frac{1}{c\,x}\right)}{x^5\,\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*asinh(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)),x)
[Out] int((a + b*asinh(1/(c*x)))/(x^5*(1 - c^4*x^4)^(1/2)), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```